The Equation of Time in Ancient Astronomy

J. Jean Ajdler

Abstract

True time is the time indicated by the position of the sun (i.e., on a sundial). It is different than mean time, the time indicated by a clock. True time is not perfectly regular and can differ from mean time by up to a quarter of an hour in either direction. In the present paper, we carefully study the relationship between these different times. In particular, we see that the ancient astronomers calibrated mean time differently than now, and the difference between their two times varied from 0 to about 31 minutes. This distinction allows us to clarify the following unsolved problems:

1. *We have studied thoroughly the notion of equation of time in ancient astronomy in general and more specifically in the work of Al-Battani.*
   We succeeded to find an analytical expression of the equation of time of the ancients in function of the sun's true and mean longitude. This allowed us to prove that Al-Battani tabulated the equation of time in function of L, the sun's true longitude.

2. *When considering observations of the antiquity or the middle ages, we must add 17.5m to the times expressed in Ptolemy mean time and 16.4m to the times expressed in Al-Battani mean time in order to get the corresponding modern mean time.*

3. *The epoch of Maimonides – the moment at which all the astronomical parameters are specified – was never known with precision. In this article, we establish this moment with precision. We show that this moment is twenty minutes after apparent sunset, at the beginning of the night, when, according to Maimonides, three stars of medium size become visible to mark the end of the Sabbath in Jerusalem.*

4. *We explain the meaning of an obscure paragraph, at the end of Chapter 29 of Al-Battani, related to the "problem of the inequality of the days and the equation of time."*

5. *We explain and justify the epoch of Savasorda (R' Abraham bar Hiya ha-Nassi).*

6. *Finally, we examine thoroughly the circumstances of the invention of the modern equation of time and we show that Flamsteed was, without any doubt, the inventor of modern mean time.*

The Equation of Time in Ancient Astronomy

J. Jean Ajdler*

I. The Equation of Time

Today, we are accustomed to the uniform time of our watches (mean time), and it is difficult to understand the concept of true time. In ancient times, it was just the opposite, as people were so accustomed to the true time (or apparent time), which is the time indicated by a sundial, that it was difficult to understand and to use the mean time.[1] Until the end of the eighteenth century, the official time was the true time.[2] It was only then that England and the Republic of Geneva officially put mean time into use,[3] and only in the beginning of the nineteenth century the mean time of Paris became the legal time in France.[4] This change was necessitated by the increasing use of watches of ever better precision,[5] but the change was not easy to implement. Indeed, in the eighteenth century, true time regulated civil life and it was common to transform the results of astronomical calculations into true time.

To explain the difference between mean time and true time, we define a day's length as the time between two consecutive upper or lower passages of the sun[6] at the meridian. An important achievement of Greek astronomy was the discovery of the variation of the length of days. Today, the length of a day is 24 hours on February 11, on May 15, on July 27, and on November 4; but it exceeds 24 hours by 13 seconds on June 20, by 29.9 seconds on December 23, and it is less than 24 hours by 18.4 seconds on March 28 and by 21.4 seconds on September 17. These differences seem negligible, but in the course of the year, these insignificant differences accumulate and become significant.

For example, if we consider the time span of 100 days beginning on November 4 (when the true longitude of the sun is about 320°) and ending on February 11 (when the true longitude of the sun is about 210°), the total time exceeds the length of 100 mean days by 30 min 41sec[7] and therefore represents 2400h 30m 41s in total. The inverse is also possible. If we consider the time span of 265 days beginning on February 12 and ending on November 3, it is 265 mean days minus 30m 41s and therefore represents 6359h 29m 19s in total. These two examples represent the extreme cases. Ptolemy had already observed that this maximum difference between true days and mean days of 30m 41s is only important for determining the moon's longitude. During this time span of 30m 41s, there is a variation of 18'=0.3° of the moon's longitude. The effect on the sun's longitude is only 1'=0.017° and is therefore insignificant. Therefore, ancient astronomers accounted for this phenomenon only for the calculation of the moon's coordinates.

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II. The Equation of Time in Modern Astronomy

The modern definitions of the mean time and of the equation of time are based on the Astronomical Dissertation “De Inaequalitate Dierum Solarium”,[8] published in 1672 in London by Flamsteed.[9] Astronomers quickly accepted his conclusions,[10] and the rest of society did so only much later, after more than a century. Flamsteed calls the difference between mean time and true time “time prosthaphaeresis” or “equation of time,” in which the words “prosthaphaeresis” and “equation” mean “correction”. Before Flamsteed, astronomers used the expression “Aequatio dierum” (translated as “the equation of the days”),[11] signifying the correction between the span of true and mean days.

To define the concept of mean time, consider two fictitious mobiles (suns):

1. The ecliptic mean sun or the mean position of the sun. It is the same mean sun as the one from the Almagest, Al-Battani, and Maimonides. Today, it is called the fictitious sun. It moves on the ecliptic at the mean solar velocity of 360° per tropical year, and it coincides with the true sun at the perigee (currently January 3) and at the apogee (currently July 4).

2. The equatorial mean sun (also called the mean sun today), which moves uniformly on the equator at the same mean angular velocity as the ecliptic mean sun.

Both of these mean suns – the mean ecliptic sun and the equatorial mean sun – coincide at the equinoctial points, which are the vernal and the autumnal points. At each moment, the right ascension α_m of the equatorial mean sun is equal to the longitude l of the ecliptic mean sun: $\alpha_m = l$. Only four times a year, both the ecliptic and the equatorial mean suns have the same right ascension at the moment of the passage of the ecliptic mean sun through the equinoxes and through the solstices.

Today, the beginning of the mean solar day is at the inferior passage of the equatorial mean sun at the meridian (midnight). The mean time T_m is the hour angle of the equatorial mean sun H_m + 12h. Until 1925, the astronomical day began at noon (in the middle of the civil day, at the upper passage at the meridian) and the astronomical mean time t_m was then the hour angle of the equatorial mean sun H_m, expressed in hours. Therefore, the equation of time is the difference $E= T_m – T = t_m – t$, where t_m is the mean time, t is the true time, $T_m=t_m + 12\text{h}$ and $T=t + 12\text{h}$.

We know that the sidereal time T_s (the hour angle of the vernal point) has the following property: $T_s = \alpha + t = \alpha_m + t_m$. Therefore: $E= t_m – t =\alpha – \alpha_m$. α is the right ascension of the sun (x-coordinate measured on the equator from the vernal point), and α_m is the right ascension of the equatorial mean sun.

Smart (1931) defines the equation of time as $E_s = t – t_m = -E$, and his definition is now the standard one used in English papers.

Therefore, we have the equation: $-E_s = E = t_m - t = \alpha - \alpha_m = \alpha - l = C + \rho$, where:

> $C = L - l$. C is the equation of the center (or in ancient astronomy, the quota of the anomaly), which is the difference between the true sun and the ecliptic mean sun. L is the true longitude of the sun and l is the longitude of the mean sun (or the mean longitude of the sun). C accounts for the sun not moving uniformly on the ecliptic.
>
> $\rho = \alpha - L$. ρ is the reduction to the equator. It accounts for the sun moving on the ecliptic, while time is measured along the equator, so that even if the sun moved uniformly on the ecliptic, true time would not be uniform. L is the true longitude and l is the mean longitude.

Ultimately, $-E_s = E = \alpha - l = (\alpha - L) + (L - l) = \rho + C$. In modern astronomy, E is calculated as a function of l, the mean longitude of the sun.[12] By contrast, ancient astronomers calculated E as a function of L in a common table with the calculation of the right ascension.[13]

On February 11,	Es = −14m 25s.	At true noon, it is 12h 14m 25s.
On May 15,	Es = 3m 47s.	At true noon, it is 11h 56m 13s.
On July 27,	Es= −6m 20s.	At true noon, it is 12h 06m 20s.
On November 4,	Es = 16m 22s.	At true noon, it is 11h 44m 38s.

III. The Equation of Time in Ancient Astronomy

Modern people no longer know “true time” in their practical lives. They only use their watches, and they have little direct contact with nature. They do not know the time of daily sunrise and sunset, and they certainly do not know the time of daily moonrise and moonset. By contrast, ancient people knew only the local true time and they did not have the notion of uniform, or mean, time, unless they used astronomical tables.

The Greeks had already recognized the two causes for the non-uniformity of true time:

1. In the ecliptic, the movement of the sun is non-uniform.
2. Even if the sun moved uniformly in the ecliptic, the true time would not be uniform because equal arcs in the ecliptic do not correspond to equal arcs in the equator.[14]

Modern astronomy makes it possible to know the mean time and the corresponding true time at any moment. Therefore, it is possible to convert the true time as read on a sundial to the mean time one would read on a watch. Ancient astronomers did not have this problem because they did not use the concept of mean time.

Ancient astronomers, however, faced another problem. At a certain moment of the day, they wanted to calculate the moon's coordinates. Generally, this moment was calculated in the units of temporary hours. They performed a first operation, which is beyond the scope of this paper, to transform the time expressed in temporary hours into a time expressed in equal, or equinoctial, hours.[15]

After this first operation, they would have a certain moment of day and would want to find the corresponding moment in the astronomical tables of the Almagest. As we already know, the length of a true day is never different from the length of a mean day by more than 30s, so we can neglect the fraction of day and think in whole days. The problem is then the following: we have a certain time span expressed in true days that we want to express in mean days. The radices (fundamental parameters at the epoch) are listed at the head of Ptolemy's tables, which give the variation of these parameters for different time spans. In other words, the beginning of all of the considered time spans is always the epoch. Nevertheless, Ptolemy explains the problem generally speaking without taking into account the fact that the beginning of the time span is usually the epoch.

As a first step, we calculate the true and the mean position of the sun at both extremities of the considered time span, ignoring the difference between the true and mean times.[16] Then, we calculate the right ascension of the arc of ecliptic included between the two true positions of the sun (the projection of the arc of ecliptic on the equator, from the Northern Pole). This arc of the equator measures the interval of time expressed in true time. We then calculate the length of the arc of ecliptic included between the two mean positions of the sun; this arc measures the interval of time expressed in mean time. The difference between the arc of ecliptic and the arc of right ascension corresponds to $\Delta T_m - \Delta T$, the difference between the time span expressed in mean time and the time span expressed in true time.

If this difference is positive (E>0), it must be added to the original time span expressed in true time to express it in mean time. If the difference between the arc of ecliptic and the arc of right ascension is negative (E<0), the difference must be subtracted from the original time span expressed in true time to express it in mean time.

The expression $E = T_m - T$ now becomes $\Delta E = \Delta T_m - \Delta T = \Delta\alpha - \Delta\alpha_m = \Delta\alpha - \Delta l$, and therefore $\Delta E = E - E_0 = \Delta\alpha - \Delta l = (\alpha - \alpha_0) - (l - l_0)$ or according to the formulation of Smart: $\Delta E_s = E_s - E_{s0} = \Delta l - \Delta\alpha = (l - l_0) - (\alpha - \alpha_0) = (l - \alpha) - (l_0 - \alpha_0)$.

The letters with subscript *0* correspond to the beginning of the time span and those without subscripts correspond to the end of the time span. These formulas demonstrate the procedure of Ptolemy and show that we can easily obtain the result of Ptolemy's calculation by calculating the modern equation of time at the extremities of the time span and their difference is ΔE.

For the spans of time beginning when the true longitude of the sun is L= 315° (the middle of Aquarius),[17] the correction from true time to mean time is always subtractive, and this negative correction is maximal when the longitude of the sun at the end of the time span is L= 210° (end of Libra). This correction is then, according to Ptolemy, 8.33 time-degrees or 33m 20s.[18] For the spans of time beginning when the true longitude of the sun is L=210°, at the beginning of Scorpio, the correction from true time to mean time is always additive, and its maximal value is reached when the longitude of the sun at the end of the time span is L=315°,[19] in the middle of Aquarius and the correction is again 33m 20s.

In the Almagest, the epoch is Toth 1 of year 1 of the era of Nabonassar: February 27, 747 B.C.E. or 02/27/-746 at noon.[20] At this epoch, the longitude of the apogee is 65°:30', the mean longitude of the sun is 330°:45',[21] and the true longitude of the sun is 333°:08'. Note that L_0 is very close to the middle of Aquarius, so that almost all the spans of time beginning at the epoch will give a negative correction from true time to mean time.

In the Handy tables, a set of tables written by Ptolemy, the epoch is Toth 1 of year 1 of the era of Philippus at noon: November 12, 324 B.C.E. or 11/12/-323.[22] At this epoch, the longitude of the apogee is still 65°:30', the longitude of the mean sun is 227°:40',[23] and the true longitude of the sun is 226°:44'. At this point, we are near the beginning of Scorpio and nearly all the spans of time beginning at the epoch will give an additive correction from true time to mean time. In the Handy Tables, there is a table of the equation of time, giving, without any calculation, the correction for the spans of time beginning at the epoch, considered at the beginning of Scorpio and ending at a moment corresponding to the true longitude of the sun L.[24]

In the formulas $\Delta E = (\alpha - \alpha_0) - (l - l_0)$ and $\Delta E_s = (l - l_0) - (\alpha - \alpha_0)$, we can see that in both the Almagest and the Handy tables l_0 and α_0 correspond to the epoch T_0. Al-Battani follows the system of the Almagest, but he fixes the limit at 2/3 of Aquarius, exactly at 318.5°. The spans of time beginning at a moment when L_0=318.5° always give a subtractive correction, and it is maximal when it ends at the moment when L=210°. Al-Battani gives a table of the equation of time as a function of the true longitude of the sun. This table directly gives the correction for a time span beginning when L_0=318.5°. Rome (1939) considered the epoch of Al-Battani to be the beginning of the era of Dhu'l quarnayn: on March 1, −311 (or 312 B.C.E.) at noon (it is actually on March 0, -310 B.C.E.), [25] and can also be used for the equation of time. In fact, Al-Battani does not give any radix relative to this date, contrary to Ptolemy; it is not possible to make any calculation relative to the time span between this epoch and a particular moment.

