A Historical outline of the understanding of Maimonides' astronomical chapters of *Hilkhot Kiddush ha-Hodesh*.

Examination of two neglected pieces of evidence.

It was generally accepted that the understanding of the astronomical chapters of Maimonides' *Hilkhot Kiddush ha-Hodesh* was progressive and faltering. The first known commentator, R. Ovadia ben David, had an incorrect understanding of the final steps of Maimonides' algorithm. R. Levi ben Haviv seemed to have been the first to have a correct qualitative understanding of the different steps of Maimonides' algorithm but it was not before the middle of the 18th century that Raphael Levi from Hanover had a complete understanding of the different steps and parameters and was able to propose an exact calculation process parallel to Maimonides' algorithm.
In the present paper we analyze two ancient documents. The first was virtually unknown until recently. It is a chapter of the Alfonsine tables, written in the 13th century, which reveals that its authors had a very precise understanding of all the steps of Maimonides' algorithm. They could even bypass all the steps of the algorithm through extant or new astronomical tables. They also noted that the main component of the quota of the geographical latitude φ, introduced in the last step of Maimonides' algorithm, is precisely tang φ. The second document is chapter 10 of the canon of Abraham Zacut transcript by Berthold Cohn in 1918, which did not receive, until now, the due attention. It also denotes a good understanding of the process. R. Levi ben Haviv knew this document and quoted it. So it appears that the main points of the understanding of *Hilkhot Kiddush ha-Hodesh*, acquired by the Jewish astronomical tradition in Spain survived through R. Levi ben Haviv and was saved from a complete vanishing with the emigration and the banishment of Spanish Jewry.

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1. Introduction.

The subject of this paper is the calculation by astronomical methods whether the moon will be visible on the considered evening after sunset. For this purpose we calculate the coordinates of the sun and the moon on the considered evening, 20 minutes after sunset, normally more than 18 hours after the true conjunction in order to calculate the first longitude λ_1 and the arc of vision b and check the criterion of visibility defined by Maimonides.

If we refer to the figures 1 to 3 representing the western horizon at sunset, we can explain the main steps of Maimonides' algorithm. S represents the sun and M the moon. M' is the apparent or topocentric moon. The arc MM' of the altitude circle is 1° and represent the horizontal parallax of the moon. B is the position of the moon M in longitude and B' is the position of the apparent moon in longitude. $SB = \gamma B - \gamma S = \lambda_M - \lambda_S = \lambda_1$ is the elongation or the first longitude. $SB' = \gamma B' - \gamma S = \lambda_{M'} - \lambda_S = \lambda_2$ is the second longitude or the elongation corrected for the parallax.

BM is the first latitude β_1 and B'M' is the second latitude β_2. B'D is the deviation of the moon's pad or *Nelizat ma'agal* and SD is the third longitude. WA is the oblique setting of SD and it is called λ_4, the fourth longitude. WT is the oblique setting of SH; it represents the moonset lag of the moon with regard to the sun. It is the arc of vision b. It should not be confused with the arcus visionis, which is an arc of the altitude circle between M' and S and is about[1] $b * \cos \varphi$. $WT = b = \lambda_4 + AT$ where AT is the quota of the geographical latitude. It is worth $\beta_2 * \text{tang}\, \varphi / \cos \alpha$. Maimonides adopted the value of $(2/3)\, \beta_1$.[2]

[1] The approximation is acceptable as long we speak of little arcs.

[2] The reason of the replacement of β_2 by β_1 is not explainable. It is no way a simplification.

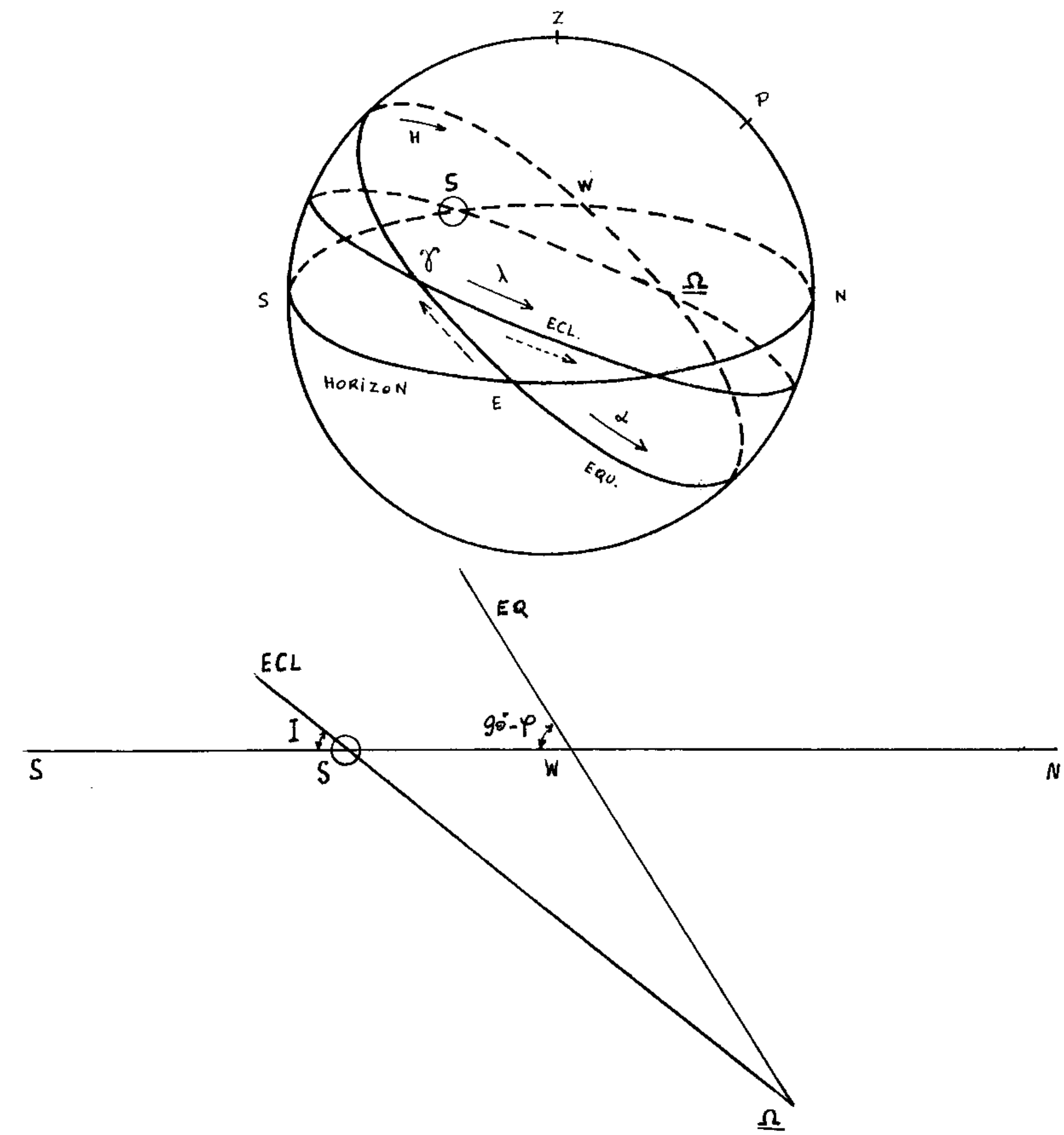

Figure 1: The linear representation of the equator and the ecliptic on the western part of the spherical vault above the western horizon. The above drawing allows understanding how we derivate the relative positions of the equator and the ecliptic from the perspective view of the celestial sphere. On this figure λ_0 is about 230° corresponding to November 15.

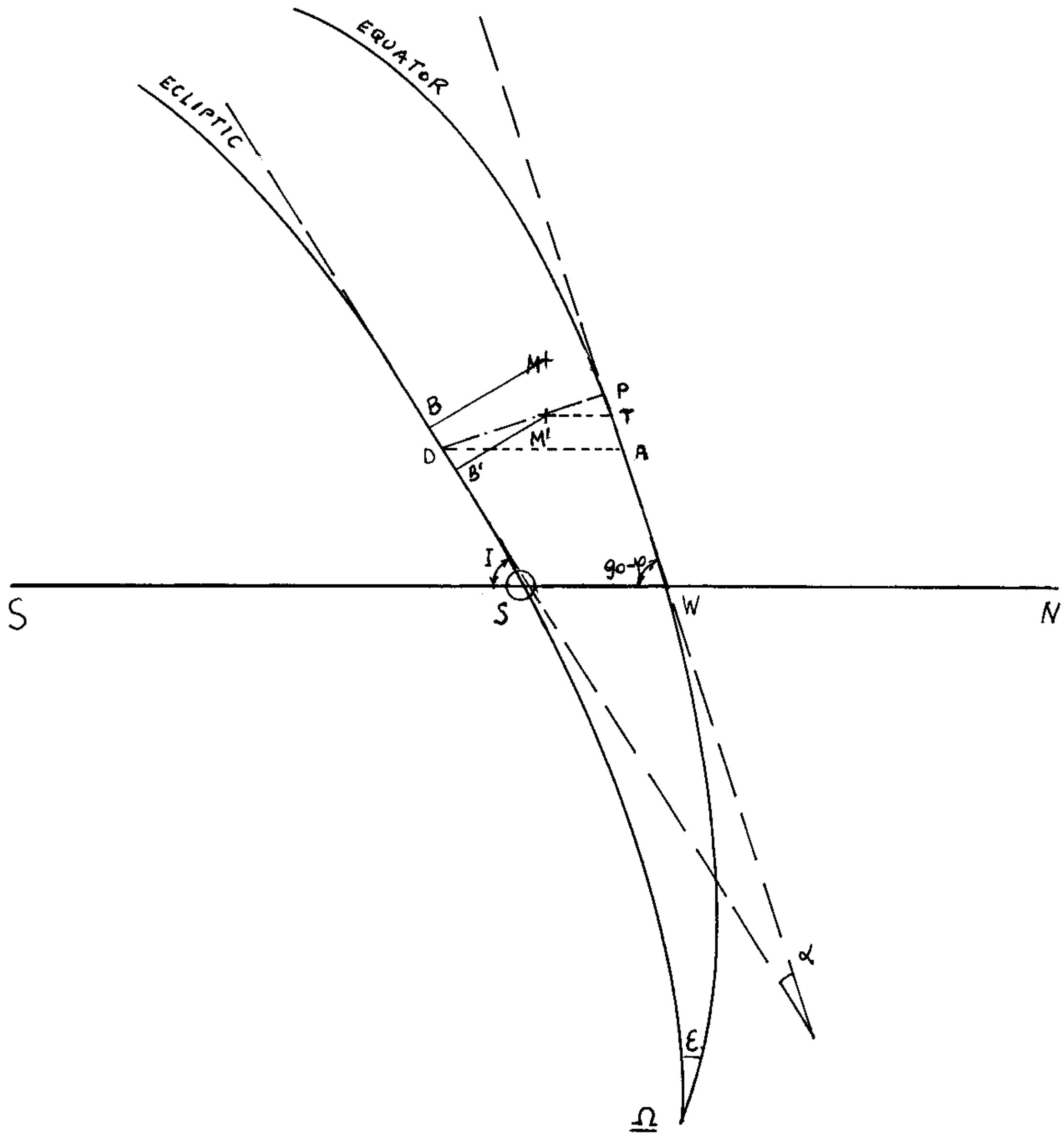

Figure 2: Representation of the equator and the ecliptic at sunset. The longitude of the sun is about 230° corresponding to the middle of November. The angular distances occurring in the problem are small enough to treat both great circles, in the area considered just above the western horizon, as straight lines. The lines of the ecliptic and the equator are the tangents to the great circles at their intersection with the horizon. M is the geocentric moon and M' is the topocentric moon.

