The Day and its Hours in Ancient Jewish Astronomy. Elucidation of Medieval Rabbinic Astronomical Quotations.

There are different ways of defining the span of time of one day.
It appears that people, along the history, did not use a unique definition.
According to their cultural background they counted the days differently. The astronomers had still their own way of counting days: it was from noon of one day until noon of the following day.
Furthermore the different authors considered that their own definition of the day was so evident that they generally did not think necessary to specify the type of day that they used and its exact definition. It is clear that the diversity of the possibilities makes sometimes the elucidation of ancient texts difficult.
We selected four sets of quotations of celebrated medieval rabbinic astronomers which are at the first glance problematic.

- R' Abraham bar Hiya, who appears to be the great specialist of Al-Battani and Ptolemy, defined his astronomical day n as beginning at noon of the Jewish civil day n – 1 and ending at noon of the Jewish civil day n. This definition seems disconcerting and in contradiction with the definition of the astronomical day which was in use from Ptolemy until the modern times.
- R' Isaac Israeli, in his book *Yessod Olam*, written in 1310 C.E. in Toledo, in Christian Spain, adopted the same position as bar Hiya.
- R' Abraham ibn Ezra, in his *Sefer ha-Ibbur*, made some statements about the moment of occurrence of the apparent vernal equinox in the year 1147 in the city of Verona in Italia and about the span of time between the apparent equinox and solstices on the one hand and the four tekufot of Adda on the other hand.
- R' Judah ha-Levi in his book *Sefer ha-Kuzari* explained that the Jewish calendar is regulated in such a way that Passover never begins before the [mean] equinox according to the calculation of Rabbi Adda, which, he wrote, is supported by the observation of Al-Battani.

We succeeded elucidating completely the apparent conundrum of the two first sets of quotations. It appears that the two Jewish astronomers followed the Arab use of counting the astronomical days by contrast with the use of Ptolemy, Al-Battani and the Christian astronomers. The Arab and Jewish astronomers considered astronomical days of the type (n – 1, n) beginning at noon of the preceding day and ending just before noon of the current day.
However the quotations of Ibn Ezra are more problematic and no satisfactory solution could be proposed. In fact it seems that he did not use the tables of Al-Battani but rather other tables like those of Al-Khwarizmi or more likely the Toledan tables of Abraham ibn Zarkali.
In the fourth quotation, R' Judah ha-Levi was apparently not worried about the shift of the tekufah of Adda with regard to the astronomical equinox. It is likely that, similarly to other Jewish astronomers, he accepted the coexistence of contradictory data, the experimental equinox of Al-Battani and the tropical year of Ptolemy, slightly equal to the length of the year of Rabbi Adda and longer than the year of Al-Battani.

The Day and its Hours in Ancient Jewish Astronomy.
Elucidation of Medieval Rabbinic Astronomers' Quotations.

I. Introduction: Problematic Quotations from Abraham bar Hiya, Abraham ibn Ezra and Isaac Israeli.

1. Sefer Mahalekhot ha-Kokhavim by R' Abraham bar Hiya.

In Sefer Mahalekhot ha-Kokhavim p. 62, it writes:

הוי יודע כי היום הנחשב בספר הזה אנו נותנים ראשיתו מחצית יום אתמולו, והוא עת היות החמה בחצי השמים על הארץ ביום אתמול שעבר עד היות בחצי השמים ביום הזה. כגון שאנו מונים שעה ראשונה מיום ראשון מהשעה השביעית מיום השבת שעבר ואומרים אותו לחשוב בסוף שעה ששית מיום ראשון וכן המנהג לנו בכל ימי השבוע וראש היום שהתחלנו ממנו לחשוב מהלכות הכוכבים בספר הזה היה סוף שעה ששית מיום רביעי כ"ט יום מירח אלול שנת ד' אלפים תתס"ד לעולם ואחרית היום ההוא סוף השעה הששית מיום חמישי שהיה יום ראשון לחדש תשרי שנת ד' אלפים תתס"ה על אופן חצי השמים במקום הרחוק ממחצית הארץ לפאת מערב שעה אחת וחצי ומרחקו מקצה מזרח שבע שעות וחצי, וזהו מרחק אמצעית ארץ ישראל באורך...............

It was generally accepted that the astronomical day "Sunday" begins on Sunday at noon and ends on Monday just before noon. Similarly the astronomical day Thursday 22 September 1104 begins on the civil day Thursday 22 September 1104 and ends on Friday 23 September 1104 just before noon.

In contrast with these generally accepted data, R'Abraham bar Hiya writes now that the astronomical day "Sunday" begins on Saturday at noon and ends on Monday just before noon and similarly he writes that the astronomical day Thursday 1 Tishri 4865 begins on Wednesday 29 Elul 4864 at noon and ends on Thursday 1 Tishri 4865 just before noon! What is the origin of this apparently revolutionary assumption?[1]

[1] The solution of this conundrum was the subject of fierce discussions with Engineer Y. Löwinger during May and June 2009. I thank him very much, because, as it is well known: אין בית המדרש בלי חידוש.The consultation of authorities like Profs. Noel Swerdlov from Chicago, Hogendijk from Utrecht and Solomon Langerman from Jerusalem did not allow sustaining the possibility that some Arab astronomers used astronomical days n of the type (n – 1, n) beginning at noon of the civil day n – 1 and ending just before noon of the day n. Engineer Jacob Löwinger noted that such a practice would make sense because this astronomical day (n – 1, n) would have 18 hours in common with the Arab civil day. Eventually the discussion turned about the understanding of Al-Battani. Löwinger championed the principle that Al-Battani used the astronomical day n of the type (n – 1, n) and he justified this principle by the fact that his epoch is at noon of the last day of the preceding year, thus 28/29 February. This epoch is then, necessarily he says, the beginning of the first astronomical day of March; it begins at noon of February 28/29. I proved vainly that the observation of four eclipses by Al-Battani and the way on which he defined them, prove that the astronomical days are of the type (n, n + 1). The discussion brought no solution: Löwinger championing that Al-Battani used always astronomical days of the type (n – 1, n) and was the authority that Bar Hiya followed. For me this answer is to simplistic; it rests on incorrect basis and it does not solve the problem (it