It seems that Al-Battani has no epoch for the equation of time, and instead of fixing T_0 of the epoch, he fixes L_0=318.5°; each year when L_0= 318.5°, the epoch of the equation of time is again reached. This expression seems to be an evolution toward the concept of mean time.[26]

Therefore, we consider that Al-Battani's table of radices, book 2, page 72 (calculated at intervals of twenty years from 931 SE until 1631 SE for March 0 at mean noon) is established in the mean time of Al-Battani. This mean time coincides with the

true time when the true longitude of the sun is L=318.5°. The corrections from true time to mean time are always subtractive. In other words, the true noon always occurs before the mean noon, except when L=318.5° when it occurs at the same time.
In the formula $\Delta E = E - E_{0,}$ we have that $\Delta E = 0$ when Lo=318.5°.
But in modern astronomy: $E_0 = (\alpha_0 - l_0)$ =16.44m for Lo=318.5°. Hence, $E_0 = T_m - T$ yields: $E_0 + T = T_m$ with E_0 =16.44m.

T_m is the modern mean time when L_0=318.5° and T is the true time when L_0=318.5°. We also know that the mean time of Al-Battani coincides with the true time when L_0=318.5. We then derive the following equation:

Mean Time of Al-Battani + 16.44m = *modern Mean Time*[27]

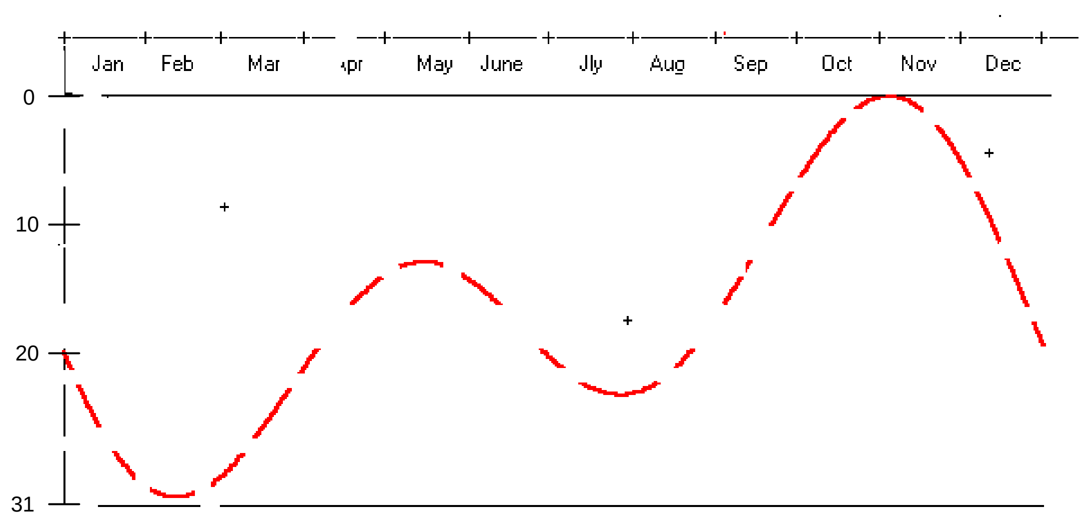

Figure 1: Equation of Time for Year 2000 Expressed by the Method of the Ancient Astronomers

- Lower x-axis = Almagest and Al-Battani mean time = 0.
- The correction from true time to mean time is subtractive from 0 to 31m (33m in Almagest).
- Upper x-axis = Handy Tables mean time = 0.
- The correction from true time to mean time is additive from 0 to 33m.
- The horizontal line at September 1 = modern mean time = 0.

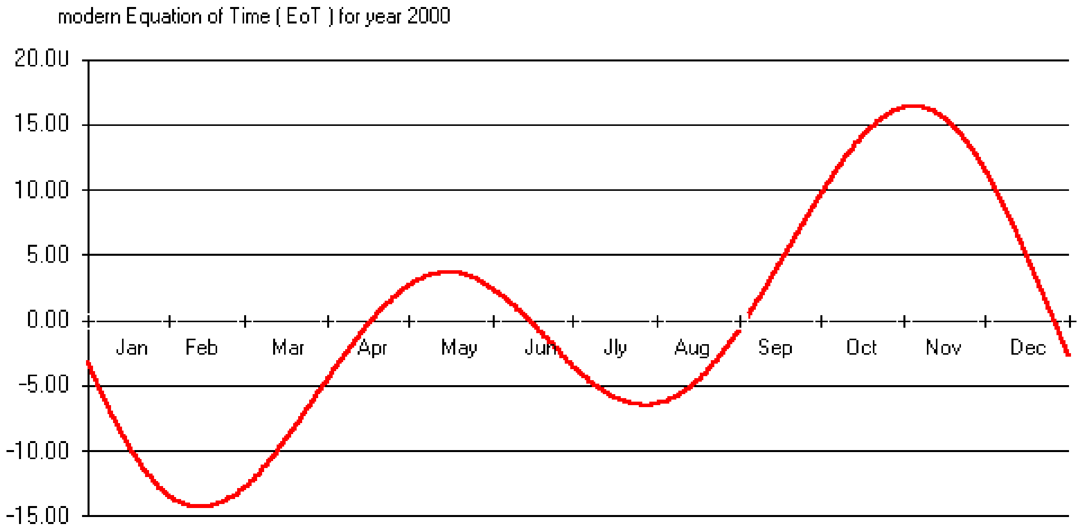

Figure 2: Equation of Time for Year 2000 Expressed by the Method of the Modern Astronomers

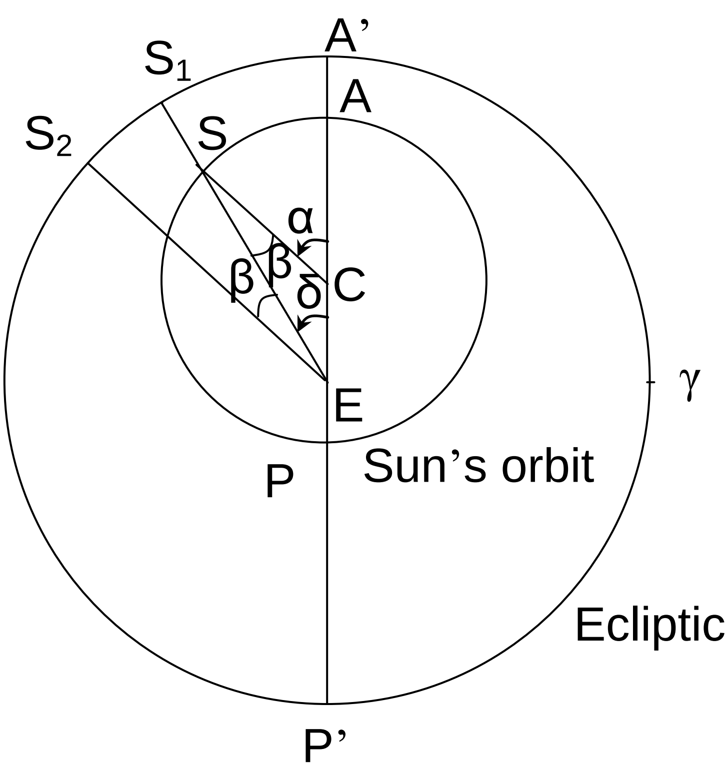

Figure 3: The sun's orbit in the ecliptic according to Ptolemy's model. The sun moves counterclockwise in its orbit

Figure 3 depicts the orbit of the sun in the plane of the ecliptic according to Ptolemy; it is a circle centered at C, and the ecliptic is a circle centered at E, the Earth. P is the point of the sun's orbit, nearest the Earth, the perigee. The furthest point of the sun's orbit is A, the apogee. The ancient astronomers could indeed not imagine another movement than a uniform circular movement. They were nevertheless aware of the non uniform evolution of the sun's longitude, which they knew by measuring the sun's declination at true noon. Therefore they imagined to bring the sun's orbit out of centre.

S is the sun on its orbit; S_1 is the true position as seen from the Earth and S_2 is its mean position.[28] Seen from E, the movement of the sun will appear fastest when closest to E in the perigee P and slowest in the apogee A. Angle α, counted from the apogee is the anomaly[29], angle $\beta = \alpha - \delta$ is the quota of the anomaly,[30] angle δ is the true anomaly[31], and e = EC/CA is the eccentricity of the sun's orbit. Point γ is the vernal point, γS_1 is the true longitude L of the sun and γS_2 is its mean longitude l, and γA' is the apogee's longitude θ.[32]

IV. The Equation of Time Expressed as a Function of the Sun's Longitude

It is interesting to express the equation of time of the **ancients** as a function of the sun's longitude.

$$E = - Es = (\alpha - L) + (L - l) = \rho + C$$

A. Function of the True Longitude

1. $-\rho = y \sin(2L) - 1/2\ y^{2} \sin(4L) + 1/3\ y^{3} \sin(6L)\ldots\ldots$ [33]

2. $-C = e \sin(L - \omega_{ap})$[34]

Therefore

$E_s = e \sin(L-\omega_{ap}) + y \sin(2L) - 1/2\ y^{2} \sin(4L) + 1/3\ y^{3} \sin(6L)\ldots\ldots\ldots$ (**1**)

B. Function of the Mean Longitude

1. $-\rho = y \sin(2l) + 2ey \sin M \cos(2l) - 1/2\ y^{2} \sin(4l)\ldots\ldots\ldots\ldots$ [35]

2. $-C = -e \sin M + 1/2\ e^{2} \sin(2M) - 1/3\ y^{3} \sin(3M)\ldots\ldots\ldots\ldots$ [36]

Therefore

$E_s = y \sin(2l) + 2ey \sin M \cos(2l) - 1/2\ y^{2} \sin(4l) - e \sin M - 1/2\ e^{2} \sin(2M)\ldots\ldots\ldots$ (**2**)

In these formulas: e is the eccentricity, $y = \tan^{2}(\varepsilon/2)$ and M is the modern mean anomaly, ω_{ap} is the longitude of the apogee (used by the ancients) and ω_{per} is the longitude of the perigee (used by the moderns). $M = l - \omega_{per} = l - \omega_{ap} + \pi$.[37]

$L-\omega_{ap}$ is the true anomaly of the ancients and $L- \omega_{per}$ is the true anomaly of the moderns.

V. Thorough Analysis of the Equation of Time of the Ancients

A. Ptolemy

$e = 2°; 30' = 2.5/60 = 0.04167$
$\varepsilon = 23°; 51' 20'' = 23.85556°$
$y = 0.04462$
$\omega_{ap} = 65°; 30'$
1 radian = 57, 295780° = 4* 57.295780 min = 13,750.98720 sec.

Equation of time expressed in function of true time.
1st term: 573.003632s; 2nd term: 613.599s; 3rd term: 13.69s and 4th term: 0.41s

E_s *in sec* = $573.0036 \sin (L-\omega_{ap}) + 613.6 \sin (2L) - 13.69 \sin (4L) + 0.41 \sin (6L)\ldots$[38]

One finds the extremes for L = 214.21°: Es = 14.3106min.
L = 317.84°: Es = - 19.2249m
Hence ΔE max = 33.5355m = 33m 32s or 8°; 23' 02''. Ptolemy gives the value of 8° 1/3 corresponding to 3° 2/3 + 4° 2/3, i.e. 14 2/3 min for L = 210° and 18 2/3 min for L = 225°

B. Al-Battani

$e = 0.034644$[39]
$\varepsilon = 23°; 35'$
$y = 0.043580$
$\omega_{ap} = 82°; 14'$

C. Equation of Time Expressed in Function of True Longitude

1st term: 476.3892s; 2nd term: 599.223s; 3rd term: 13.058s and 4th term 0.38s.
E_s *in sec* = $476.3892 \sin(L-\omega_{ap}) + 599.223 \sin (2L) - 13.058 \sin (4L) + 0.38 \sin (6L)\ldots.$

D. Equation of Time Expressed in Function of Mean Longitude

1st term: 599.268017s; 2nd term: 41.522082s, 3rd term: 13.05805s; 4th term: 476.389196s; 5th term: 8.252014s.
Es in sec = $599.268017 \sin (2l) + 41.522082 \sin M \cos (2l) - 13.05805 \sin (4l) - 476.389196 \sin M + 8.252014 \sin (2M)\ldots\ldots$

VI. Al-Battani and the Equation of Time

A. What is the maximum value of ΔE?

One finds the extreme of Es for L = 217.84°: Es = 15.1313m or 3°; 46' 58''
And for L = 319.98°: Es_0 = -16.4824m or 4°; 07' 14''.
Hence ΔE max = 31m 37s or 7°; 54'

There is a contradiction in the book of Al-Battani: in book 1, chapter XXIX, p 49 the text gives the maximum value of 7°; 48’ but in his tables the maximum value is 7°; 54’; this is the true value; the table is correct. According to the tables of Al-Battani the extremes of E_s correspond to L = 219° and L = 318-319°.

B. Does Al-Battani tabulate ΔE in function of L or l?

The formulas above allow us calculating ΔE for any L, especially near to the maximum value of the equation of the Center, when the difference between L and l is maximum.
If L = 173° we find through formula (1) above: Es (L=173°) = 1°; 24’ 22”
Es_0 = 4°; 07’ 14”
Therefore ΔE (L=173°) = 5°; 31’ 36”.
Al-Battani gives in his table, for 23° in Virgo: 5°; 31’.
Now if l =173° we find through formula (2): Es (l=173°) = 1°; 14’ 15”
Es0 = 4°; 07’ 14”
ΔE (l =173°) = 5°; 21’ 29”
Let us consider a second value L = 163°.
If L = 163° we find through formula (1) above: Es (L=163°) = 0°; 36’ 43”
Es_0 = 4°; 07’ 14”
ΔE (L=163°) = 4°; 43’ 15”.
Al-Battani gives in his table, for 13°in Virgo: 4°; 41’.
If l = 163° we find through formula (2): Es (l=163°) = 0°; 28’ 20”
ΔE (l=163°) = 4°; 35’ 34”
We see, without any doubt that Al-Battani tabulated ΔE in function of L. The error in the table of ΔE of Al-Battani can reach about 2’.

C. Al-Battani’s Operating Procedure

Al-Battani’s operating procedure was of course much different.[40]
We depart from the formula: $\Delta E = (l - l_0) - (\alpha - \alpha_0)$.
L_0 = 318°. If l_0 = 316°; 21’28”, then the mean anomaly is 234°; 07’ 28”
Quota (234°) = 1°; 38’ 23” book 2, p 79
Quota (235°) = 1°; 39’ 35” book 2, p 79
Quota = 1°; 38’ 32” by interpolation
Then $L_0 = l_0$ + 1°; 38’ 32” = 318°
For L_0 = 318°: α = 320°; 28’ [41]
$\alpha_0 - l_0$= 320°; 28’ – 316°; 21’ 28” = 4°; 06’ 32”

L = 163°
If l = 164°; 57’ 36” then the anomaly is 82°; 43’ 36” and the quota of the anomaly (equation of center) is after interpolation 1°; 57’ 36”.
L = 164°; 57’ 36” -1°; 57’ 36” = 163°
For L = 163°, α = 164°; 21’;[42] E = l – α = 164°; 57’ 36” – 164°; 21’ = 0°; 36’ 36”
Finally: ΔE (L=163°) = 0°; 36’ 36” + 4°; 06’ 32” = 4°; 43’ 08”.