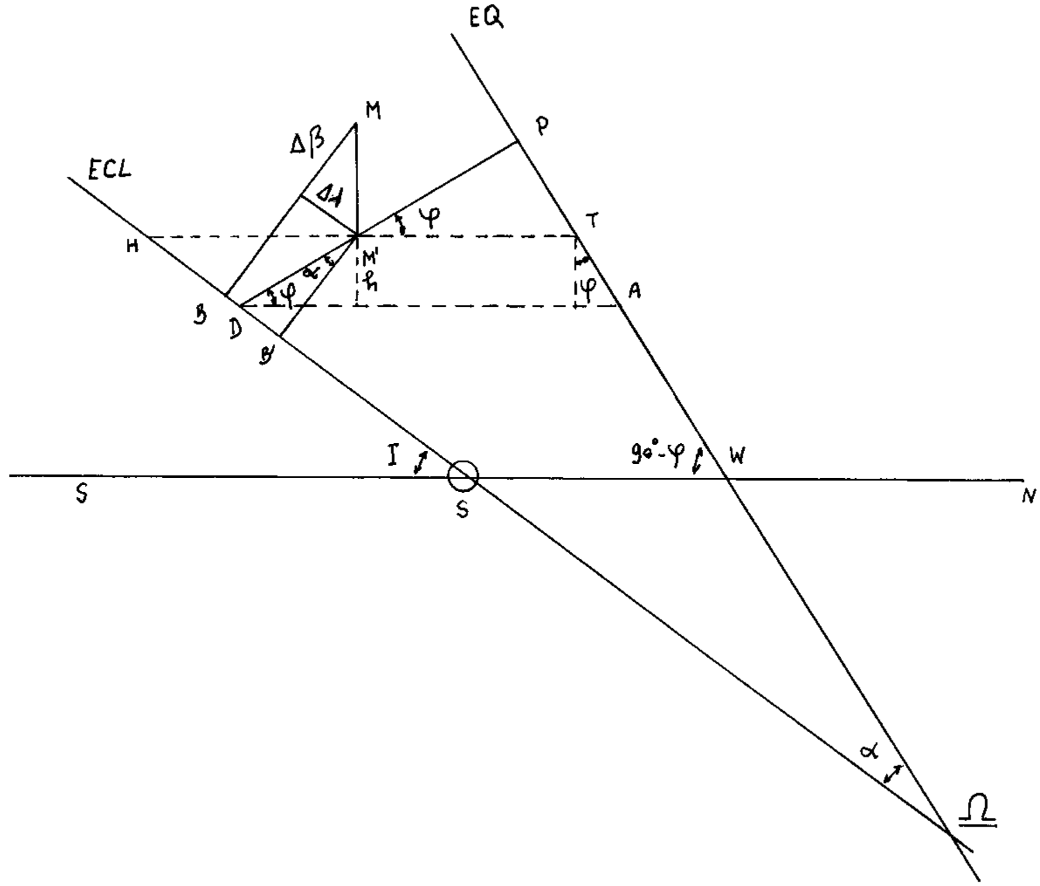

Figure 3: The linear representation of the equator and the ecliptic on the western part of the celestial vault, just above the western horizon, in the area of the first sighting of the new lunar crescent. On this figure λ_0 is about 230° corresponding to about November 15. 180°+ΩS = λ_0, 180°+ΩB = $\lambda_{(}$, 180° + ΩB' = $\lambda'_{(}$, SB = λ_1, SB' = λ_2. BM = β_1 and B'M'= β_2. B'D is *Nelizat Ma'gal*, SD = λ_3. WA = λ_4, AT is the quota of the geographical latitude and WT = b is the arc of vision.

2. The canon of the Alfonsine tables By Isaac ibn Sid and Judah ben Moses Cohen.

We give here the English translation of the chapter 36 of the canon of the Alfonsine tables, composed in Castilan in about 1272 by Judah ben Moses ha-Cohen[3] and Isaac ibn Sid[4] at the request of King Alfonso.[5] This chapter deals with the verification of the

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[3] Well read physician, author of different translations of Arabs scientific works, he collaborated with Isaac ibn Sid in the redaction of the canon of the Castilian Alfonsine tables.

[4] Isaac ibn Sid was a professional astronomer, author of different observations. Some of them are quoted in Yessod Olam. He was the responsible of the preparation of the Alfonsine tables. The Castilian tables are not more extant but one exemplar of the canon of the tables, written in Castilian by R. Judah ha-Cohen and R. Isaac ibn Sid survives in a unique manuscript in Castilian: Madrid, Biblioteca Nacional, MS 3306. The

visibility of the new crescent of the moon. Today only the canons survive in a unique manuscript. They were published recently in a book entitled: The Alfonsine tables of Toledo by José Chabas and R. Goldstein. The canons are published in the original text in Castilian,[6] without translation but with a glossary of astronomical terms. The canons include 54 chapters and chapter 36 has a particular interest for our subject because, we will see, it is directly connected to Maimonides' *Hilkhot Kidush ha-Hodesh*.

[1] [7] **Chapter 36. How to know the appearance of the moon in a certain place?**
[2] When you want to know it, take the position of the sun and the true[8] position of the moon and these positions are taken a third of an hour after sunset following the conjunction.
[3] Know the visible[9] position of the moon and its visible[10] latitude, moreover corrected for the parallax in longitude and latitude at the time of the evening sunset at the west, as it was said in its chapter.
[4] Then subtract the true position of the sun from the true position of the moon; the result is the elongation.
[5] Then check if the position of the moon is from the beginning of Capricornus until the end of Gemini and if the elongation is 9° or less, then you must not make any other calculation because the moon will not appear during this night.
[6] And if the elongation is 15° or more, don't make other calculations because it will probably appear.[11]
[7] If the position of the moon is between the beginning of Cancer and the end of Sagittarius and the elongation is 10° or less, it is worthless to make additional calculations because it will appear only on next night.
[8] And if the elongation is 24° or more don't tire you out anymore doing any other verification because it will probably appear.
[9] But when the position of the moon is between the beginning of Capricornus and the end of Gemini and the elongation is more than 9° and less than 15°, it may appear and it may not appear.
[10] Between these limits it is necessary to make additional calculations.
[11] And when the moon is between the beginning of Cancer and the end of Sagittarius and the elongation is more than 10° and less than 24° it is possible that the moon may appear or that it won't.
[12] And between these limits it is necessary to perform additional calculations.

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text of this canon was included in vol 4 of Libros del Saber de Astronomia published in 1867 by Manuel Rico Sinobas in Madrid. This canon was printed again in 2003 by José Chabas and Bernard R. Goldstein under the title The Alfonsine Tables of Toledo, Archimedes, Vol 8, Kluwer Academic Publishers, where I discovered this exceptional chapter.

[5] King Alfonso X of Castile, Leon and Galicia, born in 1221 and died in 1284. He became King in 1252. Known as the Wise King he was well read and he was the patron of scholars and of scholarly works. The Alfonsine tables must testify the greatness of his reign.

[6] I thank Mr. Ricardo Guth from Zurich, born in Argentina and of Spanish mother language, for his help in the translation.

[7] Numerotation of the lines of the manuscript mentioned in the transcript of the original text.

[8] Geocentric.

[9] Apparent or topocentric.

[10] Apparent or topocentric.

[11] In the Castilian text it writes that it will not appear.

[13] Then, when it happens that the moon is between these limits and you want to check whether it will appear or not, take the apparent longitude of the moon and enter the table of the deviation of the moon which was prepared to this aim and note the indication of the deviation in [degrees and] minutes given in the perpendicular[12] direction.
Then multiply these [degrees and] minutes by the apparent latitude of the moon and keep the result.
[14] Then take the apparent position of the moon, which you entered in the table of the deviation of the moon and if the moon is between the beginning of Capricornus and the end of Gemini and if the apparent latitude of the moon is northern, subtract the kept result [of the multiplication] from the apparent longitude of the moon.
[15] And if the apparent latitude of the moon is southern then add the kept result of the multiplication to the apparent longitude of the moon.
[16] If the apparent longitude of the moon which you entered [in the table of the deviation of the moon] is from the beginning of Cancer until the end of Sagittarius and the apparent latitude of the moon is northern, add the kept result to the apparent longitude of the moon.
[17] If the apparent latitude of the moon is southern, subtract the former kept result from the apparent latitude of the moon and the achieved result after this addition or subtraction represents the corrected longitude of the moon for this purpose.
[18] And then know the rising of the opposite of the position of the sun in the considered place and subtract the rising of the opposite of the sun from the rising of the opposite of the position of the moon. The result is the arc of setting [WA of the corrected apparent elongation SD].
[19] Afterwards take the sine of the geographical latitude of the considered place and divide it by the sine of the complement of the geographical latitude of the considered place and you multiply the result of the division [i.e. tang φ] by the apparent latitude of the moon [i.e. β_2]. Then add the result of this multiplication to the arc of setting if the latitude of the moon id northern or subtract it from the arc of setting if the latitude of the moon is southern.
[20] And what the arc of setting[WA] becomes after this addition or subtraction is the arc of vision [WT].
[21] If the arc of vision is less than 10°, the moon will not appear on this night.
[22] And if the arc of vision is 14° or more it will probably appear.
[23] But if the arc of vision is 10° or more then check the elongation which mentioned above in [4]
[24] If this arc [of vision] is 10° or more and the elongation is 13° or more then the moon will appear but if it will be less, then the moon will not appear.
[25] Moreover if the arc of vision is 12° or more and the elongation is 11° or more, the moon will appear otherwise it won't appear.[13]
[26] Moreover if the arc of vision is 13° or more and the elongation is 10° or more, the moon will appear otherwise it won't.

Astronomical commentary.

[12] See an elucidation in the astronomical explanations at the end of the translation of the original text.
[13] If the arc of vision is 11° or more and the elongation is 12° or more, the moon will be visible otherwise not. This case was apparently forgotten and it represents a lapsus calami of the scribe.

[13] The text refers to a table of the deviation of the moon. Because all the process is identical with the method of Rambam, we can assume with much certitude that the table of deviation is a table similar to Maimonides' prescription for calculating *nelizat ma'agal* literally "the deviation of the path". It represents the difference $\alpha' - \lambda'$ between the right ascension and longitude of the apparent moon.

The text refers to the perpendicular position of of the moon and it requires some explanation. In the tables of Al-Battani and in the subsequent tables, including the Alfonsine tables,[14] the table giving the rising of the points of the equator in sphaera recta or the right ascension of the points of the ecliptic, the longitudes are counted from the beginning of Capricornus.[15] The reason of this practice is unknown. Thus if we want to know the right ascension of $\lambda = 30°$ we must look for α ($\lambda = 30° + 90°$).

We find α (120°) = 117° 53' and then we must subtract 90° and finally we get α ($\lambda = 30°$) = 27°; 53'. In chapter 28 of the canon of the Alfonsine tables it uses the same term "perpendicular position" with the clear meaning that we must consider the point of the ecliptic $\lambda + 90°$, counted from the beginning of Capricornus.