D. Table of Conjunctions

1. The Conjunction of March 880 C.E. at the Epoch of Al-Battani

According to Al-Battani, book 2, p 84, the conjunction of March 880 occurred on 14^{d} 31' 49" March or 15 March 0h 43m 36s aRABMT i.e. 14 March 22h 24m UT. Indeed the longitude of ar –Raqqah is 39°; 03' and there is a difference of 16.4m between modern mean time and Al-Battani mean time. Therefore the moment of the mean conjunction expressed in UT is 0h 43m 36s – 2h 36m 12s + 16m 24s = 22h 23m 48s.
According to Meeus' Astronomical Algorithms the mean conjunction was on March 14^{th} at 23h 04m 35s dynamic time.[43] ΔT was about 36m in 880 C.E; this gives 22h 29m UT. The difference between Al-Battani and the modern value is 5m; it would be 21m if we neglect to add 16.4m to Al-Battani's mean time, as Nallino and Schiaparelli did.[44]

2. Comparison of the Tables of Conjunction of Al-Battani and Ptolemy

If we consider the first entry of Al-Battani's table of conjunctions calculated in Egyptian[45] years, we find that the first conjunction of the table occurred on Toth 22^{nd}, 14' 44", in the year 915 Dhu'l qarnaym. This year is a vague year; it corresponds to vague year 1351 of Nabonassar. 1 Toth, 1351 of Nabonassar or 915 of Dhu'l qarnaym corresponds to March 26, 603 C.E. or 914 S.E. According to Al-Battani, the conjunction occurred on 22^{d} 14' 44" Toth or on 16^{d} 14' 44" April 603 C.E. corresponding to JD 1941409.2456 ar-Raqqah.
Now the last year considered in the table of Ptolemy is 1101 of Nabonassar, but in 250 vague years, the moment of the mean conjunction of Ptolemy shifts with 0°; 27' 51".
Therefore

Al-Battani, in ar-Raqqah	conjunction occurs on 22 Toth (16 April)	22^{d} 14' 44"
Ptolemy in Alexandria	conjunction occurs on 22 Toth 1101	22^{d} 41' 45"
	Shift in 250 years	- 0^{d} 27' 51"
	Conjunction in 1351 of Nabonassar:	22^{d} 13' 54" Toth.

The difference of longitude between ar-Raqqah and Alexandria is 9.15° corresponding to 36.6m.[46]

Ptolemy
The mean conjunction was on April 16, 603CE. at 13' 54" or 17h 33m 36s Alexandria Ptolemy mean time and the longitude of conjunction was 23°; 27' 36".
Al-Battani
The mean conjunction was on April 16, 603 CE at 14' 44"or 17h 53m 36s-36m 36s = 17h 17m Alexandria Al-Battani mean time and the longitude of the conjunction was 26°; 03' 49".

The conjunction occurs, according to Ptolemy, 17 minutes later than Al-Battani. The difference in time is apparently only 17m but the difference in longitude is 2°; 36' 13".
The modern value is about 26°; 31'.

Comparison

If we want to compare Ptolemy and Al-Battani together and with modern data we must consider that both Ptolemy and Al-Battani were not aware of the irregularities of their mean time due to the irregularities of the earth's rotation.
The table of Ptolemy was written in about 150 CE when ΔT was about 135m but in 603 CE, ΔT was about 64m. Therefore the data of Ptolemy's table for the year 603 CE must be corrected by 135 – 64 = 71m. We must furthermore add 17.5m in order to get modern mean time. Therefore the mean conjunction of April 16, 603 CE, according to Ptolemy, expressed in modern mean time is:
17h 33m 36s + 17.5m - 71m – 1h 59m 36s[47] = 14h 40m 30s UT.

The table of Al-Battani was written in 880 CE when ΔT was about 36m but in 603 CE ΔT was about 64m. Therefore the data of Al-Battani's table for the year 603 CE must be corrected by 36 – 64 = -28m. We must furthermore add 16.4m in order to get modern mean time. Therefore the mean conjunction of April 16, 603 CE, according to Al-Battani, expressed in modern mean time is:
17h 53m 36s + 16.4m + 28m – 2h 36m 12s[48] = 16h 01m 48s UT.

The difference between Ptolemy and Al-Battani is then more than 81m and not 17m, as written above.

According to modern astronomy the conjunction of April 16, 603 CE was at 16h 51m Dynamic Time and 15h 47m UT. The error[49] of Al-Battani is then about 15m while that of Ptolemy is of about – 66m.

E. The Equinox measured by Al-Battani

Al-Battani has fixed experimentally true equinox on 19 September 882, 4 ¾ hours before sunrise, in ar-Raqqah.[50] Nallino writes[51] incorrectly that this was 1h 15m mean time of ar-Raqqah. This time was actually, the day of equinox, 1h 15m true time.
Al-Battani, gives in his table of ΔE, for $L = 180°$ an angle of 6°; 04' which corresponds to 24m 16s. We know further that the corrections from true time to mean time are always subtractive. Therefore true equinox was at 0h 50m 46s aRABMT. Taking into account a longitude of 39°; 03' corresponding to a time difference of 2h 36m 12s with Greenwich and a difference of mean time of 16.4m we get 22h 30m 49s UT.
According to modern astronomy equinox occurred at 23h 05m UT.[52] This represents a difference of 34m between Al-Battani and the modern value and it represents the best historical observation of equinox in ancient astronomy. During these 34m the declination of the sun varies with only 32.5"! He could hardly have measured the declination of the sun with a better precision. Schiaparelli and Nallino didn't take into account the difference of 16.4m between ABMT and our modern mean time; this would diminish the precision of Al-Battani's observation.

VII. Maimonides and the Equation of Time

A. Introduction

The Laws of the Sanctification of the New Moon (in Hebrew, *Hilkhot Kidush ha-Hodesh* H.K.H.) is the eighth part of the third book (the book of the Seasons) of the fourteen books that constitute Maimonides' famous treatise, the *Hibbur* [53]or *Mishne Torah*.[54] These laws include nineteen chapters, divided into three parts:

1. Chapters 1-5: the empirical calendar based on the observation of the new moon and the connected laws,
2. Chapters 6-10: the fixed calendar, and
3. Chapters 11-19: the astronomical chapters devoted to the calculation of the prediction of the visibility of the new crescent of the moon.

In the introduction to his treatise, Maimonides writes that his treatise constitutes the synthesis of the whole oral law.[55] He thought and hoped that it would replace the oral law and that it would be used in conjunction with the Bible without the necessity of studying the Talmud to make practical decisions. Similarly, he writes (H.K.H. 19:13) that the astronomical chapters allow readers to calculate the visibility of the new moon by applying his simplified rules without the need to consult any other astronomical books.

In both the general case and the specific case of astronomy, things did not evolve as he had hoped. Despite the tremendous impact of his treatise on Talmudic study, Talmudic studies have remained alive and his treatise has not supplanted the Talmud. Similarly, his algorithm for the prediction of the visibility of the new moon has not been widely used, but generations of scholars have tried to understand and find a justification for his rules through the use of the astronomical books he wanted readers to avoid. In fact, the understanding of Maimonides' astronomical chapters was slow and progressive, the first steps of which were the commentary by Obadia ben David (fourteenth century),[56] followed by the commentaries of Levi ben Habib (1565)[57] and Mordehai Jaffe (1595).[58] The mathematician Raphael Levi of Hanover (1685-1779) contributed significantly to the understanding of these astronomical chapters. Levi was the pupil and secretary of the famous German mathematician, G.F. Leibnitz (1646-1716). He was the first to introduce mathematics, especially spherical trigonometry, into the study of Maimonides' work on astronomy. His two books (printed in Hebrew) – the first[59] in 1756, the second[60] in 1756 and 1757, and his many, existing Hebrew manuscripts – are nearly unknown, so that his contribution[61] has unjustly been forgotten. Then, in the twentieth century, there are the decisive contributions of E. Baneth (1898, 1899 and 1903)[62] and O. Neugebauer (1949). In 1996, I published my Hebrew book *Hilkhot Kiddush ha-Hodesh al pi ha-Rambam*. I thought that it would be the final word in the long discussion of Maimonides' astronomical work. In my book, the problem of the lunar parallax, in longitude and in latitude, is indeed thoroughly examined. Nevertheless, two important problems remained unsolved. These problems are the exact epoch of Maimonides and the exact moment of vision defined by Maimonides. This paper presents a definitive solution.

B. The Problem

In astronomy, an epoch is a particular moment at which all of the astronomical parameters are specified. The epoch of Maimonides is Thursday, the third of Nissan 4938 at the beginning of the night[63] (or Wednesday evening, March 22, 1178 C.E).[64] The exact moment of this epoch has remained imprecise. Hanover has fixed this moment at 18h 20m,[65] but this moment does not coincide with the moment of vision of the moon on this evening.[66] Neugebauer has argued that the epoch occurs at 18h.[67] Nevertheless, he later corrected the moment of the epoch and fixed it at 18h 20m.[68] In his astronomical commentary on the *Laws of Sanctification,*[69] Neugebauer again fixes the epoch at 18h. Wiesenberg, in his Addenda and Corrigenda,[70] corrects this value to 18h 20m. In Ajdler (1996), I also fixed the epoch at 18h 20m, but I was forced to admit that this value was not satisfactory because it should correspond to the moment of vision of the moon on this evening. Indeed, starting from this epoch and adding a round number of days (with a small correction to account for the seasonal variation of the day's length), we must derive the moment of vision (using Maimonides' algorithm). Therefore, the epoch must also be a moment of vision, about 20 minutes after sunset.[71] The fundamental problem causing this uncertainty is that Maimonides does not specify which sunset he takes into consideration. It could be the geometrical sunset when the altitude of the sun is 0° (the normal sunset considered by Ptolemy and Al-Battani), the theoretical apparent sunset when the center of the sun is apparently at the horizon (the altitude of the sun is then −.5667°),[72] or the apparent sunset when the upper limb of the sun is apparently at the horizon (the altitude of the sun is then −0.85° according to modern knowledge).

C. Neugebauer's Proposal

Neugebauer (1949) attempts to justify and explain Maimonides' epoch and radices. Neugebauer establishes that the five radices given by Maimonides at the epoch (longitude of mean sun, sun's apogee, longitude of mean moon, moon's anomaly, and its ascending node) can be deduced nearly perfectly from the radices of Al-Battani given for the year 1471 SE, March 0 at noon, after the addition of the movements during 18 years, 22 days, and 6h 49 or 50m relying on Al-Battani's different tables of movement. In fact, neither of these two values gives a perfect justification of all Maimonides' radices together but 50m gives a better, but not perfect, coincidence.

Incidentally, Neugebauer specifies March 1, 1471 SE for the basic epoch and March 23, 1178 C.E. for the epoch of Maimonides. Both dates are incorrect by one day,[73] but errors compensate for one another and yield a correct difference of 22 days. Concerning the difference of longitude between ar-Raqqah in Mesopotamia (the place where Al-Battani's tables were established) and Jerusalem (the place where Maimonides calculated his radices), Neugebauer uses the values given by Al-Battani: ar-Raqqah: 73°:15',[74]and Jerusalem: 66°:30',[75] leading to a difference of 6°:45', corresponding to a difference of time of 27m[76] between the two towns. This leaves a difference of 23m requiring justification, to reach a difference of 6h50m between noon of Al-Battani in ar-Raqqah and about 18h 20 min in Jerusalem. As a consequence, Neugebauer fixes the epoch of Maimonides at 18h 20m[77] and considers that the last 3m are to be neglected and attributed to the approximate calculations. Indeed, from noon in ar-Raqqah until 18h 20m in Jerusalem, there is a difference of time of 6h 20m + 27m = 6h 47m, slightly inferior to the 6h 50m required.

D. My Solution

I have never been satisfied with Neugebauer's explanation, because the epoch should correspond to the moment of vision of the moon, which is approximately[78] 20m after sunset on the evening of the epoch.[79] Furthermore, it seems that Neugebauer was not yet aware in 1949 C.E. of the difference of 16m 44s between Al-Battani's mean time and our modern mean time.[80]

Let us reconsider the problem: the radices of Maimonides are the following:

Mean sun's longitude:	7°:03', 32"[81]
Sun's apogee	86°: 45', 08"[82]
Mean moon's longitude	31°: 14', 43"[83]
Mean anomaly	84°: 28', 42"[84]
Ascending node	−180°: 57', 28"[85]

In this paper, we assume that Maimonides could determine the difference between the geometrical sunset (when the altitude of the sun[86] is 0°) and the apparent sunset, as observed by any observer when the upper limb of the sun disappears at the horizon. The apparent sunset plays an important role in Jewish halakha and must be distinguished from geometrical sunset. We also assume that Maimonides considered apparent sunset when the depression of the sun is 1°.[87]

The mean sun's longitude is thus 7°: 03' 32'' and the true sun's longitude is 9°: 00' 17".[88] The true time of apparent sunset, when the upper limb is at horizon, corresponds, according to Maimonides, to a depression of the sun's center of 1°. This happens at 18h 13m 43s[89] true time. Rounding off to 18h 14m, we find the moment of vision at 18h 34m true time. This would be the moment of vision if Maimonides were considering this apparent sunset as the sunset. In the next section, we show that the moment of vision was indeed at 18h 34m true time, which proves that Maimonides' sunset was the apparent sunset.

Considering the time interval between March 0, 1489 SE at noon in ar-Raqqah, ABMT (Al-Battani mean time) and this moment 18h 14m true time (expressed in mean time (or uniform time)), we derive the true longitude of the sun, at the end of the time span, to be about 9°.[90] We find in the table of the Equation of Time, Book 2, p. 62, for 9° in Aries : Aequatio Nychtemeron:[91] 2°: 57', corresponding to 11.8m. The correction from true time to mean time is always subtractive. On the other hand, we must remember the difference of 27m between ar-Raqqah and Jerusalem, for which Maimonides accounted. Therefore, the time span is ultimately 22d 6h 34m + 27m – 11.8m = 22d 6h 49m. It is expressed in mean time and represents the time span between March 0, 1489 SE, at mean noon in ar-Raqqah, and the moment of vision in Jerusalem on Wednesday evening, March 22, 1178 C.E., at 18h 34m Jerusalem true time, twenty minutes after apparent sunset, which occurred at 18h 14m Jerusalem true time. We can now define the mean time of Al-Battani in contemporary terms. On the day of the epoch of Maimonides, the moment of vision (20m after sunset) is at 18h 34m Jerusalem true time or 18h 22m AJMT (Al-Battani Jerusalem mean time). The difference of time between noon in ar-Raqqah and twenty minutes after sunset in Jerusalem is then 6h 22m + 27m = 6h 49m, very close to the required 6h 50 min.

We now express the same reasoning in contemporary terminology. The epoch is at 18h 22m AJMT, or 18h 38.5m Jerusalem modern mean time. The modern equation of time at the epoch is 4.5m and the true time is then $T = T_m - E$, or 18h 34m. On March 22, 1178 C.E. (epoch), noon in the tables of Al-Battani, Al-Battani mean noon in ar-Raqqah

corresponded to 12h 12m true time, and because of a difference of time between the two towns of 27m, it was then 11h 45m true time in Jerusalem. Apparent sunset happened at 18h 14m Jerusalem true time and the moment of vision was 18h 34m Jerusalem true time. The difference between these two times, 18h 34m and 11h 45m, is still 6h 49m.

We now ascertain that 6h 49 m corresponds to the difference between Al-Battani's mean noon in ar-Raqqah and 20 m after the apparent sunset in Jerusalem. The difference of one minute with the better value of 6h50m used by Maimonides can come from either:

- The truncation of the Aequatio Nychtemron to 11m by a process similar to the truncation by Maimonides of 51.8 miles to 51miles,[92] or
- A different evaluation of the difference of longitude between ar-Raqqah and Jerusalem following Ptolemy instead of Al-Battani. In Ptolemy's Geographia,[93] we indeed find the following data:

Palestine:	Hierosolyma[94]	66°	31° 2/3.
Mesopotamia:	Nicephorium[95]	73° 1, 1/2	35° 1/3

The difference of Longitude between ar-Raqqah and Jerusalem is then:

73°: 1' 30"- 66° = 7°: 1' 30" = 7.025° = 28.1 m.

The calculation of Maimonides would then be: 6h 34m +28.1m -11.8m = 6h 50.3m, which is rounded to 6h 50 m.

The difference of 6h 50 m necessary to justify Maimonides' radices, as already observed by Neugebauer, now corresponds perfectly to the difference between Al-Battani's mean noon in ar-Raqqah and "the moment of vision" in Jerusalem, 20 m after apparent sunset.

Table 1: Summary Table (ABMT = Al-Battani Mean Time)

Year 1178 CE (1489 SE)	30 January		0 March		22 March
Aequatio Nychtemeron	0		4.5m		12m
Ar-Raqqah ABMT	12h	30d	12h	22d 6h 49m	18h;49m
Ar-Raqqah true time	12h	30d 4.5m	12h;4.5m	22d 6h 56.5m	19h;1m
Ar-Raqqah modern mean time	12h;16.5m		12h;16.5m		19h;5.5m
Jerusalem ABMT	11h;31m		11h;31m		18h;22m
Jerusalem true time	11h;31m		11h;35.5m		18h;34m
Jerusalem modern mean time	11h;47.5m		11h;47.5m		18h;38.5m
Modern equation of time ES	-16.5m		-12m		-4.5m

E. Conclusions

We have demonstrated that the epoch of Maimonides is not 18h 20m Jerusalem modern mean time as it has been believed since the publications of Raphael Levi in 1756. In fact, it appears that the epoch was at about 18h 22m or 18h 23m Jerusalem ABMT, or 18h 38.5m Jerusalem modern mean time, which corresponds to 18h 34m Jerusalem true time, twenty minutes after Maimonides' apparent sunset.[96]

Another unsolved problem now simultaneously finds a solution. The moment of vision, as assumed in Ajdler (1996), is about twenty minutes after the apparent sunset,[97] as demonstrated in this paper.[98]

Maimonides has perhaps neglected the problem of the equation of time in his algorithm of visibility calculation,[99] but in the calculation of his radices, he has taken it into account.[100] He demonstrates in these calculations a special mastery and professionalism, especially in the manipulation of the equation of time, the correct interpretation of the date of the epoch of Al-Battani,[101] and the correct conversion of the date of his epoch to the Julian date despite his not using this calendar.[102]

Maimonides demonstrates a remarkable coherence and precision in his treatise's calculations, even if he could have been clearer and more precise in his various definitions (such as sunset, beginning of the night, and appearance of three stars of middle size); apparently, the definitions were evident to him.

VIII. The Interpretation of Al-Battani by Abraham bar Hiya and Maimonides

A. Introduction

The problem of the equation of time is examined in *Nalino*, chap XXIX, pp. 48-49. At the end of page 49, there is an embarrassing sentence, which has not yet found a satisfactory explanation:

> "Et cum res ita sint ut diximus, loco median Lunae in initio calculi 18 minuta addidimus" ("And in order that things should be as we said, we have added 18 minutes (of a degree) to the position of the mean moon in the beginning of the calculation.")

The Arabic text of Nalino[103] confirms this translation. We already know that 18' represents the variation of the mean longitude of the moon during one half-hour, which is the difference between the mean time according to the Almagest and the mean time according to the Handy tables. Therefore, it is clear that the mean longitude of the moon at noon in the Handy tables is greater than the longitude of the moon at noon in the Almagest by 18'. When it is noon in the Handy tables, it is only about 11h 27min in the Almagest (although 11h 29m would be more correct). Recall the formula: T Almagest + 33m = T Handy tables.

Schiaparelli, in his commentary, writes that he understands that Al-Battani added 18' to his radices of the moon to make his tables compatible with the Handy tables. Rome (1943) criticizes this opinion on the ground that the tables of Al-Battani and the Handy tables are incompatible and that this small addition of 18' does not help much. Rome writes that on Choiak 27 of the year 1370 of the era of Nabonassar – corresponding to July 15, 622 C.E. (at the beginning of the era of the Hegira) – the mean longitude of the moon according to the Almagest is 130°:22'27", which corresponds to 10°:28'41" more than Al-Battani's data. Therefore, Rome says that the addition of 18' to the longitude of the moon does not help much. In reality, Rome makes the same error as Nalino and Neugebauer by assuming that the radices given by Al-Battani on page 19 of his book correspond to the first day of the different Arabic years and especially that the radices of the first line of this page correspond to the first day of the Hegira. The truth is that these radices are always given for day 0 of each year,[104] and we must add the movement of another day to get the radices of the first day of each year and for the first day of the Hegira. The mean longitude of the moon on the first day of the Hegira, calculated in Alexandria, after the subtraction for the movement of the moon during the times corresponding to the difference of longitude between ar-Raqqah and Alexandria, is then: 119°:43' 46" + 13°:10' 35" − 0°:20' 05"= 132°:34' 16". The longitude deduced from the Almagest is 130°:22' 27".

The difference is then about 2°:12'. This difference is the consequence of the overvaluation of the tropical year adopted by Ptolemy, but it is also the consequence of Ptolemy's lack of precision in the determination of the equinoxes. It is hardly conceivable that Rome could assign an error of more than 10° to either Ptolemy or Al-Battani (one of the best observers, perhaps at the same level as Tycho Brahe), which proves that the epochs of Al-Battani are always on the 0 of the month or the year under examination.
Schiaparelli thinks that Al-Battani has added 18' to his radices to make his tables compatible with the Handy tables. If this were the case, all his data would be expressed in Handy table mean time (HTMT) and the correction from true time to mean time would be additive. This contradicts Al-Battani's account. Therefore, the explanation of Schiaparelli is impossible.

B. The Reading of this Passage by Abraham bar Hiya[105]

In Abraham bar Hiya's book, *Sefer Heshbon Mahalekhot ha Kohavim* (*Book of the Calculation of the Movements of the Stars*) (written c.1122 C.E., just after the book *Zurat ha-Aretz*[106]), chapter 9 is devoted to the equation of time. In the existing Hebrew texts of the book, the original's tables were omitted.[107] Chapter 9 follows the Almagest and its numerical values. Nevertheless, in the last paragraph, which unfortunately, is absent in the Spanish edition, Savasorda refers to the last paragraph of Al-Battani, and he writes:

"And to correct this calculation, in the tables calculated in this book, add to the radix of the movement of the moon at the epoch chosen in this book, 18' [fractions of a degree], which approaches the movement of the moon during [a half-hour, which represents the maximum value of the] equation of the days."

From the text of Savasorda, we can deduce that in his text of Al-Battani, instead of "we have added," "add" was probably written. In other words, the tables of Al-Battani do not include the addition of 18', but to be compatible with HTMT, one must add 18' to the longitude of the moon.

C. Maimonides' Understanding

In the previous section, we explained the justification of Maimonides' radices and proved that the epoch of Maimonides was twenty minutes after the apparent sunset, or 6h 50m after the mean noon of Al-Battani in ar-Raqqah. Maimonides had to make a subtractive correction from true time to mean time. This subtraction proves that Maimonides knew that he was not working according to HTMT, but according to the mean time of the tables of Al-Battani, similar to the mean time of Almagest. This understanding shows that Maimonides had a similar understanding of Al-Battani to Abraham bar Hiya, and he was not bothered by the sentence relating to the addition of 18'.

D. Conclusion

This study of the two Jewish scholars of the twelfth century allows us to understand this obscure passage of Al-Battani correctly. We must conclude that this passage was corrupted in the Arabic manuscript used by Nallino.[108]

IX. The Epoch of Abraham bar Hiya

In *Sefer Heshbon Mahalekhot ha-Kohavim*, Abraham bar Hiya writes that his epoch was on Wednesday, 29 Elul 4864, at the beginning of the 257th cycle, at the end of the sixth hour in Jerusalem, which is Wednesday, September 21, 1104 C.E. at true noon. To demonstrate this point precisely, let us examine the first lines of two tables found in the Spanish translation of Millas:

Day	Completed cycles	AM	Years Egyptians	Months	Days	Hours	Min	Sec[109]
Friday	257	4883	19	0	4	16	19	40
Monday	258	4902	38	0	9	8	52	43
Thursday	259	4921	57	0	14	1	28	47
Saturday	260	4940	76	0	18	17	58	50

Table 2: Table of the conjunctions of the two heavenly bodies in Tishri, at the beginning of each cycle, beginning with the cycle 257 (1104-1123 C.E.)

The origin is 13m 23s[110] before true noon on Wednesday 9/21/1104

Begin cycle	Position of the sun and moon			Moon's anomaly			Longitude of moon's ascending node		
	degrees	min	sec	Degrees	min	sec	degrees	min	sec
257	186	59	27	351	10	19	199	31	0
258	186	59	37	298	5	52	207	6	14
259	186	59	48	245	1	25	214	41	27
260	186	59	58	191	56	58	222	16	40

Table 3: Table attached to the former, related to the conjunction of the two heavenly bodies in Tishri at the beginning of each cycle, beginning with the cycle 257

The first table verifies that the difference between the numbers of the different cycles (after the correction of misprints) is 4d 16h 33m 3s,[111] exactly the difference between the length of 235 Jewish lunations of 29d 12h 793p, representing 6,939d 16h 33m 3.33s and the length of 19 Egyptian years of 19*365=6,935 days. The beginning of each cycle is, according to the length of the Jewish cycle, 4d 16h 33m 3s after the beginning of the former cycle.

We can further see that the difference between this time span 4d 16h 33m 3s and the first line is

	4d	16h	33m	3s
—	4d	16h	19m	40s
			13m	23s

Therefore, contrary to the indication appearing in the first table, in the Spanish edition of Millas,[112] the epoch that was at first sight at true noon is really 13m 23s before true noon. The origin of time is in fact at true noon and the beginning of the different cycles is listed with regard to this origin, but the epoch, the moment at which the radices are calculated, is 13m 23s before this origin.

The first problem is to know when cycle 257 exactly begins and the astronomical meaning of this epoch, 13m 23s before true noon. The molad of Tishri 4855 is 4-18-244, or 13.56m after mean noon. Of course, Abraham bar Hiya considered his mean noon in ABMT, which on that day corresponded to about 12h 39m[113] Jerusalem true time. This time is certainly not the moment Abraham bar Hiya intended.

Because of Abraham bar Hiya's great dependence on Al-Battani, let us examine the moment of the mean conjunction of Tishri 4865 according to Al-Battani's tables. Tishri 4865 AMI corresponds to Tishri 1416 S.E. and to September 1415 Dhu'l quarnayn, (this era begins six months later).[114]
According to the table of mean conjunctions,[115] we find the following:

	year	months	days	min (1/60 day)	sec (1/3600 day)	degrees	min	sec
	1407		26	16	39	10	42	59
+	8		1	31	44	1	34	56
+		6	177	11	1	174	56	28
			204	59	24	186	56	23

The conjunction was on September 21 at 11h 45.6m aRABMT (ar-Raqqah Al-Battani mean time) slightly before mean noon. Indeed, counting 205 days from the epoch, March 0, we arrive at September 21 at noon.

A more comprehensive calculation according to the table of Al-Battani[116] shows the following for September 21 at noon, 1415 Dhu'l quarnayn, using the entries 1411 years, 4 years, August and 21 days in Al-Battani's tables:

	Sun's mean longitude			Moon's mean longitude			Moon's anomaly			Longitude of moon's ascending node		
	degrees	min	sec	degrees	Min	Sec	degrees	min	sec	degrees	Sec	Min
1411	344	51	42	195	11	22	185	1	51	111	15	17
4	0	2	14	170	43	7	7	56	23	77	21	41
August	181	21	36	264	27	27	243	57	25	9	44	34
21 days	20	41	55	276	42	16	274	21	53	1	6	44
Total at 12h	**186**	**57**	**27**	**187**	**4**	**12**	**351**	**17**	**32**	**199**	**28**	**16**
Minus			33		7	18		7	15			2
Total at 11h 46.7m	**186**	**56**	**54**	**186**	**56**	**54**	**351**	**10**	**17**	**199**	**28**	**14**

At mean noon, the difference of longitude is 6’ 45”= 405”. In one hour, the variation of elongation between the sun and moon is 1976 − 148 = 1826”/hour. The mean conjunction was 405/1826 h = 0.221796 h before mean noon, at 11h 46.7m aRABMT. The sun’s mean longitude is 186° 56’ 54”, the sun’s true longitude is about 185°, and the equation of time according to the table *Aequatio Nychtemeron*[117] of Al-Battani is 6° 28’ or 25.86m. As for the distance between ar-Raqqah and Jerusalem, Al-Battani gives the following longitudes: 66.50° for Jerusalem and 73.25° for ar-Raqqah. The difference between these longitudes is 6.75°, corresponding to 27m.