We assume that the table of deviation of the path of the moon considered in the canon is constructed on the same basis and therefore we must enter this table with the entry $\lambda + 90°$. Otherwise this table is probably similar to the table published by Neugebauer in *Sanctification of the New Moon* in Yale Judaica series Volume XI, p. 141. But it could have been more precise and more detailed by the calculation, through the table of rising in sphaera recta, of $\alpha' - \lambda'$.

[18]. In this article, we want to find the oblique setting λ_4 of the arc of ecliptic λ_3 whose extremities are the sun S and the point of the ecliptic D with the same right ascension as the apparent moon. Normally we search the points of the equator which set together with each of these two points and by subtraction we find the length of the arc λ_4. However all the ancients astronomical tables, beginning with Almagest, the tables of Al-Battani and the Alfonsine tables, did not contain a table of oblique setting but a table of oblique rising. Nevertheless the table of oblique rising allows performing the same calculation. Indeed: arc of setting (λ) = arc of rising ($\lambda + 180°$) – 180°.

This can be demonstrated because arc of setting = $\alpha + \Delta$ and arc of rising = $\alpha - \Delta$ with $\sin \Delta = \tan \varphi * \tan \delta$. Therefore arc of rising ($\lambda + 180°$) – 180° = $\alpha + 180° - \Delta_{180} - 180°$ = $\alpha - \Delta_{180}$. But $\sin \Delta_{180} = \tan \varphi * \tan \delta_{180} = - \tan \varphi * \tan \delta = - \Delta$.

Therefore = $\alpha - \Delta_{180} = \alpha + \Delta$. And arc of rising ($\lambda + 180°$) – 180° = arc of setting (λ).

[19]. Calculation of the arc of vision.

We want to calculate the arc AT that must be added to WA = λ_4 in order to get the arc of vision b = WT (see fig 3). Maimonides called it the quota of the geographical latitude. We see on the figure that AT = M'D * tang φ / cos α. = β_2 * tang φ / cos α.

Tang φ = tang 32 = 0.62.

[14] Chapter 28 of the canon.

[15] See Al-Battani, vol 1, pp. 61 – 64.

α varies between 0 and 23°; 35’[16] and cos α varies between 1 and 0.92; average value 0.96.
Maimonides adopted (2/3) $\beta 1$ after replacing for an unknown reason β_2 by β_1.
Hence the quota of the geographical latitude is (tang φ / 0.96) * $\beta_2 \sim$ (tang φ) * β_2 if we neglect 1 / 0.96 = 1.04.
This explains the prescription of the Canon of the Alfonsine tables which neglects the factor cos α.

Conclusion.

As soon as in 1276 Isaac ibn Sid and Judah ben Moses Cohen introduced in the canon of the Alfonsine tables, a chapter about the visibility of the new lunar crescent. This chapter is deeply influenced by the treatment of the subject by Maimonides. The detail of the method of calculation proves that they understood completely the method but they did not copy servilely the detail of Maimonides’ algorithm. By contrast they made use, when it was possible, of existing tables and constructed even a table of the “deviation of the lunar path”. The quota of the geographic latitude (2/3) * β_1 was replaced by tang φ * β_2 which is not only more precise than Maimonides’ quota but the presence of tang φ proves us also that this arc AT belongs to the equator. We have thus an irrefutable proof of the perfect understanding, by the two Spanish scholars, of the method of Maimonides, as soon as 1276.

3. The Hebrew canon of the astronomical tables of Abraham Zacut.[17]
Hebrew text.[18]

הפרק העשירי בראיית הירח החדש וכן בראיית [הירח] הישן בוקר יום עשרים וט' לחודש שעבר היה קודש ליי. דע שזה הפרק הייתה כוונתנו בלוחות שעשינו ודברנו עד עתה והנה כשתרצה לידע אם יראה הירח תוציא מקום השמש האמיתי ומקום הירח האמיתי על הדרך שביארנו למעלה וכן רוחב הירח, ואם הוא צפוני או דרומי. וכל זה תעשה ביום שלאחר הדבוק [5]
האמיתי או ביום הדבוק על מנת שלפחות יהיה רחוקה [ב]עת הראייה האמיתי מעת הדבוק האמיתי, י"ח שעות. וכל זה תוציא לשליש שעה אחר בוא השמש, ומתוקן בתיקון הימים, והם מגרע, כמו שביארנו בפרקים שעברו. ואחר שיהיה לך מתוקן כל זה, גרע השמש האמיתי

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16 Value of Al-Battani.

17 Abraham Zacut, Salamanca (1452 – 1515). The canon and the astronomical tables were written in Hebrew, this composition was called *ha-Hibbur ha- Gadol*. The tables were translated into Latin and a canon was drafted by José Vinzinho under the global name of Almanach Perpetuum. José Vinzinho was a Jewish pupil of Zacut, he converted later to Christianity. The Almanach Perpetuum was also adapted in Arabic were it remained in use until the 19th century. When Zacut composed his astronomical tables, he used plentifully the Alfonsine tablesand he was also influence by the tables of Al-Battani. He used also materials derived from another Jewish astronomer: Jacob ha- Poel of Perpignan and probably also indirectly from Levi ben Gerson (1280 – 1344). See Julio Samso: Astronomy and Astrology in al-Andalus and the Maghrib, Ashgate Variorum 2007.

18 This text was copied by Berthold Cohn from the manuscripts of Munich and Vienna. The third manuscript of Lyon was not available to Cohn because of the war 1914 – 1918.

ממקום הירח האמיתי והנשאר הוא הנקרא אורך ראשון. וכן לאחר שידעת רוחב כמה הוא ואם הוא צפוני או דרומי, הוא הנקרא רוחב ראשון.

והזהר בזה האורך הראשון וכן ברוחב הראשון ויהיו שניהם מוכנים לפניך. ואף על פי שכוונתי בזה הפרק לידע ראיית הירח לכל האקלימים, הנה לא חששתי לכתוב בכאן גדרי ראיית הירח לכל האקלימים, והנה לא חששתי אלא לאקלים ירושלם שהיא קצה אקלים ד', כי שם העיקר לנו באורך שכתב לירושלם כמו הקושמוגרופיאה לא היקבלה הגדרים, שהגדרים הם כפי הרוחב. ואם חכמת מזה הפרק תוכל להבין וללמוד גדרי

ראיית הירח לכל האקלימים. אלא לאקלים ירושלם והגדרים שכתב הרמב"ם ז"ל לאקלים ירושלם, הם אלה. התבונן באורך הזה הראשון, אם יצא לך מט' עד ט"ו מעלות וכל שכן יותר, אז אפשר שיראה הירח ואם פחות מט' אז אין לה אור מהשמש שיהיה אפשר לה שיראה . ואם יותר מט"ו מעלות, אז אינך צריך חשבון כי בודאי יראה. וכל זה אם היה הירח מתחילת גדי עד סוף תאומים יען שאלו המזלות שוקעים במתינות. הנה

זה גדר הראשון שנתן הרב ז"ל. והתימה עליו מרוב זריזותו ובקיאותו בזאת החכמה , איך נתן את הגדר הזה עד ט"ו מעלות כי בודאי הגדר הוא כפי הדרכים שהוא נתן לראיית הירח עד י"ג מעלות ולא יותר. ולפי הנראה לי שהוא טעה בזה כי (ש)חשב שהחלופים שיבואו במאזנים הם עצמם יבואו בטלה ר"ל החלופים שהם מלבד המצעדים. ואינו כן, כי אם הרוחב דרומי במאזנים הוא ה' מעלות אז יבוא בחשבון לגרוע

מהאורך ה' מעלות וכ'[19] דקים דהיינו ב' מעלות בעבור נליזת המעגל וב' מעלות וכ'[20] דקים בעבור האופק ועוד דקי חילוף ההבטה שיהיה הכל בקירוב [ה'] מעלות [וכ' דקים]. ובטלה לא יהיה אלא מעלה א' וב'[21] דקים לגרוע וכן בכל המזלות שהם מגדי עד סרטן לא יבוא לגרוע על הרוב מהאורך יותר מג' מעלות וב' דקים ועוד דקי חילוף ההבטה שיגיע הכל לד' מעלות ולא יותר והנה אם כן הגדר הוא י"ג מעלות ולא יותר. אבל

בגדרים האחרונים הם באים בסדר ובשורה ואלה הם: אם יהיה מקום הירח מתחילת סרטן עד סוף קשת שאלו המזלות שוקעים במהירות, אז גדרי ראיית הירח הם מי' מעלות עד כ"ד מעלות. ואם היה פחות מי' מעלות אז לא יראה ואם יותר מכ"ד אז אינך צריך חשבון כי בודאי יראה. ודע כי הגדרים הראשונים שאמרנו הם שווים בכל הישוב אבל הגדרים האחרונים משתנים כפי רוחב המדינות. ותדע הגדרים לשאר האקלימים

שתרצה, במזל טלה עם י' מעלות ממצעדים כמה יעלו מהחגורה ועוד תוסיף ו' מעלות וב'[22] דקים, וזה יהיה גדר הראייה לאיקלם ההוא. וכשיהיה בין הגדרים ההם האמורים תצטרך לדרוש ולחקור בחשבונות הראייה עד שתדע אם יראה (אותם) אם לא יראה. ואלה הם

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[19] נ"א: ב'
[20] נ"א: ב'
[21] נ"א: ב'
[22] נ"א: כ'

חשבונות הראייה, התבונן וראה הירח באיזה מזל הוא ותדמה כאלו אז לקות השמש. ותכנס בלוח חילוף ההבטה שעשיתי. וכבר אמרתי לך איך תידע חילוף ההבטה לכל האקלימים. ותכנס במזל ההוא לעת בוא השמש שהקדים שתמצא שם בלוח חילוף ההבטה לזה העת. הוא עצמו יהיה לירח לשליש שעה אחר בוא השמש. ואע"פ שזה העת אינו עת הדבוק, לא ישתנה כל כך חילוף ההבטה בעבור גלגלו ביום אחד. ואם תרצה לדייקו עוד, כבר אמרתי לך בפרק לקות השמש (ש)דקדוקו. והדקים ממעלה שתמצא שם מחילוף האורך בלוח חילוף ההבטה תגרע מהאורך הראשון והנשאר הוא הנקרא אורך שני. וכן לזה העת עצמה תיקח שם חילוף ההבטה הרוחב ותגרע אותו מרוחב הירח הראשון אם היה צפוני, או תוסיפהו עליו אם היה דרומי, אם היה רוחב המדינה מכ"ד מעלות ומעלה. והנשאר או הנקבץ כן הוא הנקרא רוחב שני. ואם היה רוחב הירח הצפוני בשיעור חילוף הרוחב הדרומי, אז אין לירח מרחב כלל לפי הנראה. וכן אם לא היה לירח רוחב כלל, אז יהיה רוחב הירח דרומי כשיעור חילוף הרוחב. והנה זה רוחב הדרומי לא פירש הרמב"ם בפירוש. ואחר כך ראה אם היה רוחב לירח, עם אי זה מעלה באמצע השמים [יעבור], מפני שלא יעבור המעלה שהיתה בה הירח בחגורה עם המעלה ההיא עצמה באמצע השמים כשיהיה לה רוחב,אלא מעלה אחרת, או פחות או יותר, חוץ אם היתה בראש גדי או בראש סרטן, כמו שביארנו זה היטב בפרק הט' שעבר בקרוב. וזה מה שקרא הרמב"ם ז"ל נלוז במעגלו. ובזה הדרך תדעהו, ואם היה מקום הירח מתיחלת טלה או תחילת מאזנים עד כ' מעלות מכל אחד מהם או מ' מדגים עד סופו או מ' מבתולה עד סופו, תקח מן הרוחב הב' שני חומשים ואם מכ' מטלה עד י' משור או מכ' ממאזנים עד י' מעקרב או מכ' מאריה עד י' מבתולה או מכ' מדלי עד י' מדגים, תקח מן הרוחב השני שלישיתו. ואם מי' משור עד כ' ממנו, או מי' מעקרב עד כ' ממנו או מי' מאריה עד כ' ממנו, או מי' מדלי עד כ' ממנו תקח ממרחב השני רביעיתו. ואם יהיה הירח מכ' משור עד סופו או מכ' מעקרב עד סופו או מתחילת אריה עד י' ממנו או מתחילת דלי עד י' ממנו תקח מן הרוחב השני חמישיתו ואם יהיה הירח מתחילת תאומים עד י' ממנו או מתחילת קשת עד י' ממנו וא מכ' מסרטן עד סופו או מכ' מגדי עד סופו תקח מהרוחב השני שתותו. ואם מי' מתאומים עד כ' ממנו או מי' מקשת עד כ' ממנו או מי' מסרטן עד כ' ממנו או מי' מגדי עד כ' ממנו תקח מהמרחב השני חצי שתותו. ואם מכ' מתאומים עד כ"ה או מכ' מקשת עד כ"ה או מה' מסרטן עד י' ממנו או מה' מגדי עד י' ממנו תקח מהרוחב השני רביע ששיתו. זה הטעם לכל אלה הגבולים הם כפי מרחקם מההפוכים שהם ראש סרטן וראש גדי. ואם היה הירח ה' מעלות קודם ראש סרטן או ה' אחריו וכן בראש גדי ה' לפניו או ה' לאחריו אז לא תקח כלום לפי שאז אין בו נליזות מעגל. וזה המקצת שתקח