The mean conjunction occurs then at		11h 46,70m	
Equation of time	+	25,87m	
Difference of longitude	−	27,00m	
		11h 45,57m	(Jerusalem True Time)

This time differs by about 1m from 11h 46.62m, the epoch of Abraham bar Hiya. When we compare the radices given by bar Hiya, with the values deduced from the tables of Al-Battani, we verify a very good, but not perfect, coincidence.

Note that bar Hiya writes that the longitude of Jerusalem is 67.5°, different from the value of 66.5° given by Al-Battani and from the value of 66° given by Ptolemy. Further, we do not know the table of equation of time of bar Hiya and we do not precisely know the difference of longitude he considered between Jerusalem and ar-Raqqah. Bar Hiya’s explanation of the theory of the equation of time is practically the same as in the text of al-Battani, but the value of its maximum is Ptolemy’s figure. Therefore, the former calculation remains conjectural. When we compare the values of Al-Battani with modern values, we observe that in 1104 C.E., the mean conjunction of Al-Battani was twenty minutes too late and the longitude of the two heavenly bodies at conjunction was 26’ too great.

In conclusion, the epoch and the radices of Abraham bar Hiya are compatible with the tables of Al-Battani. Nevertheless, slight differences remain inexplicable. It is certain that the equation of time was taken into account in the determination of the epoch. While the tables of Al-Battani had an unbelievable precision at the beginning of the tenth century, in the beginning of the twelfth century, they had lost their accuracy. Nevertheless, it appears that these tables were still used as reference despite the existence of new tables like those of Abraham ibn Zarkali[118] (the Toledan Tables). The epoch of Abraham bar Hiya is 13m 23s before true noon of Wednesday, September 21, 1104 C.E., and it can be confirmed by the use of the tables of Al-Battani, accounting for the equation of time and the distance between ar-Raqqah and Jerusalem, according to his tables. The epoch is the mean astronomical conjunction according to Al-Battani. The beginning of the following cycles is derived from the epoch by the addition of the length of Jewish cycles of 235 Jewish lunation of 29-12-793.

X. About the Invention of the Modern Equation of Time

It is generally accepted that Flamsteed is the inventor of the modern equation of time and of the modern mean time which he expounded in his little booklet: De Temporis Aequatione Diatriba. London 1672. He relates in it, that the equation of time which was used by the ancients, from Ptolemy until the sixteenth century, began to pose a problem to the astronomers of the seventeenth century. Tycho Brahé used only the second part of the equation, which depends of the obliquity of the ecliptic. Kepler used it completely. But he was much hesitating because he thought that the irregularities of the rotation of the earth had to be taken into account in the equation of time. Longomontanus, Lansberge and Morin didn't agree about the form and the cause of the equation of the days. Street, in his Caroline tables was mistaken about the sign of the first part of the equation.[119]

Bailly[120] writes: « C'est lui (Flamsteed) qui a publié le premier en 1672 les idées saines qu'on devait avoir sur ce point d'astronomie. C'est son écrit qui a été la première règle et l'époque de l'usage non interrompu de l'équation ; c'est donc à lui qu'est due la gloire de cette restauration. Nous employons ce mot, parce que Flamsteed n'a réellement rien produit de nouveau. Les deux causes avaient été indiquées par Hipparque ; elles ont été connues de Kepler et Flamsteed n'aurait eu rien à réformer si Kepler n'avait pas tout gâté, en y mêlant une cause imaginaire. Mais il faut observer que quand la vérité est incertaine et méconnue, la retrouver est invention.» Similarly, Delambre[121] writes : « Cette doctrine (de Ptolémée) est saine et claire ; les astronomes qui sont venus depuis sont parvenus à l'embrouiller et même à la rendre défectueuse; Flamsteed les a ramenés aux vrais principes si bien établis par Ptolémée.» Nevertheless Bailly and Delambre neglect to consider the invention of an equatorial mean sun which allows defining modern mean time which will become a century later the civil and legal time. As we see the main French astronomers agree that Flamsteed has the paternity of modern mean time but they minimize the importance of Flamsteed's contribution and the originality of his invention. This mean time was adopted very soon; it was already used in the Connaissance des Temps from the beginning.[122] Nevertheless we read in the "Connaissance des Temps pour l'année 1721" in the explanations of the table of the equation of time according to Flamsteed's principles: « Monsieur Cassini a donné dans ses Tables des Satellites de Jupiter une Table (de l'Equation du temps) pour l'année 1668, faite sur les mêmes principes que celle-ci de laquelle elle ne diffère que du nombre de secondes que demande le mouvement de l'apogée et la différence du lieu du soleil surtout depuis le retranchement d'un jour qui se fit en 1700 » This affirmation raises the problem of the paternity of the modern equation of time. I checked in the "Mémoires de l'Académie des Sciences" [123]

p. 87 : année 1670 : « Cette année on fit plusieurs recherches sur l'équation des jours, sur les réfractions et sur les parallaxes.»

p. 173 : année 1678 : « Les anciens calculaient cette équation sur deux principes. L'un est l'excentricité du soleil à la terre qui fait que quand il en est plus éloigné, son mouvement propre paraît plus lent. L'autre est l'obliquité du zodiaque par rapport à l'équateur, qui fait que des parties égales du zodiaque rapportées à l'équateur, qui est la mesure du temps, y répondent à des parties inégales et par conséquent à des temps inégaux. Tycho pour accommoder mieux, à ce qu'il prétendait, ses calculs aux observations, ne faisait rouler l'équation du temps que sur le dernier de ces deux principes. Kepler y avait encore fait quelques changements et Monsieur Cassini compara ces trois méthodes d'équation pour voir lequel représentait le mieux le temps de cette éclipse de lune, tel qu'on l'avait observé. Après avoir fait encore la même comparaison sur d'autres éclipses et même sur celles des satellites de Jupiter, il se confirma dans la préférence qu'il avait toujours donnée à l'équation des anciens».

Therefore we can conclude, without any doubt, that Jean-Dominique Cassini I was not yet aware of the new approach of Flamsteed when he studied thoroughly the problem of the equation of time in 1678. It appears that the concept of the equation of time was very much in the news in the scientific world both in England and in France. Nevertheless, Picard the older fellow of Cassini, introduced very soon the new definition of the mean time in the "Connaissance des Temps" which he edited from 1679 until 1684, the edition of 1685 being edited by Jean Lefebvre. Finally I could check that Cassini introduced indeed, à posteriori, a table of the modern equation of time calculated on the basis of the sun's parameters for year 1668 in his book "Les Hypothèses et les Tables des satellites de Jupiter, réformées sur des nouvelles observations, par M. Cassini: Paris 1693". This book is based on a first Italian edition of 1668 and contains tables for the year 1668.[124] This removes the last doubts on this question: Flamsteed is the only inventor of the modern mean time; he discovered it while revisiting the problem of the equation of time which was very much in the news at that time.

XI. Conclusions

In the present paper, we studied the relationship between the different times. In particular, we saw that ancient astronomers calibrated mean time differently than now, and the difference between their two times, true and mean times, varied from 0 to about 31 minutes. This distinction allows us to clarify the following unsolved problems:

1. *We have studied thoroughly the notion of equation of time in ancient astronomy in general and more specifically in the work of Al-Battani.*
   We succeeded to find an analytical expression of the equation of time of the ancients in function of the sun's true and mean longitude. This allowed us to prove that Al-Battani tabulated the equation of time in function of L, the sun's true longitude.

2. *When considering observations of the antiquity or the middle ages, we must add 17.5m to the times expressed in Ptolemy mean time and 16.4m to the times expressed in Al-Battani mean time in order to get the corresponding modern mean time.*

3. *The epoch of Maimonides – the moment at which all the astronomical parameters are specified – was never known with precision. In this article, we establish this moment with precision. We show that this moment is twenty minutes after apparent sunset, at the beginning of the night, when, according to Maimonides, three stars of medium size become visible to mark the end of the Sabbath in Jerusalem.*

4. *We explain the meaning of an obscure paragraph, at the end of Chapter 29 of Al-Battani, related to the "problem of the inequality of the days and the equation of time."*

5. *We explain and justify the epoch of Savasorda (R' Abraham bar Hiya ha-Nassi).*

6. *Finally, we examine thoroughly the circumstances of the invention of the modern equation of time and we show that Flamsteed was, without any doubt, the inventor of modern mean time.*

With the solution of these problems, it becomes clear that to understand ancient astronomy, one must understand thoroughly the equation of time.

XII. Appendix

A. Definitions

anomaly of the sun: the longitudinal distance, i.e., the difference of longitude, between the sun's apogee at a given point of time and the sun's mean position at the same time.
anomaly, mean: a term serving to qualify the uniform moon's rate of motion in the epicycle.
anomaly, true: the mean anomaly increased by a certain correction depending on the size of the double elongation, permitting definition of the true position of the moon.
conjunction: the moment when the longitude of the moon and sun are equal.
double elongation: twice the distance between the mean position of the sun and that of the moon, serving to determine the value of the true anomaly and the true position of the moon.
eccentricity: the ratio EC/CA, referring to Figure 7 and 7bis.
elongation: the longitudinal distance between the true position of moon and that of the sun.
epoch: point of time for which astronomical positions of sun, moon and planets have been established, and which serves as the starting point for the computation of such positions at other points of time. The value of these positions at the starting point, are the radices.
equinoctial hours: equal hours of 1/24 solar mean days.
Era of Contracts: ancient Jewish era beginning on Tishri 1, 3450 AMI.
Era of Nabonassar: era used by Ptolemy in connection with Egyptian years of 365 days.
Hegira: era of the Arabs, beginning July 15, 622 C.E. at sunset.
lunar parallax: difference between the actual position of the moon, as seen from the Earth's center, and its apparent position (as seen from the Earth's surface).
mean conjunction: the moment when the longitude of the mean sun and that of the mean moon are equal.
mean moon: fictive moon having a uniform velocity in longitude.
mean sun (ecliptic): fictive sun having a uniform velocity in longitude (point S_2 in Figure 5).
mean synodic lunation: the average value of the synodic months. It is also the synodic lunation of the mean moon.
radices: the values of the positions of sun, moon and planets at the epoch.
synodic month: the interval between two successive true conjunctions, when the longitudes of the sun and moon are the same. The average value of the synodic month is 29.5306 mean solar days.
solstices: points of the ecliptic corresponding to a longitude of 90° for the summer solstice (the right ascension is 90° and the declination is about 23.5°) and to a longitude of 270° for the winter solstice (the right ascension is 270° and the declination is about -23.5°). The sun passes through these points on about June 21 and December 21.
temporary hours: hours representing 1/12 of the length of the day, i.e., the time between sunrise and sunset.

B. Special Abbreviations

ABMT: Al-Battani mean time.
AJMT : Jerusalem Al-Battani mean time;
AMI : Aera Mundi I. This is the generally used era of the Jewish years. Its epoch corresponds to the beginning of the year preceding the creation.
AMII: Aera Mundi II. It was the generally used era of creation during the Gaonic period. Its epoch corresponds to the beginning of year 2, AMI, following the beginning of the creation.
aRABMT: ar-Raqqah Al-Battani mean time.
B.C.E.: Before Common Era. The Julian calendar was introduced by Julius Caesar at the occasion of an important calendar reform. He adjusted the year to the sun's course by making it 365 days, abolishing the intercalary month, and adding one day every fourth year. All the dates anterior to January 1, 45 B.C.E. correspond to a fictitious Julian calendar.
Beharad: the molad of Tishri year 1, AMI; it means ב ה ר״ד, Sunday evening, 11h 204p p.m. corresponding already to 5 hours and 204 halakhim in the 2nd Jewish day of the week, it was 1 Tishri 1, AMI. i.e. Sunday, October 6, -3760 at 11h 11m 20s p.m.
C.E.: Common Era. Until October 4, 1582, the dates are expressed in the Julian calendar. The day following October 4, 1582 C.E. was October 15, 1582. This day was the first day of the Gregorian calendar (new style). The Gregorian reform was made necessary because it appeared that both the Julian calendar and the Julian ecclesiastical lunar calendar no longer agreed with reality. During the sixteenth century the vernal equinox fell on the 11th instead of the 21st of March and full moon came three days earlier than was computed (according to the Ecclesiastical computation). During the twelfth century (period of Maimonides) the difference between the Julian calendar and the fictitious Gregorian calendar amounts to 7 days, if one adds 7 days to the Julian dates, one gets the corresponding fictitious Gregorian dates, allowing a first comparison with the present solar horary (vernal equinox, sunrise and sunset).
HTMT: Handy Table mean time
L_0: The letters with subscript 0 correspond to the beginning of the time span and those without subscripts correspond to the end of the time span. The beginning of the time span also represents the epoch.
S.E.: Seleucid Era or Era of Contracts. The era of Dhu'l quarnayn, which refers also to Alexander the Great, begins about six months later, on March 0, -310.[125]

XIII. Bibliography

Abraham bar Hiya, (1720). Tsurat ha Arets, Offenbach.
Abraham bar Hiya, (1852). Sefer ha Ibbur, ed Filipowski, London.
Abraham bar Hiya, (1959). Libro de Calculo de los Movimientos de los Astros, con Traduccion y Notas por José Ma. Millas Vallicrosa. Barcelona.
Ajdler, J.J. (1996). Hilkhot Kidush ha Hodesh al pi ha Rambam. Jerusalem.
Bailly, J. S. (1785) Histoire de l'Astronomie Moderne. Paris.

Baneth, E. (1898, 1899, 1902 and 1903). Maimuni's Neumondsberechnung. Berichte uber die Lehranstalt fur die Wissenschaft des Judenthums in Berlin.
Cassini, J.D. (1693). Les Hypotheses et les Tables des Satellites de Jupiter. Paris.
Cohn, B. (1906). Zur Frage Der Sichtbarkeit Der Neuen Mondsichel. Astronomische Nachrichte n° 4083.
Connaissance des Temps. Annualy from 1679 until today. Paris.
Delambre, J.B. (1817). Histoire de l'Astronomie Ancienne. Paris.
Delambre, J.B. (1821). Histoire de l'Astronomie Moderne. Paris.
Danjon, A. (1959). Astronomie Générale. Seconde édition, tirage 1986
Delmedigo, J.S. (1629). Sefer Elim. Amsterdam
Flamsteed, J. (1672). De Temporis Aequatione Diatriba. London.
Goldstein, B.R. (1985). Theory and Observation in Ancient and Medieval Astronomy. London
Halma. (1813-1816). La Composition Mathématique de Claude Ptolémée, (traduction française) + commentaire de Delambre. Paris.
Halma. (1823). Tables Manuelles Astronomiques de Ptolémée et de Théon. Paris.
Halma. (1825). Tables Manuelles (Handy Tables). Paris.
Hannover, R. (1756). Tekhunat ha Shamaym. Amsterdam.
Hannover, R. (1756-1757). Luhot ha Ibbur. Hannover and Leiden.
Houzeau, J.C. (1882). Vade-Mecum de l'Astronomie. Bruxelles.
Katz, T. (1986). Igueret be Amoud ha Shahar. Tora U Mada Vol II, Iyar 5746.
Katz, T. and Weiss, J.G. (1996) Igueret be Amoud ha Shahar. Makor le Divrei ha Rambam be Iniyan Amoud ha Shahar. Moriah, Tevet 5746, p 7-18.
Lalande, J. (1792). Astronomie. Paris, 3rd edition.
Meeus, J. (1991). Astronomical Algorithms. Willman Bell, Richmond, Virginia.
Mémoires de l'Académie Royal des Sciences, tome 1. Paris 1733.
Mucke, H and Meeus, J. (1983). Canon of Solar Eclipses. Wien.
Nalino, C.A. (1903-1905). Al-Battani Opus Astronomicum, Milano.
Neugebauer, O. (1949). The Astronomy of Maimonides. HUCA, Vol. XXII.
Neugebauer, O. (1967) Astronomical Commentary on Sanctification. Yale Judaica Press, Vol. XI.
Neugebauer, O. (1975). History of Ancient Mathematical Astronomy, Springer.
Pedersen, O. (1974). A Survey of The Almagest. Odense.
Petersen, V.M. & Schmidt, O. (1968). The Determination of the Longitude of the Apogee of the Orbit of the Sun according to Hipparchus and Ptolemy. Centaurus XII, pp 73-96.
Ptolemaeus, Claudius. (1513). Geographia. Strasburg. Facsimile published by Theatrum Orbis Terrarum ltd. Amsterdam 1966.
Rome, A. (1939). Le Problème de l'Equation du Temps chez Ptolémée. Bruxelles, A.S.S.B, serie I, 59 (pg 211- 224).
Rome, A. (1943). Commentaires sur les Livres 3 et 4 de l'Almageste. Studi et Testi 106..
Smart, W.H. (1931). Textbook on Spherical Astronomy. Cambridge University Press.
Stahlman, W.D. (1959). The Astronomical Tables of Codex Vaticanus Graecus 1291, Doctoral thesis, Brown University.

Tihon, A. & Mogenet, J. (1978-1985) Le Grand Commentaire de Théon d'Alexandrie aux Tables Faciles de Ptolémée. Vatican.

Tannery, P. (1893). Recherches sur l'Histoire de l'Astronomie ancienne. Paris.

Toomer, G.J. (1984). Ptolemy's Almagest. Duckworth..

Van Der Waerden, B.L. (1958). Die Handlichen Tafeln des Ptolemaios. Osiris 13, pp 54-78

von Dalen, B. (1994). Ptolemy's Table for the Equation of Time. Centaurus, 1994, vol 37, 97-153.

Wiesenberg, E. Addenda et Corrigenda to treatise VIII(Yale Judaica SeriesVol XII), Yale Judaica Series, Vol XIV

Ysraeli, I. (1848). Yessod Olam, Berlin.

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1 Practically, they did not know the mean time, but they could imagine a regular or uniform time, corresponding to their astronomical tables.

2 Civil life and astronomical almanacs were based on true time.

3 In the year 1780 C.E. See Lalande (1792) p. 341.

4 Lalande (*id.*) pleaded for this transformation, which had already happened in England. From 1816 onward, the public clocks of Paris were regulated by the mean time. The unification of time was necessitated as soon as rapid means of communication developed throughout France. The use of Paris mean time extended to all of France due to the railway companies, which regulated the clocks of all the railway stations on Paris time. The adoption of the Paris mean time was already an established fact when the law of March 15th, 1891 made it obligatory. See Danjon (1986), chap. 36, pp. 72 and Ginzel III, p. 335.

5 Huyghens introduced the main innovation. In the beginning, a table of the equation of time was attached to each clock to transform the clock's indications (mean time) into true time.

6 Ancient astronomers used two consecutive upper passages, and the day began at noon.

7 The main differences between Ptolemy and Al-Battani come from the differences of the characteristics of the sun's path: eccentricity and the sun's apogee. The maximum difference is 33m 20s for Ptolemy, while for Al-Battani it is 31m 12s; in modern astronomy, it is 30m 41s. The daily motion of the moon is 13°: 10' 35", 03534. The motion during 33m is then 13, 1763987056*(33/60)*(1/24) =0:30°=0°:18'.

8 Flamsteed introduces the expression "equation of time." He also uses the expression "equation of the clocks." The older expressions were "aequatio diebus" or "prosthaphaeresis."

9 Flamsteed was the first director of the observatory of Greenwich and had the title of Astronomer Royal. He was still an astronomer of the ancient generation, as he preceded the Newtonian revolution. He was an exceptional observer.

10 The French astronomer Picard, who founded the "Connaissance des Temps" in 1679 and was responsible for it until 1684, directly introduced the new notion of Flamsteed's equation of time. But the new equation of time coexisted with the old equation - which was similar to the equation of the handy Tables of Ptolemy - for many years, until approximately the mid-seventeenth century. The correction from true time to mean time was always additive, and the equation of time was zero on November 1 through November 3.

11 See *supra* note 5.

12 See Smart, Textbook on Spherical Astronomy, 1980, Chapter VI, Time.

13 Delambre (1813) and Delambre, in his astronomical commentary on Ptolemy, Halma (1813-1816), affirm it without any remark. This remark can be demonstrated empirically in the tables of equation of time of either Ptolemy or Al-Battani, in the area of the maximum of the equation of the center. See also von Dalen (1994).

14 Therefore, even if the movement of the sun in the ecliptic was uniform (i.e. that the diurnal increase of the longitude $\Delta\lambda$ was always equal) the length of the true solar day would not be uniform. Indeed, near to the equinoxes the arc $\Delta\lambda$, projected on the equator gives an increase of right ascension $\Delta\alpha$ smaller than $\Delta\lambda$. In contrast, near to the solstices, $\Delta\alpha$ is greater than $\Delta\lambda$. Now, as the length of the days depends on the length of the arcs $\Delta\alpha$ of the equator, it appears that length of the true days would not be uniform even if the movement of the sun was uniform on the ecliptic.

15 See Toomer (1984), p. 171.

[16] This calculation is made with the time span expressed in true time, as if it was mean time. The error, due to this approximation, on the position of the sun is negligible.
[17] This is the value given by Ptolemy. Al-Battani indicates 320° in his text, but in his tables it is 318.5°.
[18] This value is too high, but it depends on the parameters adopted. It is only 7°: 48'= 31m 12s in Al-Battani, Nallino Vol.1, p. 49, but in his table (t2, p. 64), he gives 7°: 54'=31m 36s.
[19] See note 17.
[20] See Pedersen (1974) p. 127.
[21] See Toomer (1984) p. 142.
[22] See Pedersen (1974) p. 127.
[23] See Rome (1943).
[24] See note 13. In the Handy Tables and in Al-Battani, the equation of time is tabulated as a function of the true longitude of the sun. This equation of time corresponds to the difference between the modern equation of time of the final moment and that of the epoch. This table of Ptolemy can also be considered as a table of equation of time with respect to the fictitious mean time or the mean time of the Handy Tables, coinciding with the true time when L=210°.
[25] This is the general principle of the tables of Al-Battani. They begin on March 0 i.e. the last day of February. This starting date allows the table to be independent of the length of February, and we can directly use the date of the month for additional days. It was generally admitted that the epoch of Al-Battani was on March 1, 880 (Nallino, Schiaparelli, and Rome). By comparing the radices of Al-Battani with the calculations based on modern astronomy, Loewinger has observed, see Ajdler (1996, 125), that the epoch of Al-Battani must be on March 0, the last day of February. However, he wanted to justify March 1 on the basis that in Arab astronomy the astronomical day would be calculated, not from noon of the day until noon of the following day, but from noon of the preceding day until noon of the day. Thus, March 1 would in fact be what we now call March 0. This affirmation is based on following references: Ysraeli, Yessod Olam, Maamar 2, Chapter 10, fol. 26 col. 4 : veharega ha dalet and Savasorda Mahalekhot, Gate 9, p. 62, line 12, in which they write that the days begin from yesterday's noon. Nevertheless, this ingenious solution is not true. I have demonstrated through the comparison of the data of Al-Battani about four lunar eclipses and an equinox and the corresponding modern data, that Al-Battani counts his astronomical days from noon of the day until noon of the following day in the same way as modern astronomers did until 1925. The conclusion is then that the epoch is on March 0, 880 C.E., and that, March 1, considered by Nallino, Schiaparelli, Rome, and Neugebauer is a mistake. March 0, means the last day of February, which has 28 or 29 days. Now what about the year of Al-Battani's epoch mentioned by Rome, it is not – 311, as written by Nallino and Schiaparelli but -310, six months after the beginning of the era of Contracts. This can be demonstrated in two different ways:

A) The epoch of Maimonides is March 22, 1178 C.E. or March 22, 1489 S.E. We have further seen that Maimonides derives his radices from the tables of Al-Battani for year 1489 Dhu'l quarnayn + 22d 6h 50m. Necessarily year 1489 Dhu'l quarnayn began six months after that of Maimonides. Indeed, if it began six months before, then we would already be in year 1490 Dhu'l quarnayn and Maimonides should have used the line of year 1490 in Al-Battani's tables.

B) A second and independent proof is given by the consideration of a solar eclipse. Al-Battani writes in Vol. I, p. 56, that he observed a solar eclipse in ar-Raqqah on August 8, 1202 Dhu'l quarnayn at 1h 7.5m p.m.

Now Mucke-Meeus (1983) gives for this eclipse, the date of August 8, 891. Therefore:

August 8, 891 = August 8, 1202 Dhu'l quarnayn.

August 8, -310 = August 8, 1 Dhu'l quarnayn

and finally March 1, -310 = March 1, 1 Dhu'l quarnayn. About six months after the beginning of the era of Contracts of Maimonides.

[26] A more conservative approach, parallel to Ptolemy's point of view, is to consider that Al-Battani also had an epoch for the equation of time. According to this point of view, Al-Battani would have neglected the difference of Aequatio Nychtemeron (the difference of equation of time) between the date corresponding to L=318.5°, when Aequatio Nychtemeron is 0 and the date of March 0. If we consider that the epoch of the equation of time is March 0, 880 C.E. (the most likely date, which is also the main epoch for his other data), then the true longitude Lo=344°:46' 56" and Aequatio Nychtemeron is 3.8m (which is far from negligible). This is why Rome chose March 0, 311 C.E. or March 0, 1 S.E. for the epoch of the equation of time. At this date, Lo=335.6° and Aequatio Nychtemeron is about 2m, which could be considered

negligible. Nevertheless, this entire approach seems highly artificial because Al-Battani does not mention such an approach and he never gives any information about the radices corresponding to this epoch.

[27] For Al-Battani, the correction from true time to mean time is subtractive, so the true time is generally ahead, except when *L* is 318.5° when the mean time and the true time are equal. Therefore, the modern mean time is ahead with respect to the mean time of Al-Battani. If *l*=316°: 21' 28", the anomaly is 234°: 12' 48", the quota of the anomaly or equation of the center is 1°: 38' 32", and *L*=318°. The right ascension is then 320°: 28'and the equation of time is then E= 320°:28' − 316°: 21' 28" =4°:06' 32", corresponding to 16.4356 m.

[28] ES_2 is parallel to CS.

[29] It corresponds to the modern mean anomaly but it is measured from the apogee while M is measured from the perigee, therefore $M = 180° + \alpha$.

[30] In modern astronomy, this little angle is called the equation of the centre. Nevertheless $\beta = \alpha - \delta$, i.e. the mean anomaly minus the true anomaly but $C = v - M$, i.e. the true anomaly minus the mean anomaly. Therefore $C = -\beta$

[31] It corresponds to the modern true anomaly v but $v = 180° + \delta$.

[32] We can write $L = \delta + \theta$ and $l = \alpha + \theta$.

Ptolemy considered that θ, the longitude of the apogee is constant and equal to 65.5°. Al- Battani had fixed the value of θ for 1 March 880 C.E. to 82° 14', θ increasing with 1° in 66 years.

In triangle ECS of figure 7bis, let EC = c and CA = a. We have then the following relations:

$ES^* \sin\beta = c^* \sin\alpha$

$ES^* \cos\beta = a + c^* \cos\alpha$

By division: $\tan\beta = (c^* \sin\alpha)/ (a + c^* \cos\alpha) = e^* \sin\alpha / (1+ e^* \cos\alpha)$.

But β is less than 2° and therefore $\tan\beta \approx \beta$ (expressed in radian).

Using the Maclaurin serie $1/ (1-x) = 1+x + x^2 + x^3 + \ldots$

we can derive by substituting $-e^*\cos\alpha$ for x

$1/ (1+e \cos\alpha) = 1 - e \cos\alpha + e^2 \cos^2\alpha - e^3 \cos^3\alpha + \ldots$

Multiplying by $e^* \sin\alpha$:

$e \sin\alpha / (1 + e \cos\alpha) = e \sin\alpha - e^2 \sin\alpha \cos\alpha + e^3 \sin\alpha \cos^2\alpha - e^4 \sin\alpha \cos^3\alpha + \ldots$

Now $M = \alpha + 180°$ therefore $\sin\alpha = -\sin M$ and $\cos\alpha = -\cos M$.