מרוחב הירח הב' אם היה רוחב הירח צפוני, גרע אותו מהאורך הב' ואם הוא דרומי, הוסיף אותו על האורך הב'. וזה תבין אותו אם היה הירח בתחילת גדי עד סוף תאומים אבל אם היתה הירח מתחילת סרטן עד סוף קשת, אז תעשה להפך שאם היה רוחב הירח צפוני תוסיף ואם היה דרומי תגרע. ומה שיהיה האורך השני אחר שתוסיף עליו או תגרע ממנו, הוא הנקרא אורך ג' ואם לא תוסיף או לא תגרע על האורך השני, זה יהיה כאשר לא יהיה לירח רוחב או כאשר כאשר יהיה הירח בראש גדי או בראש סרטן, אז תקרא לאורך השני גם כן אורך ג'. וכל זה שאמרנו הוא שוה בכל העולם כולו ר"ל בידיעת מעת מעלת אמצע השמים. ואחר כך תחזור ותראה האורך הג' הזה והוא המעלות שבין השמש והירח עם כמה מצעדים ירדו באותו אקלים. וזה תדע אותו בלוח המצעדים האופקיים שכתבתי לאקלימים והוא שתקח נוכח מעלת השמש ותגרעם ממצעדי נוכח מעלת הלבנה שהוא בה באורך ג'. ומה שיעלה ממצעדים לאלו המעלות הוא הנקרא אורך רביעי. ואחר כך תחזור ותקח ממרחב הירח הראשון שני שלישיו לעולם ותוסיפהו על האורך הד' אם היה אורך הירח צפוני, או תגרעהו ממנו אם הוא דרומי וזה התיקון הוא בעבור האופקים הנוטים. ומה שהיה האורך הרביעי אחר שתוסיף עליו או תגרע ממנו הוא הנקרא קשת הראייה . ואחר כך תקח האורך הראשון ותחברהו עם קשת הראייה ואם יעלו לכ"ב מעלות וה' דקים יראה הירח ואם פחות מזה לא יראה, על מנת שכל אחד מהם לא יהיה פחות מט' מעלות, או האורך ראשון או קשת הראייה. הנה זה לדעת הרמב"ם ז"ל, אבל לדעת הראב"ע ז"ל בספר כלי הנחושת נראה שם שגבול הראייה הוא כשתחובר שניהם ויהיה כ"ד מעלות על מנת שלכל אחד מהם לא יהיה פחות מי' מעלות. ואולי אמר זה ונתן גבול יותר ארוך מהרמב"ם ז"ל בעבור שהוא לא דקדק שם בענין חילוף ההבטה וכן בענין מעלת חצי השמים וכיוצא בו כמו שדקדק הרמב"ם ז"ל ואם כן אולי יסכימו שניהם. הנה כללתי לך בפרקהבזה מבואר היטב ויותר לכל הדברים שכתב הרמב"ם ז"ל בהלכות קדושת החודש אם תדקדק היטב בדרכיו ובדבריו. וכל זה שאמרנו הוא לדעת אלו ולדעת רוב התוכנים אבל לדעת חז"ל במסכת ראש השנה וכן הנסיון ששמעתי מאנשי אמת, אפשר שיראה הירח הישן בבוקר והירח החדש באותו היום בעצמו בערב במקומות ויהיה לפי זה נולד קודם חצות ואחר חצות כפשוטו. וכל אלו הדברים תעשה לא פחות ולא יותר לראיית הירח הישן בבוקר ביום כ"ט לחודש זולת שהגדרים יהיו להפך וכן תקח המצעדים מהמעלות שבין השמש והירח במעלותיהם עצמם, לא במעלות הנוכחיות להם כמו שהוא זה בערב. ודברינו גם כן בזה הפרק בראיית הירח הישן כי מצינו שצריך זה גם כן למסכת ראש השנה כמ"ש במשנה העדים שאמרו: ראינוה שחרית במזרח וערבית במערב. ונפל המחלוקת בין החכמים שאמרו שם במשנה. אע"פ שכבר פסק הרמב"ם ז"ל שאין נזקקים לזה העת אלא במה שכתב בערבית, אעפ"כ טוב הוא לידע אותו. וכבר אמר הרמב"ם

שאם יבואו אלו הגבולים שאמרו בצמצום שאז ומפני כן לא נאמר שיראה הירח בכל מקום אלא במקומות גבוהים ויהיה האויר זך. אבל אם היה
[105]
קשת האורך וקשת הראייה ארוכים, כי אז יראה יותר בפרסום לכל. וכן גם כן אם הראייה היה במזלות גדולי הנטיה כמו טלה ומאזנים לא יראה כמו במזלות ההם דרומי הנטיה כמו גדי וסרטן. הנה כבר השלמנו מה שיעדנו לבאר בזה הפרק המועיל לנו מאוד בה, יעזרנו לגמרו אמן.

English translation.[23]

The tenth chapter is devoted to the visibility of the new moon and also the visibility of the old moon in the morning of the 29th day of the last month. The tenth [chapter] will be holy to God.[24] Know that we had this chapter in mind in the tables that we constructed and about which we spoke until now.
If you want to know if the moon will be visible, get the true[25] position of the sun and the true position of the moon according to the rules that we explained above. Get also the latitude of the moon and look whether it is northern or southern. Do all this for the day following the true conjunction or, for the day of the true conjunction itself if the moment of vision is removed by at least 18 hours from the moment of the true conjunction. This calculation of the coordinates of the two celestial bodies must be done for 20 minutes after sunset. Moreover you must correct that moment by the equation of the days[26]which is always negative as we explained it in the preceding chapters.[27]
Then take the true positions of the sun and the moon and subtract the position of the sun from that of the moon, this difference is called the first longitude. Take also the latitude of the moon after you determine if it is northern or southern, it is called the first latitude and behold these two results. Although I had in mind to study in this chapter the visibility of the new moon in all the climates of the earth, nevertheless I did only give the limits of the visibility for Jerusalem which lies at the limit of the fourth climate[28] because we consider that the origin of the longitudes lies there although the cosmography did not accept it. But the limits of the climates depend only on the latitude and if you understand this chapter you will be able to deduce the limits of the visibility for all the climates. Indeed the limits of Maimonides are valid only for the climate of Jerusalem.

[23] Based on the transcript and a German translation published in 1918 by Berthold Cohn in Der Amanach Perpetuum des Abraham Zacut, Ein Beitrag zur Geschichte der Astronomie in Mittelalter, Schriften der Wissenschaftlichen Geselschaft in Strasburg, 32 Heft, Strasburg 1913, Karl, J, Trübner.
[24] Leviticus 27:32.
[25] By contrast with the mean position.
[26] Today we use the expression "equation of time" but we understand it according to new definition given by Flamsteed. See J. Ajdler: The equation of time in ancient Jewish astronomy. BDD 16.
[27] The ancient considered always true time and they needed to correct the true time to get mean time and use the astronomical tables. In the tables of the Almagest and Al-Battani, the correction from true time to mean time is subtractive.
[28] Ptolemy considers 7 climates. The 4th climate corresponds to the latitude of Rhodes of 36°, where the longest day has 14.5 hours. The 3rd climate corresponds to Lower Egypt, latitude 30°; 22' where the longest day has 14 hours. The latitude of Jerusalem of 33° is thus at the inferior limit of the fourth climate. Note that Zacut, like ibn Ezra, considers that the latitude of Jerusalem is 33° by contrast with Al-Battani and Maimonides who adopted 32°.

Now consider the first longitude, if it is comprised between 9° and 15° and more than 15°, then it is possible that the moon will be visible but if it is less than 9° then it does not receive enough light to be visible. If the difference is greater than 15° then you don't need any other calculation because it will certainly be seen. This is only valid if the moon is between the beginning of Capricornus and the end of Gemini because these signs of the zodiac are setting slowly in all the climates. This is the first limit that the Rabbi [Rambam] gave.
Now taking into account his great knowledge and precision in this science [of astronomy], it is not understandable that he extended the limit until 15 degrees because the limit that he gave for the visibility of the moon must be 13 degrees and not more.
It seems to me that he erred and made the following mistake: he considered that the corrections which are applicable to the sign of zodiac of Libra are also valid for the sign of the zodiac of Aries, i.e. the corrections which are beside those for the calculation of the oblique setting [in order to measure the span of time corresponding to the considered arc of the ecliptic]. This is in fact incorrect, because if the latitude of the moon is 5° southern in the sign of Libra, then we must subtract 5°; 2' from the longitude i.e. 2° because of the deviation of the path and 3°; 02' because of the horizon. But in Aries we must subtract only 1°; 02' and similarly in the other signs of the zodiac which are between Capricornus until Cancer we must generally not subtract from the longitude more than 3°; 02' without taking into account the minutes [of a degree] for the parallax so that the correction to subtract from the longitude, should not reach more than 4 degrees.
But the other limits [10° and 24°] are likely and correct as follows: if the position of the moon is between the beginning of the sign of zodiac of Cancer and the end of Sagittarius, which are the signs of the zodiac which are setting fast then the limits of the visibility of the moon are from 10° until 24°. If the first longitude is less than 10° the moon will not be visible on this night and if it is greater than 24° then you don't need any calculation because it will certainly be visible. Moreover know that the inferior limits that we mentioned are valid for any place in the world but the superior limits change according to the latitude of the place of observation. And you will be able to find these limits for other climates, you will see what is the oblique setting of 10 degrees of the ecliptic in the sign of the zodiac of Aries [for example] and you will add to this arc of oblique setting the value of 6°; 02'. The final result will be the limit of visibility for that climate.
If [the first longitude] is between these limits [9° and 15° or 10° and 24°] then you will have to examine and perform the visibility calculations in order to know if the moon will be visible or not.
And those are the visibility calculations: examine and see in which sign of the zodiac the moon is, and consider the situation as if we were in the case of a solar eclipse and enter the table of parallax that I established. I told you already how to know the parallax for each climate. You enter this table at the moment of sunset and the number of minutes indicated in those tables, are also valid for the moon 20 minutes after sunset. Although the considered moment is not exactly the moment of the conjunction, this will not affect much the value of the parallax because of the moon's movement on its path during a day. If nevertheless you want to perform an exact calculation, I explained it to you already in the chapter of the solar eclipse. The minutes of a degree that you find in the table of parallax must be subtracted from the first elongation and the result is called the second elongation. For the same moment you find the parallax in latitude and you subtract it

from the first latitude if it is northern or you add it to the first latitude if it is southern, if the latitude of your place is more than 24°. The result of this difference or addition is called second latitude.