$C = -\beta = e \sin M - e^2 \sin M \cos M + e^3 \sin M \cos^2 M - \ldots$

$C= -\beta = e \sin M - 1/2\, e^{\wedge 2} \sin 2M + 1/3\, e^3(\sin 3M + \sin^3 M)\ldots$

This is the Equation of the centre according to the ancients. Comparing this relation with the equation Of the center according to the moderns

$C = (2e - 0.25\, e^3) \sin M + 2.5\, e^2 \sin M \cos M \ldots$

we conclude $e_{ancients} = (2e - 0.25\, e^3)_{moderns}$.

Now e = 0.01673 and therefore $e^3 = 0.00000468$ is negligible, therefore

$e_{ancients} = 2e_{moderns.}$

We understand now why Ptolemy calculated through the measure of the length of the seasons an eccentricity e = 0.04167 and Al-Battani calculated a more accurate value e = 0.034644.

An important consequence of these considerations is the following: the sun's orbit of the ancients is not the principal circle, circumscribed to the ellipse of the moderns. The circular path which allowed the ancients accounting for the evolution of the sun's longitude is a circle for which (today) e = EC/CA = 0.03346. Its centre is above the centre of the ellipse and assuming that the radius of the circle is a, the semi-major axis of the ellipse, the points A and P are slightly different.

[33] See Smart p 148, formula 23.

[34] In triangle ECS of figure 7bis, let EC = c and CA = a. we have then the following relation :

$(\sin\beta)/c = (\sin\delta)/a$ and therefore $\beta = \sin\beta = (c/a)^* \sin\delta$

Now $\delta = \alpha - \beta$ or $L = l + C$; $\delta + \omega_{ap} = L$ and $\alpha + \omega_{ap} = l$

Finally $C = -\beta = e \sin (L - \omega_{ap})$

[35] We depart from the relations: $-\rho = y \sin (2L) - 1/2\, y^{\wedge 2} \sin (4L) + 1/3\, y^{\wedge 3} \sin (6L)\ldots$

$L - l = C = e \sin M - 1/2\, e^{\wedge 2} \sin (2M) + 1/3\, e^{\wedge 3} \sin (3M)\ldots$ see remark 32

We substitute the value of L deduced from the second relation on the right-hand side of the first relation. As we have already pointed out y is about 0.04358 and e is about 0.034644; regarding y and e as small quantities of the same order of smallness and keeping only terms up to the second order in the value of $-\rho$, we can write with the accuracy indicated:

$\sin(2L) = \sin(2l + 2e \sin(M)) = \sin(2l)\cos(2e \sin(M)) + \cos(2l)\sin(2e \sin(M))$

$= \sin(2l) + 2e \sin M \cos(2l)$.

Indeed: $\cos(2e \sin(M) \sim 1$ and $\sin(2e \sin(M) = 2 \sin(e \sin(M))^* \cos(e \sin(M)) \sim 2 e \sin(M)$.

Similarly we have, with the limitations imposed: $\sin(4L) = \sin(4l)$.

Hence $-\rho = L - \alpha = y \sin(2l) + 2ey \sin M \cos(2l) - 1/2\, y^{\wedge 2} \sin(4l)$…………

[36].Let us refer to fig 3.

We can write $L = \delta + \theta$ and $l = \alpha + \theta$.

Ptolemy considered that θ, the longitude of the apogee is constant and equal to 65.5°. Al- Battani had fixed the value of θ for 1 March 880 C.E. to 82° 14', θ increasing with 1° in 66 years.

In triangle ECS of figure 3, let EC = c and CA = a. We have then the following relations:

$ES^* \sin\beta = c^* \sin\alpha$

$ES^* \cos\beta = a + c^* \cos\alpha$

By division: $\tan\beta = (c^* \sin\alpha)/ (a + c^* \cos\alpha) = e^* \sin\alpha / (1+ e^* \cos\alpha)$.

But β is less than 2° and therefore $\tan\beta \approx \beta$ (expressed in radian).

Using the Maclaurin serie $1/ (1-x) = 1+x + x^2 + x^3 + \ldots$

we can derive by substituting $-e^*\cos\alpha$ for x

$1/ (1+e \cos\alpha) = 1 - e \cos\alpha + e^2 \cos^2\alpha - e^3 \cos^3\alpha + \ldots$

Multiplying by $e^* \sin\alpha$:

$e \sin\alpha / (1 + e \cos\alpha) = e \sin\alpha - e^2 \sin\alpha \cos\alpha + e^3 \sin\alpha \cos^2\alpha - e^4 \sin\alpha \cos^3\alpha + \ldots$

Now $M = \alpha + 180°$ therefore $\sin\alpha = - \sin M$ and $\cos\alpha = - \cos M$.

$C = -\beta = e \sin M - e^2 \sin M \cos M + e^3 \sin M \cos^2 M - \ldots$

$C= -\beta = e \sin M - 1/2\, e^{\wedge 2} \sin 2M + 1/3\, e^3(\sin 3M + \sin^3 M)\ldots$

This is the Equation of the centre according to the ancients.

[37] π radians = 180°.

[38] Delambre in his Histoire de l'Astronomie Ancienne p 140 has given, without demonstration, the same relation. His coefficients are slightly different but the calculations were less easy.

[39] $\mathrm{Tg}\,\beta = (e \sin(M+180°))/1+ e \cos(M+180°)$. Al_Battani gives β = 1°; 59' 03" for the anomaly of 90°, hence e = Tg (1°; 59' 03") = 0.03466.

[40] See also an example of calculation of the equation of time in Toomer (1984) p.651.

[41] α = 50°; 28' – 90° = 320°; 28'; book 2, p 61.

[42] α= 254°; 21' – 90° = 164°; 21'.

[43] The Universal Time UT, or Greenwich Civil Time, is based on the rotation of the earth. The UT is necessary for civil life and for the astronomical calculations where local hour angles are involved. However the earth's rotation is generally slowing down and moreover, this occurs with unpredictable irregularities. For this reason UT, is not a uniform time. This fact was unknown to the Ancients.

The astronomers need a uniform time for their calculations. From 1960 to 1983, they used the *Ephemeris Time* ET and defined it by the laws of the dynamics. In 1984 the ET was replaced by the *Dynamical Time*. It is defined by atomic clocks. One can consider that the dynamical Time TD is, in fact, a prolongation of the Ephemeris Time. The relation is: UT = TD – ΔT. The exact value of ΔT can be deduced only from the records of ancient observations.

[44] In their commentaries of Al-Battani.

[45] The Egyptian year or vague year is a year of 365 days contrary to the Julian year of 365.25 days.

[46] According to Al-Battani, this difference was 10° or 40m.

[47] The longitude of Alexandria is 29.9°.

[48] The longitude of ar-Raqqah is 39°; 03'.

[49] We must be very careful because the modern value is based on ΔT; its value is quite uncertain.

[50] See Al-Battani book 1, p 210.

[51] See Al-Battani book 1, p 210.

[52] Letter of October 22nd, 1996 from Jean Meeus.

[53] Maimonides never used the title "Yad ha-Hazaka", or "the strong hand" based on Deut. 34: 12 (the last sentence of the Pentateuch) and alluding on the 14 (numerical value of Yad) books of the work. It postdates the books and is polemical.

[54] The copy of the Torah, based on Deut. 17: 18.

[55] Apparently, Maimonides considered the Talmudic discussion secondary and exhausting. His position loses the richness of the original work and constitutes an intellectual impoverishment. Some rabbis (mainly in Spain), who judged cases by only consulting the codes and neglecting the Talmudic sources, were criticized.

[56] In his commentary on chap.12:8, he mentions the year 1341 C.E. His definitions of the fourth elongation and the arc of vision are incorrect. He is the first to present evidence of the dependence of Maimonides on Al-Battani (see his commentary on chap. 12: 1). The importance of Al-Battani in ancient Jewish astronomy had already been recognized earlier. Al-Battani is already mentioned by R' Judah ha-Levi in *Sefer ha-Kuzari*, book 4, chap 29, for the justification of the Tekufat R' Adda, and in *Yessod Olam*, book IV, chap 7, p. 12, col 1, to demonstrate that the Jewish Molad corresponds to the mean conjunction at the central meridian (Tibbur ha Aretz), situated 24° east of Jerusalem.

[57] Levi ben Habib (1483-1545) is the first to give a correct, but nevertheless intricate, definition of the four elongations and the arc the vision. At the end of chap.17, he mentions Abraham Zacut, the Spanish "Astronomer Royal" , whose tables in Zacut's Canon (The Great Composition) influenced him.

[58] Mordehai Jaffe (1530-1612) follows the commentary of Levi ben Habib.

[59] *Tekhunat ha-Shamaym*, (the astronomy of the heaven) was printed in Amsterdam, by Moses of Tiktin, who bought in an auction a manuscript of Hanover's lectures intended for his pupils and edited it with some additional notes. In the introduction to his second book (see note 7), Hanover mentions that this book was printed without his authorization.

[60] It consists of two parts. The first part includes tables for the calculation of the visibility according to the algorithm of Maimonides, and the second is a set of tables based on Newton's theory, for the calculation of conjunctions, equinoxes, and eclipses.

[61] Among his contributions:

- All the visibility's calculations of Maimonides' algorithm are performed under the assumption that the double elongation is 31°.
- The tables of parallax of Maimonides are calculated 20m after sunset with a double elongation of 31°.
- A correct interpretation of the visibility limits.
- The difference between mean conjunction and Molad is put into evidence.
- The difference between the mean synodical lunation and the conventional month of 29-12-793 is emphasized.

[62] Although his contribution is remarkable, it should be noted that many of his achievements seem very similar to those of Raphael Levi of Hanover. Although he never mentions him, it is difficult to believe that he did not know about his printed books – the book *Naavah Kodesh* (1786), Berlin, written by Simon Weltch, the pupil of R' I.B. Bernstein (1747-1802), Hanover's own pupil, or the book *Yrat Shamayim*, by Meir Furth, (1820), Dessau, devoted to the explanation of Luhot ha Ibbur, Hanover (1756-1757).

[63] The translation of the Hebrew date is (as always) ambiguous. The fifth Jewish day of the week begins on Wednesday evening at 18 h. The epoch is then at the beginning of the fifth day, which is still the civil Wednesday. Apparently, this point has escaped Neugebauer (1949).

[64] This date, as all the dates in this paper preceding October 4, 1582 C.E. is a Julian date. In the twelfth and thirteenth century, the difference between the Julian calendar and the fictitious Gregorian calendar amounts to 7 days.

[65] 18h is not far from sunset, because it is close to the equinox. The moment of the vision is about 20m after sunset (H.K.H 14, 6). 18h 20m is then not far from the time of vision on the same day. We thus explain Raphael Levi Hanover's choice of 18h 20m. See Hanover (1756-1757) book 2, p. 20, and the first example in Hanover (1756), chap 89, p. 30a and table p. 34a. Apparently, he does not account for the equation of time, although he is fully aware of it. In his manuscript *Sefer ha-Tehunah* about spherical astronomy, he details the relation $E = C + \rho$. He seems to ignore that the mean time of Al-Battani is different from our modern mean time.

[66] On the basis of the coordinates of the sun given by Maimonides, it appears that sunset, according to modern astronomy, occurred at 18h 13m Jerusalem true time. Sunset occurs when the upper limb of the sun is just at the horizon when accounting for refraction. The moment of vision would then be at 18h 33m true time.

[67] See Neugebauer (1949) p. 342. This reading is the literal interpretation of Maimonides' text; Maimonides fixes the epoch at the beginning of the night. See *Hilkhot Kiddush ha-Hodesh*, 11:16; 12:2; 12:4 (Yale Judaica Series, Volume XI, 1967), 14:4; 14:5; 16:4; 16:19.
[68] See Neugebauer (1949) p. 344.
[69] See Neugebauer (1975) p. 125.
[70] See Wisenberg, p. 579.
[71] See H.K.H. 14: 6. Maimonides writes "about twenty minutes."
[72] In modern almanacs, sunrise and sunset are calculated when the upper limb of the sun is at the horizon, accounting for the refraction of 0°: 34'; the altitude of the sun's center is then −0.85°. Until the eighteenth century, sunrise and sunset were calculated when the sun's center was at the horizon, accounting for a refraction of 0°: 32'; the altitude of the center of the sun was then −0,5333°. Even today, in the "Annuaire du Bureau des Longitudes," sunrise and sunset are still calculated according to this definition, but accounting for a refraction of 0°: 34' and an altitude of the sun's center of −0.5667°.
[73] The basic epoch is on March 0, and the epoch of Maimonides is on March 22.
[74] Nalino p. 41.
[75] Nalino p. 54.
[76] The time in ar-Raqqah is 27m ahead of Jerusalem.
[77] See Neugebauer (1949) p. 344.
[78] See H.K.H. 14: 6. This is the only place where Maimonides mentions that the moment of vision is about twenty minutes after sunset. Why does he say "about 20m"? The answer is probably that the vision requires a certain degree of darkness. At the equinox, 20m after apparent sunset represents a depression of the sun of 5.1°. In summer or in winter, the moment of vision must correspond to the same degree of darkness of the sky and to the same depression of the sun. This is the reason for a slight variation of the twenty minutes throughout the year. In the following table, all times are expressed in true time.

	Geometrical Sunset	Apparent Sunset	Moment of Vision	Delay
Equinox	18h	18h 04m	18h 24m	20m
Summer Solstice	19h 02m 31s	19h 07m 03s	19h 30m 11s	23m
Winter Solstice	16h 57m 29s	17h 02m	17h 24m 15s	22m

[79] Indeed, Maimonides counts from the epoch a whole number of days, either 29 or 30 days, taking into account the variation of the length of the days with respect to the seasons to reach the moment of vision. That means that the epoch must also have been a moment of vision.
[80] Therefore, 18h 20m must be understood in Al-Battani mean time and corresponds to 18h 03m 30s of our modern mean time. It does not represent 20m after 18h, mean sunset near to the equinox. This position is untenable!
[81] H.K.H. 12 :4.
[82] H.K.H. 12 :5.
[83] H.K.H. 14 :4.
[84] H.K.H. 14 :4.
[85] H.K.H. 16 :3.
[86] i.e. the altitude of the sun's center.
[87] See point 2, *infra* note 97.
[88] See Ajdler (1996) p. 116.
[89] The declination is given by $\sin\delta = \cos\beta * \sin\varepsilon * \sin\lambda$. $\varepsilon = 23°{:}\ 35'$, $\lambda = 9°{:}\ 00'\ 17''$ and $\beta = 0$.
We find $\delta = 3.5902°$. The required hour angle is given by $\cos H = (\cos 91° - \sin\Phi * \sin\delta) / (\cos\Phi * \cos\delta)$.
The hour angle is then 93,4298° corresponding to 18h 13m 43s.
What about the value of ε, see Nallino I, pp. 12 and 160.
[90] It is exactly 9°: 00' 17".
[91] This expression is synonymous with the equation of the days.
[92] See point 2, *infra* note 97.
[93] I have used the facsimile of the Strasbourg Geographia of 1513 C.E.