If the latitude of the moon was northern and had the same value as its parallax in latitude, then the apparent moon has no latitude at all. Similarly if the moon has no latitude [and is on the ecliptic] then the apparent latitude of the moon will be (northern) [southern] and it will be equal to this parallax in latitude. Maimonides did not mention explicitly this southern latitude.

Now if the moon has a [second] latitude, then examine which point of the ecliptic crosses the meridian together with it. Indeed if the moon has a [second] latitude, then the position of the moon on the ecliptic [the point of the ecliptic B' with the same longitude as the apparent moon] does not cross the meridian together with the moon but it is another point [D] with a greater or smaller longitude which crosses the meridian together with the moon except when the moon is at the beginning of the signs of the zodiac Capricornus or Cancer as we explained it recently.

. And this is what Maimonides called the "deviation of its path" and you will be able to quantify it according to the following indications. If the moon was between the beginning of the sign of the zodiac of Aries or the beginning of Libra until 20° of each of these signs or from 10° of the sign of the zodiac of Pisces or from 10° of Virgo until its end then take 2/5 of the second latitude. If the moon is between 20° of Aries until 10° in Taurus or between 20° in Libra until 10° in Cancer or between 20° in Leo until 10° Virgo or between 20° in 20° in Aquarius until 10° in Pisces then take 2/3 of the second latitude. If the moon is between 10° in Taurus until 20° in Taurus or between 10° in Scorpio until 20° in Scorpio or between 10° in Leo until 20° in Leo or between 10° in Aquarius until 20° in Aquarius, then take1/4 of the second latitude.

If the moon was between 20° in Taurus and its end or 20° in Scorpio and its end or between the beginning of Leo until 10° in Leo or between the beginning of Aquarius and 10° in Aquarius, take 1/5 of the second latitude. If the moon is between the beginning of Gemini and 10° in Gemini or between the beginning of Sagittarius until 10° in Sagittarius or between 20° in Cancer and its end or between 20° in Capricornus and its end take 1/6 of the second latitude. If the moon is between 10° in Gemini and 20° in Gemini or between 10° and 20° in Sagittarius or between 10° and 20° in Cancer take 1/12 of the second latitude. If the moon is between 20° and 25°in Gemini or between 20° and 25° in Sagittarius or between 5° and 10° in Cancer or between 5° and 10° in Capricornus take 1/24 of the second latitude.

And the reason for each of these limits depends on its distance from the two diametrically opposite points of the zodiac, which are the beginning of Cancer and the beginning of Capricornus. If the position of the moon is between 5° before and 5° after the beginning of Cancer and similarly between 5° before and 5° after the beginning of Sagittarius, then you don't take anything because we don't note then any deviation of the path. Now this quota of the second latitude that you found, you must subtract it from the second latitude if the latitude of the moon is northern and add it to the second longitude if the latitude of the moon is southern. This is the rule if the moon is between the beginning of Capricornus and the end of Gemini but if the moon is between the beginning of Cancer until the end of Sagittarius then it is the contrary, you add the quota of the second latitude if the latitude of the moon is northern and you subtract it from the second latitude if the

latitude of the moon is southern. What the second longitude becomes after this addition or subtraction is called the third longitude. Now if you don't add to the second longitude or subtract from it any correction because it is zero and this happens when the moon has no latitude or when it is at the beginning of Capricornus or at the beginning of Cancer. In these cases you call the second longitude also the third longitude. And all that we established about the transit at the meridian is valid for the whole world [the principle of the deviation according which the position of the moon on the ecliptic D does not cross the meridian together with the apparent moon M']. Consider now the third longitude, which represents the number of degrees of the ecliptic between the sun [S] and the position [on the ecliptic D] of the apparent moon and determine the number of degrees of oblique setting of this distance in the considered climate. You will find that through the table of oblique rising at the horizon, which I drew up for the different climates. You find the oblique rising of the point opposite to the sun [S] and of the point opposite to the moon [D] and you subtract the oblique rising of the point opposite to the sun from that of the oblique rising of the point opposite to the moon and the difference is an arc of the equator[the oblique setting of λ_3] which is called the fourth longitude.
Then you take 2/3 of the first latitude and add it to the fourth longitude if the latitude of the moon is northern and subtract from the fourth longitude if the latitude is southern. This correction is caused by [the latitude of the moon but it depends also on] the obliquity of the horizon. The result of this addition to the fourth longitude or the subtraction from it is called the arc of vision.
Now take the first elongation and add it to the arc of vision. If the total reaches 22° 05' the moon will be visible on the condition that each of these two arcs, the first longitude and the arc of vision, is not smaller than 9° and if the total is less, it will not be visible. This is the position of Maimonides. However according to Ibn Ezra in his book "the copper astrolabe" it seems that the limit when you add both arcs [λ_1, the first longitude and b the arc of vision] is 24° on the condition that each of these two arcs is not smaller than 10°. It is possible that he gave a greater limit than Maimonides because he did not take into account the effects of the parallax and of the difference of transit between the positions on the ecliptic of the moon [D] and the apparent moon [B'] and other details as Maimonides did. Therefore it is possible that they agree.
I have included in this chapter, with great details and precision, all the rules that Maimonides wrote in his laws of the sanctification of the new crescent, if you examine the words of Maimonides. All what we wrote in this chapter is according to this opinion and according also to the opinion of most of the astronomers. However according to the opinion of Hazal in the gemara Rosh ha-Shanah and according to the observations about which I heard from reliable people, it is possible to see the old moon in the morning and the new moon on the evening of the same day in particular places where the air is specially pure. And you should also apply all these prescriptions meticulously, no more or no less, to check the visibility of the old moon in the morning. Of course you must use the coordinates of the moon for that moment in the morning and not those for the evening used before. But Maimonides' rules related to the visibility of the new moon are also valid for the study of the visibility of the old moon. This study has also its importance because we need it in the study of the treatise of Rosh ha-Shanah in the Mishnah about the witnesses who saw the old moon in the morning toward the east and in the evening toward the west and there was a discussion between the Tanaïm of the Mishnah about

these two testimonies. Although Rambam ruled that we pay no attention to the first part of the testimony about the morning and consider only the last part of the testimony about the evening, it is nevertheless good to know it [in order to be able to check such a statement]. Maimonides also said that when the limits of visibility are just reached, then the moon is not visible anywhere but only in high places with pure air. But if the arc of the first longitude and the arc of vision are long then the moon will be seen anywhere more clearly. Similarly if the vision occurs when the moon is in signs of the zodiac presenting a greater inclination [with regard to the horizon] like Aries and Libra, it will not be as easy as when the moon is in a sign of the zodiac presenting a smaller inclination like Capricornus or Cancer. So we finished all what we intended to elucidate about this chapter which presents much importance for us. May God help us to finish [this book] Amen!

Astronomical commentary.[29]

[6]

The presumed moment of vision should be at least 18 hours after the true conjunction. The variation of the elongation is 0.51° / hour. It is assumed that $\lambda_1 = \lambda_{(} - \lambda_0$ must be greater than 9° corresponding to about 9 / 051 = 17.65° when we consider the mean movement. Zacut adopted an average value of 18 hours.

[7]

The moment of sunset is calculated through the formula $\cos H = -\text{tang}\ \varphi * \text{tang}\ \delta$.
The ancients used generally the equivalent formula $H = 90° + \Delta$ where $\sin \Delta = \text{tang}\ \varphi * \text{tang}\ \delta$.
The moment of sunset is, on this manner, calculated in true time.
We know that the mean time of Almagest and Al-Battani is equal to the true time around 11 February and the correction true times towards mean time is always subtractive.
This is what Zacut writes in line 7: the correction of the equation of the days is subtractive.

Lines [20] until [30].

We try to elucidate this passage by using Maimonides' algorithm of the determination of the arc of vision, although Abraham Zacut referred partially to his own tables for the calculation of the effects of the parallax and the oblique setting. This could be the cause of insignificant differences.
In the period winter – spring, the more favorable period for a fast seeing of the new moon, corresponding to $\lambda_{(}$ included between 270° or the beginning of Capricornus and 90° or the end of Gemini, the limits given by Maimonides are $\lambda_1 = 9°$ and $\lambda_1 = 14°$ with

[29] Following the numbering of the Hebrew text.

the meaning that under 9° the visibility is impossible and above 14° the visibility is certain (if there are no clouds) and no additional calculation is needed.[30]
In the period summer – autumn, the less favorable period for a fast seeing of the new moon, corresponding to $\lambda_{(}$ included between 90° or the beginning of Cancer and 270° or the end of Sagittarius, the limits given by Maimonides are $\lambda_1 = 10°$ and $\lambda_1 = 24°$ with the meaning that under 10° the visibility is impossible and above 24° the visibility is certain (if there are no clouds) and no additional calculation is needed. [31]
Zacut writes that the inferior limits are intrinsic limits valid everywhere, related to the insufficient illumination of the new crescent by the sun. The superior limits are depending on the latitude of the considered place. Furthermore he considers that these superior limits are rough approximations of the true limits in Jerusalem.
The following calculations are based on the algorithm described in *Hikhot Kiddush ha-Hodesh.*[32]

Period winter – spring.

Let us consider the sign of the zodiac of **Capricornus** (beginning). We assume that $\lambda_1 = 9°$ and $\beta_1 = -5°$.
$\lambda_3 = 9° - 44' + (1/24) * (5°+36') = 9 - 0.73 + (1/24) * 5.6 = 9 - 0.73 + 0.23 = 8.5°$
$\lambda_4 = (7/6) * \lambda_3 = 9.92°$ and $b = \lambda_4 + (2/3)\ \beta_1 = 9.92 - (2/3) * 5 = 9.92 - 3.33 = 6.59°$
Note that the term $(2/3)\ \beta_1$ used by Maimonides at the end of the calculation is an approximation and the correct estimation of this term, called the quota of the geographical latitude is $(2/3)\ \beta_2 = -(2/3) * 5.6 = -3.73°$ and then $b = 6.19°$
In fact the minimum value compatible with a visibility is $b = 9°$.
The criterion of visibility given by Zacut is $\lambda_1 + b > 22°; 05'$.
If we consider $\lambda_1 = 12°$ then
$\lambda_3 = 12° - 44' + (1/24) * (5°+36') = 12 - 0.73 + (1/24) * 5.6 = 12 - 0.73 + 0.23 = 11.5°$
$\lambda_4 = (7/6) * \lambda_3 = 13.42°$ and $b = \lambda_4 + (2/3)\ \beta_1 = 13.42 - (2/3) * 5 = 13.42 - 3.33 = 10.09°$
Thus $\lambda_1 = 12°$ and $b = 10.09°$, $\lambda_1 + b = 22.09 > 22°; 05'$ as required by Zacut.[33] We have just reached the limit of visibility.
If we consider a quota of the geographical latitude of $(2/3)\ \beta_2 = -3.73°$ then $b = 13.42 - 3.73 = 9.69°$ and $\lambda_1 + b = 21.69° < 22°; 05'$ and there is no visibility. We will need $\lambda_1 = \sim 12.20°$ to satisfy the criterion of visibility.

[30] HKH 17:2.
[31] HKH 17:3.
[32] For details about this algorithm, see standard editions of *Hilkhot Kiddush ha-Hodesh* from chapter 17 onwards. See also an English translation and an astronomical commentary in Sanctification of the new moon, Yale Judaica Series XI, Yale University Press 1956 and J. Ajdler, *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Sifriati 1996. In the next calculations λ_1 is the first longitude (elongation moon – sun), λ_2 is the second longitude (the first longitude after correction for the parallax), .λ_3 the third longitude after taking into account of the "deviation of the path" or "*nelizat ma'agal*"), λ_4 the fourth longitude, the oblique setting of λ_3, β_1 the first latitude or the latitude of the true moon M or geocentric moon, β_2 the second latitude or the latitude of the apparent moon M' or topocentric moon and b the arc of vision corresponding to the moonset lag, the delay between the apparent setting of the moon and the sun.
[33] The plain understanding of the texts would require only 22°. The value of 22°; 05' is from Zacut and must probably correct the approximations of the calculations.

Let us now consider the sign of the zodiac of **Gemini** (end). We assume that λ_1 = 9° and β_1 = – 5°.
λ_3 = 9° - 58' + (0) * (5°+16') = 9 – 0.97 + (0) * 5.27 = 9 – 0.97 + 0 = 8.03°
λ_4 = (7/6) * λ_3 = 9.37° and b = λ_4 + (2/3) β_1 = 9.37 – (2/3) * 5 = 9.37 – 3.33 = 6.04°
Note that the term (2/3) β_1 used by Maimonides at the end of the calculation is an approximation and the correct estimation of this term, called the quota of the geographical latitude is (2/3) β_2 = – (2/3) * 5.27 = –3.51° and then b = 5.86°
In fact the minimum value compatible with a visibility is b = 9°.
The criterium of visibility given by Zacut is λ_1 + b > 22°; 05'.
If we consider λ_1 = 12.40° then
λ_3 = 12.26° - 58' + (0) * (5°+16') = 12.26 – 0.97 + (0) * 5.27 = 12.26 – 0.97 = 11.29°
λ_4 = (7/6) * λ_3 = 13.17° and b = λ_4 + (2/3) β_1 = 13.17 – (2/3) * 5 = 13.17 – 3.33 = 9.84°
Thus λ_1 = 12.26° and b = 9.84°, λ_1 + b = 22.10° = 22°; 06' > 22°; 05' as required by Zacut.[34] We have just reached the limit of visibility.
If we consider a quota of the geographical latitude of (2/3) β_2 = –3.51° then b = 13.17 – 3.51 = 9.66° and λ_1 + b = 21.92 < 22°; 05' and there is no visibility. We will need λ_1 = ~ 12.35° to satisfy the criterion of visibility.

The argument of R. Abraham Zacut is thus clear: the limit of λ1 = 14° as minimum value of λ_1 allowing to give the guarantee of visibility without checking the criterion λ_1 + b > 22°; 05' is much to great as we check that λ_1 > 12°; 26' or even 12°; 35' is sufficient to give this guarantee of visibility if there is no climatic impediment..

Period summer – autumn.

Let us consider the sign of the zodiac of **Libra** (beginning). We assume that λ_1 = 10° and β_1 = – 5°.
λ_3 = 10° - 34' – (2/5) * (5°+46') = 10 – 0.57 – 0.4 * 5.77 = 10 – 0.57 – 2.31 = 7.12°
λ_4 = (2/3) * λ_3 = 4.75° and b = λ_4 + (2/3) β_1 = 4.75 – (2/3) * 5 = 4.75 – 3.33 = 1.42°
Note that the term (2/3) β_1 used by Maimonides at the end of the calculation is an approximation and the correct estimation of this term, called the quota of the geographical latitude is (2/3) β_2 = – (2/3) β_2 = – (2/3) * 5.77 = –3.85° and then b = 0.90°
In fact the minimum value compatible with a visibility is b = 9° and if b >14° then the moon is certainly visible.[35] We can then calculate λ_1 by the former formula considered in the opposite direction. We depart from b = 9° the minimum admissible value of b.
λ_4 = 9 + 3.33 = 12.33°, λ_3 = (3/2) * 12.33 = 18.50°, λ_1 = 18.50 + 0.57 + 2.31 = 21.38°
Thus λ_1 = 21.38° and b = 9°, λ_1 + b = 21.38 + 9 = 30.38 > 22°; 05' as required by Zacut.[36]
If we consider a quota of the geographical latitude of–3.85 then λ_4 = 9 + 3.85 = 12.85°, λ_3 = (3/2) * 12.85 = 19.28°, λ_1 = 19.28 + 0.57 + 2.31 = 22.16°

[34] The plain understanding of the texts would require only 22°. The value of 22°; 05' is from Zacut and must probably correct the approximations of the calculations.
[35] HKH 17 :15.
[36] The plain understanding of the texts would require only 22°. The value of 22°; 05' is from Zacut and must probably correct the approximations of the calculations.

Let us now consider the sign of the zodiac of **Virgo** (end). We assume that $\lambda_1 = 10°$ and $\beta_1 = -5°$.
$\lambda_3 = 10° - 37' - (2/5) * (5°+44') = 10 - 0.62 - 0.4 * 5.73 = 10 - 0.62 - 2.29 = 7.09°$
$\lambda_4 = (2/3) * \lambda_3 = 4.73°$ and $b = \lambda_4 + (2/3)\ \beta_1 = 4.73 - (2/3) * 5 = 4.73 - 3.33 = 1.40°$
Note that the term (2/3) β_1 used by Maimonides at the end of the calculation is an approximation and the correct estimation of this term, called the quota of the geographical latitude is $(2/3)\ \beta_2 = -(2/3)\ \beta_2 = -(2/3) * 5.73 = -3.82°$ and then $b = 0.91°$
In fact the minimum value compatible with a visibility is $b = 9°$. We can then calculate λ_1 by the former formula considered in the opposite direction. We depart from $b = 9°$ the minimum admissible value of b.
$\lambda_4 = 9 + 3.33 = 12.33°$, $\lambda_3 = (3/2) * 12.33 = 18.50°$, $\lambda_1 = 18.50 + 0.62 + 2.29 = 21.41°$
Thus $\lambda_1 = 21.20°$ and $b = 9°$, $\lambda_1 + b = 21.20 + 9 = 30.2 > 22°; 05'$ as required by Zacut.
If we consider a quota of the geographical latitude of–3.85° then $\lambda_4 = 9 + 3.85 = 12.85°$, $\lambda_3 = (3/2) * 12.85 = 19.28°$, $\lambda_1 = 19.28 + 0.62 + 2.29 = 22.19°$.

Let us now consider that we are in the beginning of Libra, in northern Israel with latitude of 35°. The influence of the latitude in Maimonides' algorithm is limited to the quota of the geographical latitude if we can recognize that its true expression is $(\text{tang}\ \varphi / \cos \alpha)\ \beta_2$. As the sun is near to the autumnal point (opposite to the vernal point γ) the angle α is near to 23.5° and the quota of the geographical latitude is 0.76 β_2 instead of 0.68 β_2 for a latitude of 32°. It gives a difference of $0.08 * 5.77 = 0.47°$.
Let us depart from $b = 9°$. $(\text{tang}\ \varphi / \cos \alpha)\ \beta_2 = (0.70 / 0.92) * 5.77 = 4.39°$.
$\lambda_4 = 9 + 4.39 = 13.39°$, $\lambda_3 = (3/2) * 13.39 = 20.09°$, $\lambda_1 = 20.09 + 0.57 + 2.31 = 22.97°$
Thus $\lambda_1 = \sim 23°$.

The statement of Zacut that the limit $\lambda_1 = 24°$ is the minimal value of λ_1 allowing giving the guarantee in the area summer – autumn of visibility of the new moon in the absence of climatic impediment, surprised R. Levi ben Haviv who made already this remark, in the harshest terms, in his commentary[37] on *Hilkhot Kiddush ha-Hodesh*. But when we take the factor latitude into consideration we get values of λ_1 close to 24°. The exact calculation shows that we can even overstep the limit of 24°.[38]

Lines [23] until [26].

Zacut explains that if we are in **Libra**, we must subtract from λ_1, 5°; 20' i.e. 2° for *nelizat ma'agal*, the deviation of the path, 2°; 20' for the oblique setting and also the parallax in longitude which amounts in total to 5°; 20'.[39]
We found 34' for the parallax, 2.31° = 2°; 18' for the *nelizat ma'agal* and 2.37 = 2°; 22' for the oblique setting, and in all 5°; 14'. Apparently Zacut makes another decomposition, he considers 2° for *nelizat ma'agal* as if he considered (2/5) * β_1 instead

[37] See in the Vilna edition pp. 171a – 171b.
[38] See below, annex b. See also J. Ajdler, Hilkhot Kiddush al-pi ha-Rambam, p. 501.
[39] There is a certain contradiction between the extant manuscripts about the 20'. In the manuscript of Munich we have 20 but in the manuscript of Vienna we have 2'. The letters ב and כ are similar. In the transcription of this passage in the commentary of R. Levi ben Haviv, we have 20' but here we have another problem: he writes 3°; 20' instead of 2°; 20'an this is certainly a misprint.

of β_2. But then he adds (2/5) * ($\beta_1 - \beta_2$) to the parallax, giving 34' + 18' = 52' for the consequences of the parallax.
Hence: 2° + 2°; 20' + 52' = 5°: 12'. Therefore the readings 2°; 20' for the oblique setting and 5°; 20' for the total, seem the most likely readings, instead of 5°; 02'.
In Aries we should write:
λ_3 = 9° - 59' + (1/3) * (5° + 9') – (2/3) *5 = 9 – 0.98 + (1/3) * 5.15 = 9°– 0.98° + 1.72°
λ_3 = 9.74° and λ_4 = (7/6) * 9.74 = 11.36°.
Thus the numbers to subtract from λ_1 are 0.98 – 1.72 + 1.68 = 0.94° = 0°; 56'. Therefore the reading 1°; 02' in the text of Zacut seems the most likely reading.

[36]

6°; 02' or 6°; 20' according to the readings. In fact the correct reading must be, as we saw above, 5°; 20'. The figure 6° is not understandable and contradicts the former figures.

[47] If the latitude of the place is more than 24°.