[94] Maimonides indeed follows Ptolemy when he fixes the longitude of Jerusalem at 24° in H.K.H. 11: 17. In H.K.H, he works with respect of the principal meridian, which was considered to be in the middle of the inhabited countries and was called the center of the world. The longitude of this meridian is 90°. Jerusalem has a longitude of 66° east with respect to the origin meridian (longitude 0°) crossing the Atlantic Ocean. With respect to the meridian center of the world, the longitude of Jerusalem is 24° west.

[95] On the map (quarta asiae tabula), we find Nicephorium on the Euphrates, corresponding without any doubt to ar-Raqqah. It must be added that this reading is not absolutely sure, as there is another possible reading for the longitude of Nicephorium: 73°: 1/12. Ultimately, this gives no significant difference.

[96] Sunset mentioned in H.K.H. 14:6 is exactly the modern apparent sunset, when the sun disappears at the horizon and the upper limb of the sun is at the horizon, accounting for the refraction; according to modern astronomy, the depression of the sun is then 0°: 51'= 0.85°, but Maimonides used a sun's depression of 1°. This definition of sunset and apparent sunset is in contradiction with the general definition of sunset and apparent sunset in ancient astronomy, according to which sunset is the moment when the center of the sun is at the horizon and apparent sunset is the moment when the center of the sun is apparently at the horizon. At this moment, the center of the sun has a depression of 0°: 34'. Nevertheless, the value adopted for the refraction was different in ancient astronomy. In the eighteenth century, they still used a refraction of 0°: 32'. This value can be found in the "Connaissance des Temps" 1721, p. 125. Similarly, in a table intended for Sabbath and daily prayer times, printed in 1766, Raphael Levi Hanover (the first – or probably the second after Joseph Solomon Delmedigo – to define the times of Jewish life on a scientific basis and to define the time of the end of Sabbath on the basis of a solar depression), considers sunrise and sunset when the center of the sun is apparently at the horizon.

[97] One can raise the following objections: 1. on what basis do we try to make the distinction in the interpretation of Maimonides and the explanation of his epoch between true sunset and apparent sunset, when the classical astronomical books used by Maimonides, the Almagest, and the composition of Al-Battani, never mention the apparent sunset? 2. On which basis have we chosen a depression of 1° for the apparent sunset according to Maimonides? 3. Were the ancient astronomers able to measure the difference between geometrical sunset (altitude=0°) and the apparent sunset (upper limb of the sun disappearing at the horizon)?

1. It is certain that Maimonides, in his non-astronomical works, always refers to the apparent sunset, as observed by laymen. The definition of halakhik (legal) sunset is also ascribed by Maimonides' son, R' Abraham, as the apparent sunset (Responsa 96 of R' Moses Alashkar).
   On the day of the epoch, with the radices of Maimonides, we have following data:

Geometrical sunset	18h 09m true time; 17h 57m AJMT; 18h 13.5m Modern MT
Geometrical sunset + 20m	18h 29m true time; 18h 17m AJMT; 18h 33.5m Modern MT
App. sunset (-.85°) +20m	18h 33m true time; 18h 21m AJMT; 18h 37.5m Modern MT
App .sunset (-1°) +20m	18h 34m true time; 18h 22m AJMT; 18h 38.5m Modern MT

   Maimonides has adopted his epoch at 18h 50m – 27m= 18h 23m AJMT. If he had geometrical sunset in mind, the epoch would have been 18h 17m AJMT. It is certainly not quite by chance that he refers to a moment near to apparent sunset, removed by 6m from geometrical sunset.
   We can further assert that he certainly did not have 18h; 20m AJMT, 20m after mean sunset, in mind, because our mean sunset of 18h happens at 17h; 43.5m AJMT. Furthermore this notion was unknown to them, as they ignored our equatorial mean sun; their mean sunset was at 18h true time.
2. There are good reasons to believe that Maimonides and other astronomers of his time considered that apparent sunset corresponded to a depression of 1°. Indeed, in his commentary to Berakhot I.1, Maimonides mentions that astronomical dawn (at the equator) has duration of 72m and that the thickness of the atmosphere is 51 miles (the altitude of the surface limiting the atmosphere, which reflects the light of the sun). Thanks to these two values, Delmedigo was already able to establish the dependence of Maimonides on the Book of Dawn (erroneously ascribed to Alhazen; this book has been translated to English: Goldstein (1985) and the 14th century Hebrew manuscript was published in Katz (1986) and to correct the altitude of 51miles to 52 miles. We are dealing with the same miles as those mentioned in Maimonides' introduction to treatise Zeraim, of which he says that the equator has a length of 24,000 miles. For more details on this Arabic Book of Dawn, see Katz (1986) and Katz and Weiss (1996). According to the theory developed in the Book of Dawn, the atmosphere's thickness depends on the sun's depression at the end of the astronomical dawn or twilight. It can be proved that a depression of 19° corresponds to an altitude of 51.8 miles, while a

depression of 18.85° corresponds to 51 miles and a depression of 18° to 46 miles. We could deduce from Maimonides' given value of 51 miles that he considered a depression of 0.85° at apparent sunset, as in modern astronomy, but this would be naïve. In reality, Maimonides follows the Arabic treatise and considers a depression of 19° at the end of the astronomic twilight and an altitude of 51.8 miles for the atmosphere. The value of 51 miles has been rounded off by truncation. Because Maimonides considers the length of the astronomical dawn or twilight to be 72m, corresponding to 18°, his astronomical twilight necessarily begins when the solar depression is 1°, at apparent sunset. It is then very likely to consider that his apparent sunset corresponds to a depression of 1°. The difference between apparent sunset and geometrical sunset is then about 5 minutes instead of 4. Katz and Weiss (1996) p. 9 arrives at the same conclusion.

3. The ancient astronomers, Archimedes, Ptolemy, and especially Alhazen in his book on optics (see Houzeau (1882) p. 299) were aware, at least in principle, of the phenomenon of astronomical refraction and its consequences: enlarging of the heavenly body at the horizon and inflection of the rays in the atmosphere. Stars under the horizon seem to be above it. These astronomers could appraise the phenomenon only by the measure of the short time span between the theoretical geometrical sunset, transformed into true time, and the apparent sunset. The ancient astronomers, when they had a good water clock, could measure spans of time with good precision. But this phenomenon can be measured even with an imprecise clock. Assume that the clock loses 16m per day. The direct measure on the day of the equinox of the difference between half an apparent day of 6h: 4m and 6h will give 0. But one quarter of the difference between the length of apparent day and apparent night will give .25 * (11h: 59m 55s – 11h: 44m 05s) = .25 * (15m 50s) = 3m 57.5s. Furthermore, the sum of day + night is 23h: 44m, which gives us the possibility to correct and improve the results. In other words, even with an imprecise clock, on the day of equinox it is possible to measure the phenomenon with precision by comparative measures and derive the correspondent depression of the sun. The inaccuracy of the clocks can therefore be eliminated. The true problems are a) that the true equinox does not fall exactly at 6h (or 18h) and therefore the night and day after (or before) are not exactly equal. b) An error of 12h (and even 14h) in the determination of the equinox was common (see Ajdler (1996) p. 179). Therefore, the operation is somewhat more complicated. One must determine the minimum of the difference (day-neighboring night) on the supposed day of the equinox and on the days before and after. Even this minimum can still have an error, which can reach about 4m according to the exact moment of the equinox, leading to an error of 1m in the appreciation of the difference between geometrical and apparent sunset. In fact, a depression of 1° in place of .85° of the sun at apparent sunset represents an error of about 1m in the appreciation of this difference of time (5m instead of 4m). A good knowledge of this phenomenon requires observations around more than one equinox.

[98] Raphael Levi Hanover, in the different numerical examples developed in his *Luhot ha-Ibbur*, neglects this problem and makes his calculation of vision twenty minutes after geometrical sunset when the depression of the sun is zero. On the contrary, in his table for religious objectives, he considers the sunset when its depression is 0°: 32' (center of the sun at the horizon, taking into account a refraction of 0°: 32').

[99] Why, indeed, did Maimonides take the equation of time into consideration in the calculation of the epoch and the radices and why did he neglect it in his algorithm of the calculation of the arc of vision? When calculating the epoch and the radices, Maimonides follows the instructions given by Al-Battani. This is particularly justified as he gives his radices with a (probably illusory) precision of the angular second. If we consider that the longitude of the moon increases in 24h with 13°; 10' 35.03" and therefore in one minute with 0.00915° or 0°; 00' 33", it appears that when the epoch is determined with a precision of 1 minute, the longitude of the moon is known with a precision of only half an angular minute. Hence, we understand that Maimonides must calculate his epoch with the greatest precision, at least to the minute, and therefore he must use the equation of time. On the contrary, in Maimonides' algorithm in HKH 14:6, he calculates a correction for the moon's elongation, to account for the variation of the length of daylight in Jerusalem. It varies during the year between 10 and 14 hours, and consequently, sunset is delayed by about one hour at the summer solstice and is advanced by about one hour at the winter solstice. During one hour, the mean motion of the moon is about 0.5°; therefore, if we compute the mean position of the moon, twenty minutes after mean sunset, we must apply a correction that depends on the seasons and ranges from −0.5° to +0.5°. In this simplified algorithm, Maimonides replaces the continuous function correction by a step function correction. By doing that, he often neglects delays up to 15 and even 18 minutes. We can then understand

that he neglected the effect of the equation of time, of about the same order of size, although the two errors can be cumulative. Indeed, at the epoch, the modern equation of time is 4.5 minutes and therefore the mean time of the algorithm, coinciding with the true time at the epoch, is close to our modern mean time; the error due to the negligence of the Aequatio Nychtemeron (equation of time in the Latin translation of Al-Battani) ranges from +13m to −18m (in place of 0 to −31m). The main reason of this simplification is that accounting for the Equation of Time would have complicated this algorithm so much that it would not have been as useful.

[100] This opinion is subjective, but when we compare the calculations of the epoch and the radices made by Maimonides with those of Abraham ibn Ezra or with those of Abraham bar Hiya (also called Savasorda), using parameters of different origins and presenting a lack of homogeneity, we observe much more precision in Maimonides' calculations. In his *Sefer ha-Ibbur*, Ibn Ezra indicates that true equinox was on Friday, March 14, 1147 C.E. at seven hours in the day (probably seven hours after sunrise), that is to say 13h true time of Verona. The precision of this equinox is clearly less than the precision of Maimonides' calculation of the equinox.

[101] He has correctly fixed it on March 0. He lived in Fostat, and he did not use the Julian calendar.

[102] He may have used for this purpose the *Sefer ha-Ibbur* written by Abraham bar Hiya in 1123 (see the introduction written by the editor Filipowski in 1851). Maimonides mentions probably this book, without naming it explicitly, in his commentary on the Mishnah Arahim, 2:2.

[103] I express my thanks to Dr. Benezri, a specialist in classical Arabic.

[104] i.e. the last day of the year before.

[105] He is also called Savasorda.

[106] See the introduction of Filipowsi in Abraham bar Hiya (1852).

[107] There is a complete edition in Spanish titled *Libro del calculo de los moviementos de los astros* containing a Spanish translation and the astronomical tables by Jose Millas Vallicrosa of Barcelona (1959).I express my thanks to Eng. Yakov Loewinger for sending me a copy of this book. We had interesting discussions about the astronomical tables in this book.

[108] Concerning the interest of the study of the old Jewish texts, there is another interesting example. Al-Battani (Nallino, Vol.1, p. 57) describes the eclipse, of July 23, 883 C.E., which occurred a little later than eight equinoctial hours in the afternoon. Using the tables of Al-Battani, Schiaparelli has shown that this moment was 8h 07m in the afternoon. In *Yessod Olam*, book IV, chap 7, p. 12, col. 1, Ysraeli refers to this eclipse and writes that Al-Battani observed it in ar-Raqqah 8h and 6m after noon. Unfortunately, there are many misprints in this edition of *Yessod Olam*.

[109] In the book of Millas, it says "frac," which we demonstrate are in fact seconds.

[110] In the tables of the Spanish edition of Millas, the text is 13m 200 frac, but this is a misprint or a copying error.

[111] The difference is in fact 3.33s; three * 3.33s is transformed into 3s+3s+4s. We deduce also that the indication "frac" on the last column right is expressed in seconds.

[112] See note 109.

[113] See below. The equation of time is 25.86m. Therefore, in Jerusalem, the Molad occurs at about 12h 14m ABMT or 12h 14m+26m= 12h 40m Jerusalem true time.

[114] See remark 25.

[115] See Nallino 2: p. 84

[116] See Nallino 2: p. 72.

[117] i.e. equation of time.

[118] Abraham ibn Zarkali is also known under the Latin name of Arzachel. He lived from about 1029 until about 1087 C.E. He published the Toledo Tables which served as a basis for the Alphonsine Tables. Ysraeli mentions one of his observations in Yessod Olam IV: 15, namely the autumnal equinox of year 1076 C.E.

[119] Lalande in his Astronomy, n° 972 p. 340, writes about the same.

[120] Histoire de l'Astronomie Moderne, tome 2, p. 430.

[121] Histoire de l'astronomie Ancienne, tome 2, p. 140.

[122] I could check the oldest exemplar of the Connaissance des Temps available in Belgium, namely the "Connaissance des temps pour l'année 1685", and it contains a short table for the equation of time according to the principles of Flamsteed. In the "Connaissance des temps" for 1721 and following, available at the library of the Observatory of Brussels, there are two tables, the first according the principles

of Flamsteed and a second following the equation of time of the ancients, namely the system of the Handy Tables: ΔE is 0 at the beginning of November. This situation will last until about 1755.

[123] Tome I: 1666 – 1686.

[124] The edition of 1693 is based on the first Italian edition; the tables refer to year 1698. Picard discovered Cassini at the occasion of this publication of 1668 and he persuaded him to come to Paris.

[125] See remark 25.