The angle between the equator and the southern part of the horizon is 90° - φ.
The angle between the ecliptic and the southern part of the horizon is variable and is given by the formula:
Cos I = cos ε sin φ – sin ε cos φ sin T_s. where T_s is the sidereal time. The angle I varies constantly and during the course of a sidereal day I varies between two extreme values: 90° - φ + ε and 90° - φ – ε. This happens for Ts =90° and Ts = 270°.
Ts =90° then. cos I = cos ε sin φ – sin ε cos φ = cos (90° - φ + ε) and I = 90° - φ + ε.
Ts = 270° then cos I = cos ε sin φ + sin ε cos φ = cos (90° - φ – ε) and I = 90° - φ – ε.
If φ < 23.5° and Ts = 90° then I = 90° - φ + ε > 90° and the parallax in longitude of a celestial body with a northern latitude will become additive, instead of subtractive. This will be valid during the span of time when I = 90° - φ + ε > 90°. Then the length of this span of time will increase when φ diminishes.
The remark of Zacut is an incident remark without a fundamental importance. However R. Levi ben Haviv assaulted Zacut very harshly and claimed, incorrectly, that this happens already for a latitude of 28°; 33' corresponding to 23°; 33' + 5°. Zacut had considered ε = 24° and R. Levi ben Haviv adopted 23°; 33' but this was not the subject of their altercation.

[50] until [55]. *Nelizat Ma'agal*.

Let us refer to figure 3. B' is the point of the ecliptic with the longitude λ' and SB' is the second longitude β_2. D is the point of intersection of the ecliptic with the circle of declination of M'. DB' = $\alpha' - \lambda'$ = *nelizat ma'agal*, literally the deviation of the path. Maimonides gave simplified rules in order to evaluate DB': see HKH 17; 10 – 11. Zacut followed rigorously the same simplified rules. Neugebauer presented these results in a very simplified and convenient table.[40]

[40] See Sanctification of the New Moon, Yale Judaica series XI, p. 141.

[80] and [81]: נוכח מעלת השמש and נוכח מעלת הלבנה .

We want to find λ_4, an arc of the equator corresponding to the oblique setting of λ_3, an arc of the ecliptic. Zacut, as most of his predecessors, used a table of oblique rising. We rest on the relation, demonstrated above: arc of setting (λ) = arc of rising (λ + 180°) – 180°.
We can then write: arc of setting (λ) = arc of rising (λ + 180°) – 180°.
Thus arc of setting (D) = arc of rising (D + 180°) – 180°.
arc of setting (S) = arc of rising (S + 180°) – 180°.
Subtracting these tow relations we get:
Arc of setting (SD) = arc of rising (D + 180°) – arc of rising (S + 180°)
where D + 180° = נוכח מעלת הלבנה. and S + 180° = נוכח מעלת השמש

[82]

We want to calculate the term that must be added to λ_4 in order to get the arc of vision b = WT (see fig 3). Maimonides called it the quota of the geographical latitude.
We see on the figure that AT = M'D * tang φ / cos α. = β_2 * tang φ / cos α..
Tang φ = tang 32 = 0.62.
α varies between 0 and 23°; 35' and cos α varies between 1 and 0.92; average value 0.96.
Hence the quota of the geographical latitude is (0.62 / 0.96) * β_2 = 0.65 β_2.
Maimonides adopted (2/3) β_1 after replacing for an unknown reason β_2 by β_1.

Zacut follows exactly the value adopted by Maimonides although the formula would be more precise with β_2, without any additional difficulty.
Furthermore, Zacut tried in some instances, to generalize the procedure and to make it independent from the location of Jerusalem. Therefore he should also here have put the emphasis on the signification of the coefficient 2/3 and on the importance of tang φ.

Conclusion.

R. Abraham Zacut had a complete understanding of the calculation of the visibility of the new moon. He had even the ambition of generalizing the method to other latitudes.
Zacut represents, because of the exile of Spain, the last generation of the Jewish Spanish astronomers and the end of the Jewish astronomical tradition. In the introduction to his canon, Zacut made an allusion to this tradition and paid homage to his predecessors. He mentioned successively in his introduction, R. Jacob Poel,[41] R. Jacob ibn Tibbon and his almanach,[42] R. Isaac ibn Sid,[43] R. Levi ben Gershon,[44] R. Isaac Israeli,[45] R. Judah ben

[41] Jacob ha-Poel or Jacob ben David Bonjorn of Perpignan, was a famous astronomer. He drew up tables for the year 1361 for the latitude of his town Perpignan.
[42] R. Jacob ben Makhir ibn Tibbon (~1236-1307) author, translator and astronomer. Inventor of a new device, the "quadrans novus" also called "quadrans Judaicus". He played a leading part in in the defense of the philosophy during the controversy over Maimonides in 1304 – 1305.
[43] Second half of the 13th century. Leading astronomer of the team responsible of the Alfonsine tables.
[44] Levi ben Gershon (1288 – 1344) lived in Orange and Avignon in Provence. He was a remarkable astronomer, philosopher and Bible commentator.
[45] R. Isaac Israeli (first half of the 14th century) was a Spanish astronomer living in Toledo. His *Yessod Olam* was written in 1310.

Asher the martyr,[46] R Isaac Alhadib[47] and R. Judah ibn Verga.[48] Happily the Hebrew canon of Zacut fell in the hands of R. Levi ben Haviv, as it is mentioned in the monograph entitled *Derekh ha-Kodesh* that he wrote at the end of his responsa and which was printed partially in the Vilna edition of Rambam. It is through this way that the achievements of the Jewish astronomical Tradition in Spain in the understanding of the astronomical chapters of Maimonides' *Hilkhot Kiddush ha-Hodesh,* was transferred to the rabbinical tradition and was not forgotten. It is also through this process and thanks to the development during this period of the printing, that this astronomical knowledge became known by the whole rabbinical and intellectual Jewish society.

4. History of the evolution of the understanding of the astronomical chapters of *Hilkhot Kiddush ha-Hodesh*.

~ 1276. Publication of the canon of the Alfonsine tables written by Isaac ibn Sid ans Moses Cohen.
Thanks to the use of a technical vocabulary and of extant astronomical tables, the text devoted to the visibility of the new moon in chapter 36 of the canon is clear and concise and proves, above any doubt, the perfect comprehension of Maimonides' algorithm. In fact most of Maimonides' empiric tables were replaced by astronomical tables, giving at the same occasion the real meaning of the operation. The only question that remains is why did Isaac ibn Sid follow the algorithm of Rambam, using a table of deviation (nelizat ma'agal) and the term of the geographical latitude and didn't he calculate the right ascension of the point of the equator setting together with the apparent moon, as would do Raphael Levi from Hanover 450 years later. By contrast with the generally accepted idea, the ancients knew how to calculate the equatorial coordinates of a celestial body not on the ecliptic.[49] The answer is probably that these calculations were too intricate. The trigonometric lines were still given in sexagesimal notation. They did not know the use of logarithms which made possible the calculation of multiplications and divisions. This seems the main reason why only in the eighteenth century, astronomers drew up tables giving $\Delta\alpha$ and $\Delta\delta$ for celestial bodies with a latitude with regard of celestial bodies without latitude.

~ 1310. R. Isaac ben Joseph Israeli and the book *Yessod Olam*.

This book was considered the most important contribution to Hebrew literature in the field of astronomy. The book gives a very clear representation of the stage of astronomy at that time and explains the fundaments of the astronomy of Ptolemy with the improvements brought by the observations of Al-Battani and Arzachel. The book includes a chapter about the visibility of the new moon, but in fact, this book does not refer to the chapters of Maimonides ' *Hikhot Kiddush ha-Hodesh* and does not constitute

[46] Great grand-son of the Rosh. He was Rabbi of Toledo, astronomer and had interest in philosophy. He died as a martyr in 1391 during the riots. He was a close friend of R. Isaac bar Sheshet Perfet. See details in Rabbi Isaac bar Sheshet by R. Abraham Hershman, Mossad ha Rav Kook, 1956, p 109.
[47] Rabbi, poet and astronomer of Spanish origin (~ 1370 – ~ 1427) who lived in Sicily from 1396 onwards.
[48] First half of the 15th century.
[49] See below in the annex a.

a commentary or an elucidation of it. It gives nevertheless important and clear information about the movement of the moon and sun in Ptolemaic astronomy.

~ 1341. R. Ovadia ben David wrote at about this time the first commentary on *Hilkhot Kiddush ha-Hodesh*.
The commentary was illustrated by numerous figures. Unfortunately the printers, probably because of the difficulty to reproduce them, omitted them from the successive editions. Moreover these figures are generally difficult to understand because of their primitive character, the absence of a perspective representation, because of their incapacity to represent it or to reproduce it. The text is long and often unclear and difficult to understand. The author committed an important mistake; he understood that the fourth longitude, an arc of the equator, represents the complete moonset lag and the arc of vision is the corresponding arc of the altitude circle. The quota of the geographical latitude would thus allow passing from λ_4, an arc of the equator to b an arc of the circle of altitude. If we refer to figure 3, we see that this is impossible. Indeed, he understands that λ_4 is the arc WT. Now if β_1 is + 5° then we must add 3.33° to λ_4 while in fact the arc of the altitude circle is WT * cos φ; it should be less than λ_4. The commentary of R. Ovadia remains the standard commentary and noted authors like R. Levi ben Haviv, R. Mordekhai Jaffe, R. Solomon Demedigo (manuscript) and even R. Jonathan of Raznai in the beginning of the eighteenth century, continued to analyze and discuss his statements.

~ 1478. The canon of R. Abraham Zacut.

R. Abraham Zacut addressed the problem of the visibility of the moon in chapter 10 of his canon. He was certainly aware of the writing of Isaac ibn Sid but his treatment of the subject is much closer to Maimonides' algorithm than the report of ibn Sid. From the other side, Zacut considered the generalization to other latitudes and the seeing of the old moon. Furthermore he discussed some data given by Maimonides and questioned one of them. He introduced also the notion of equation of time in order to transform the time of sunset into mean time for the calculation of the coordinates of moon and sun. The works of ibn Sid and Zacuto were intended mainly for astronomers, professional or lovers. Nevertheless, the work of Zacut, written in Hebrew, had certainly a greater distribution among Jewish intellectuals.

~ 1565. Responsa of R. Levi ibn Haviv (~ 1483 – 1545).

R. Levi ben Haviv is the second[50] rabbinic authority who wrote a commentary on *Hilkhot Kiddush ha-Hodesh*.
R. Levi ben Haviv was a Talmudic scholar interested in astronomy who devoted a monograph to the commentary of *Hilkhot Kiddush ha-Hodesh*. Fortunately he got acquainted with the book of Zacut, as he testified himself, when he claimed on two occasions, that Zacut was wrong. It allowed him to write the most advanced and perfected commentary on *Hilkhot Kiddush ha-Hodesh*. It is on this manner that the Jewish astronomical tradition of Spain entered the Jewish public intellectual property instead of remaining confined among astronomers.

[50] We don't know anythink about R. Ovadia; was he a rabbinical authority or an astronomer connoisseur?

~ 1738[51] – 1756.[52] Raphael Levi from Hanover.

When we examine the works of the authors following R. Levi ibn Haviv until the eighteenth century, more particularly R. Mordekhai Jaffe, R. Solomon Demedigo (manuscript), R. Jonathan of Radunia we note no progress at all. All these authors tried again and again to express in better words the different notions considered. Unfortunately, without a mathematical support and clear figures, these texts remained unclear and often difficultly comprehensible. Raphael Levi from Hanover made decisive progress in the understanding of the methods of calculation proposed by Maimonides, whether in the calculations of the coordinates of the sun and moon or in the performance of the visibility calculation. In his *Luhot ha-Ibbur*, he introduced tables giving the equatorial coordinates of the moon when it is not on the ecliptic and allowing to performing easily the exact calculation of the moonset lag without Maimonides' algorithm. He reached in the elaboration of his tables an unmatched precision thanks to s systematic use of logarithms.

~1898 – 1903: Baneth, E., Maimuni's Neumondsberechnung.

Important contribution to the study and the understanding of Maimonides' *Hilkhot Kiddush ha-Hodesh.* For the reader aware of the work of Hanover, the book has not a great originality. The book gives nevertheless a detailed and original justification of all the parameters adopted by Maimonides in his algorithms. Baneth, like Hanover, thought that Maimonides had proposed new parameters for the movement of the sun, leading to a length of the tropical year of an unequalled precision. The book is precise but it is difficult and therefore not available to anyone.

1949. Neugebauer, O. The Astronomy of Maimonides and its sources.

The author tried to explain the methods of calculation used by Maimonides. He showed that Maimonides followed Al-Battani faithfully but some of his parameters are the result of the use of Al-Battani's data and of their rounding off. He tried explaining the table of the components in longitude and latitude of the parallax and apprehending their difficulties but he did not succeed solving the problem, which remains a conundrum.

5. Annex.

5a. The ancients knew how to find the equatorial coordinates from the ecliptic coordinates.

The classical formulas of transformation

$\text{tang}\ \alpha = [\sin \lambda \cos \varepsilon - \text{tang}\ \beta \sin \varepsilon] / \cos \lambda$

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[51] Date of the completion of the manuscript of *Tekhunat ha-Shamayim*.
[52] Date of the publication of *Luhot ha-Ibbur*, Vol 2.

$\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda$.

were not known before the seventeenth century, after that FrançoisViete had published in 1593 the fundamental formula of spherical trigonometry for the ordinary spherical triangles. Nevertheless the problem of the transformation of coordinates was already solved theoretically by Ptolemy in the Almagest,[53] by Al-Battani[54] and in the Alfonsine tables.[55]

In the spherical triangle γBC, rectangle in B: $\tan \beta' = \tan \varepsilon \sin \lambda$. Hence β'.
The ancients could find β' through the table of the rising times at sphaera recta used in the opposite direction. They looked for the longitude corresponding to a given right ascension equal to λ. With this right ascension they could find the declination in the table of the declination of the points of the ecliptic.

In the spherical triangle MPC rectangle in P: $\sin(\beta + \beta') = \sin MP / \sin C = \sin \delta / \sin C$.
In the spherical triangle CDQ' rectangle in D: $\sin C = \sin DQ' / \sin Q'C = \cos \varepsilon / \cos \beta'$
Thus $\sin(\beta + \beta') = \sin \delta * \cos \beta' / \cos \varepsilon$ and $\boldsymbol{\sin \delta = \sin(\beta + \beta') \cos \varepsilon / \cos \beta'}$.

In the spherical triangle Q∏M:
$\gamma B = \lambda$, $MB = \beta$, $BC = \beta'$, $MP = \delta$, $Q\Pi = \varepsilon$, $EF = \varepsilon$, $\gamma P = \alpha$, $BE = 90° - \lambda$, $\Pi M = 90° - \delta$.
$\text{Sin}\ \Pi MQ / \sin \varepsilon = \sin \Pi QM / \sin \Pi M = \cos \lambda / \cos \delta$.
$\text{Sin}\ \Pi MQ = \sin \varepsilon \cos \lambda / \cos \delta$.
In the spherical triangle MPC rectangle in P:
sin PC = sin sin PMC sin MC. $PC = \lambda - \alpha$, PMC= ∏MQ and $MC = \beta + \beta'$ thus
$\sin(\lambda - \alpha) = \sin PMQ * \sin(\beta + \beta') = \sin \varepsilon \cos \lambda \sin(\beta + \beta') / \cos \delta$
$= \sin \varepsilon \cos \lambda \sin \delta \cos \beta' / \cos \delta \cos \varepsilon$.
Therefore $\boldsymbol{\sin(\lambda - \alpha) = \tan \varepsilon \tan \delta \cos \lambda \cos \beta'}$.

5b. Two examples of visibility calculation.

1. Let us consider the case $\lambda_0 = 170°$, $\lambda_(= 192.5°$ and $\beta_(= -5°$ with $\varepsilon = 23.5°$ and $\varphi = 32°$.
We find then $\alpha_0 = 170.81°$, $\delta_0 = 3.97°$, $\Delta_0 = 2.49°$ and the point of the equator setting at the horizon together with the sun: $\alpha_0 + \Delta_0 = 173.30°$.
The moon is in Libra and therefore Δλ = 0°; 34' = 0.57° and Δβ = 0°; 46' 0.77°
We have then $\lambda'_(= 192.5 - 0.57 = 191.93°$ and $\alpha'_(= 188.67°$
$\beta'_(= - 5 - 0.77 = -5.77°$ and $\delta'_(= -10.05°$, $\Delta'_(= -6.36°$
The point of the equator setting at the horizon together with the apparent moon is:
$\alpha'_(+ \Delta'_(= 182.31°$.
The arc of vision is then $(\alpha'_(+ \Delta'_( ) - (\alpha_0 + \Delta_0) = 182.31 - 173.30 = 9°; 01'$.
This confirms the simplified calculation, made according to Maimonides' algorithm, in the paper and proving that a first elongation of 22.5° is sufficient to guarantee the

53 Toomer chapter II and in a more intelligible terms: Pedersen, O., A survey of the Almagest, pp. 95 – 99.
54 Al-Battani Vol I pp. 31 – 32: calculating the declination from the longitude and the latitude.
55 Chapter 39: calculating the declination from the longitude and the latitude.

visibility of the new moon when the moon is in Libra. This was the argument of R. Levi ben Haviv against Zacut in his *Derekh ha-Kodesh*

2. Let us consider the case $\lambda_0 = 170°$, $\lambda_(= 194°$ and $\beta_(= -5°$ with $\varepsilon = 23.5°$ and $\varphi = 35°$.

We find then $\alpha_0 = 170.81°$, $\delta_0 = 3.97°$, $\Delta_0 = 2.79°$ and the point of the equator setting at the horizon together with the sun: $\alpha_0 + \Delta_0 = 173.60°$.

The moon is in Libra and therefore $\Delta\lambda = 0°; 34' = 0.57°$ and $\Delta\beta = 0°; 46' = 0.77°$

We have then $\lambda'_(= 192.5 - 0.57 = 191.93°$ and $\alpha'_(= 190.07°$

$\beta'_(= -5 - 0.77 = -5.77°$ and $\delta'_(= -10.62°$, $\Delta'_(= -7.55°$

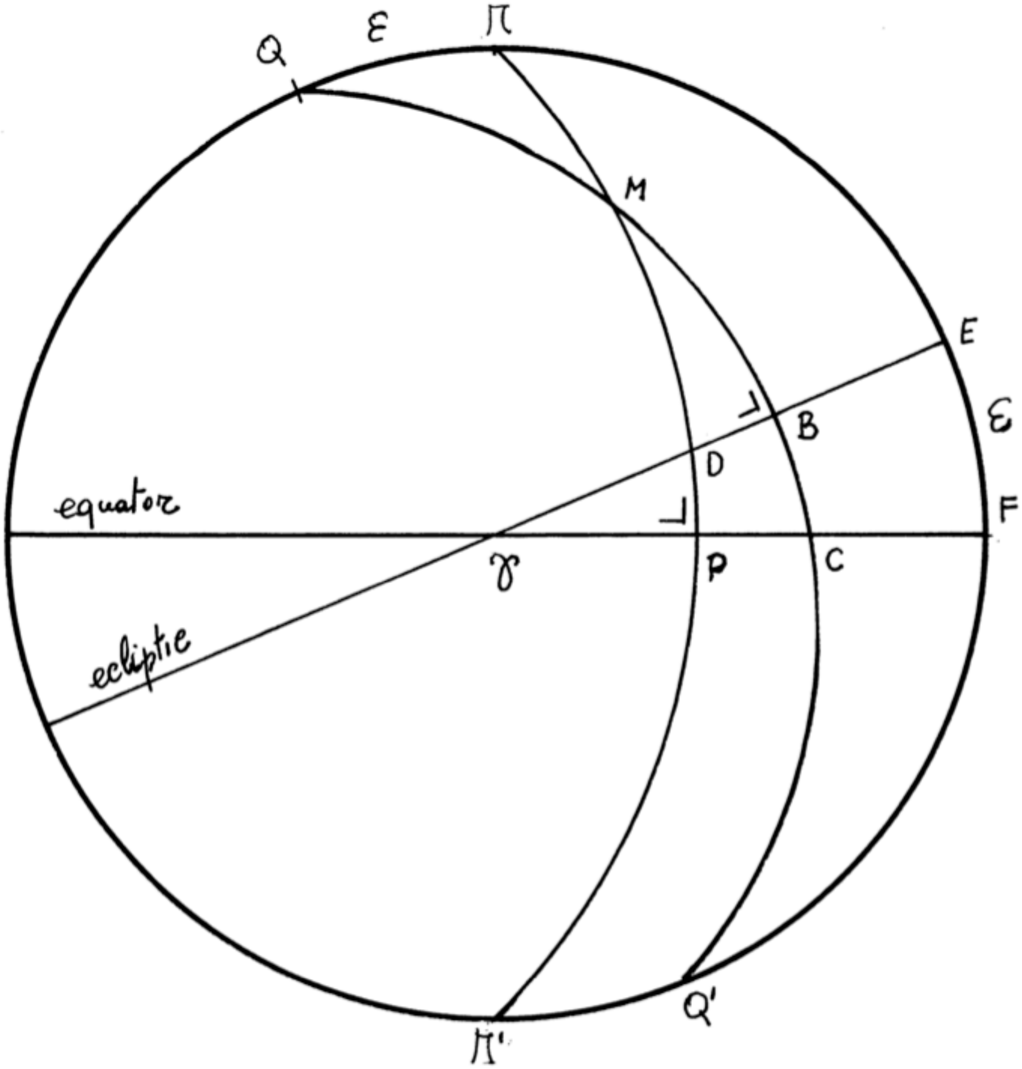

Figure 4. Transformation from ecliptic to equatorial coordinates. The plane of the page contains ∏, the pole of the equator, Q, the pole of the ecliptic and O, the center of the celestial sphere. This plane is perpendicular to the intersection of the equator and the ecliptic, the line joining the vernal and autumnal points, γ on the figure. M is the celestial body, γP is its right ascension α and MP is its declination δ. γB is the longitude λ and MB is the latitude β. C is a point of the equator which has the same longitude γB as the celestial body BC is an auxiliary arc called β'. Arcs Q∏ = EF = ε.

The point of the equator setting at the horizon together with the apparent moon is: $\alpha'_{(} + \Delta'_{(} = 182.52°$.
The arc of vision is then $(\alpha'_{(} + \Delta'_{(}) - (\alpha_0 + \Delta_0) = 182.52 - 173.60 = 8°; 93'$.
There is no vision of the new moon. It is only for a first elongation of $\lambda 1 = 24.21°$ that we will get an arc of vision of 9°.

This is probably the justification for the limit of 24° introduced by Maimonides and the approbation given by Zacut: the calculations of visibility are valid for all Israel from $\varphi = 29°$ until $\varphi = 35°$. This fact could have escaped to R. Levi ben Haviv. Moreover we have shown that the ancients could in principle perform the exact calculation because they knew how to find the equatorial coordinates from the ecliptic coordinates. The difficulty that they faced was not theoretical; it was only practical because of the length and the difficulty of the calculations. But the calculation was not insurmountable.