2. Sefer Yessod Olam.

In *Sefer Yessod Olam* book II, chap. 10, p. 26d, at the bottom, it writes:

והרגע הד' הוא עת חצות היום במקום מקום ידוע כלומר משיגיע מרכז החמה לחנות על אופן חצי היום ממעלה ועד שישוב שנית לשם ועי"ז הסכימו רוב המחשבים למהלך הכוכבים לשום אותו תחילת כל יום מימי השבוע וראשיתו כלומר שיום שבת ע"ד משל לפי סברתם יחשבו אותו מרגע חצות יום ששי במקום ידוע ועד רגע חצות יום שבת לשם וכן ה"ה בשאר ימי השבוע ר"ל שכל מיום מהם יתחילו אותו בחשוב זה מרגע חצות יום אתמולו.............

Isaac Israeli explains that most of the astronomers define the astronomical day as beginning at noon of the preceding day and ending on the present day just before noon. Again this assumption is in contradiction with the generally accepted definition of the astronomical day since the definition adopted by Ptolemy in the Almagest.[2]

3. Sefer ha-Ibbur by R' Abraham ibn Ezra.

In *Sefer ha-Ibbur*[3] of R' Abraham ibn Ezra, on p. 9a, at the bottom, it writes:

......ובעבור ששנת הגוים כשנות שמואל אזכור דבריהם כי הנה בשנה החמישית למחזורינו שהיא שנת תתק"ו[4] לבריאת העולם עם ארבעת אלפים תהיה תקופת ניסן על דעת שמואל בתחילת יום ד' כ"ג יום ..... בניסן, והנה יהיה ט"ו יום לחודש בי"ח יום לחודש מרסו והשמש לא נכנסה עוד בטלה כפי חשבון הגוים, ע"כ הוצרכו להיות מועדם אחר חצי חדש האחר ולפי תקופת רב אדא כאשר הראיתיך בשער הראשון תהיה התקופה ביום ראשון של חג המצות וזאת התקופה היא כנגד גלגלה גם היא צריכה שני תיקונים כי תקופת האמת תהיה בשעת ז' מיום ששי שהוא י"א בניסן והעד כלי הנחשת למראה עינים גם בצל כי אז יהיה בוירונא בלונברדיאה שאני דר בה היום, צל כל דבר כמוהו בעבור..

On p. 10a row 21, it writes

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contradicts the dates of the eclipses observed by Al-Battani). I proved that Al-Battani, without any doubt used astronomical days of the type (n, n+1). Afterwards I had the opportunity to consult Prof. Georges Saliba, Professor of Arabic and Islamic science at the University of Columbia. He confirmed Löwinger's intuition that generally Arab astronomers used astronomical days of the type (n – 1, n). The solution to our problem would then be that R' Abraham bar Hiya and R' Isaac Israeli followed the general Arabic practice and considered astronomical days of the type (n – 1, n). However Al-Battani followed the Christian practice and considered astronomical days of the type (n, n+1) because he gave the precedence to his Roman tables constructed on the Julian calendar. Afterwards I found new evidence of the existence of two different practices: the Arabic practice of astronomical days of the type (n – 1, n) and the Christian practice of astronomical days of the type (n, n+1).

[2] See the Almagest, Toomer p. 170 bottom.

[3] This book was written in 1147. The book was edited by Solomon Zalman Halberstamm and printed in Lyck in 1870 by the *Hevrat Mekizei Nirdamim* with a historical introduction by the editor. The transcription of manuscript Ouri 438, which contains only two chapters, was made in Oxford in 1852 by Baer Goldberg (1800-1884); He is mentioned at the end of the book by his initials. Apparently the third chapter is no more extant.

[4] The figure 4906 must be corrected to 4907. The editor noted it already in the list of corrections.

והנה היום תקופת תשרי באמת היא תקופת רב אדא בעצמה ולא כן תקופת ניסן כי יש בין שתי התקופות ד' ימים ובין תקופת תמוז ותקופת טבת ב' ימים ושעות ....

These two quotations raise different problems related to the exact moment of the Tekufah of Adda which occurred in the year 4907 on the eve of Pessah at about 4 p.m. and not during the first day of Pessah.
The common point of these different quotations is that they deal with the notion of day and seem to define the day differently than us. This will be the subject of this paper, defining the notion of day and examining whether R' Abraham bar Hiya followed other Arab astronomers or innovated alone in adopting this new assumption for the definition of his astronomical day.

II. The definition of the day and its hours in Ancient astronomy.

1. The beginning of the day.

Al-Biruni in the beginning of his book[5] , R' Abraham bar Hiya in Sefer ha-Ibbur[6] and R' Isaac Israeli in his book Yessod Olam[7] examined, all the three, which moment is the best for the beginning of the day. The day can begin at sunset; this is the practice of the Jews and the Arabs. It can also begin at sunrise; this was the practice of the Romans and the Greeks. These days have a big disadvantage: their length changes each day. For this reason the astronomers preferred to choose another beginning. They had two possibilities; they could choose midnight when the sun is on the inferior meridian. This solution corresponds to our modern practice for the civil day and also for the astronomical day since 1925. Al-Biruni mentioned that only one Arab astronomer adopted this assumption for his astronomical days.
The last possibility is to begin the day at noon. This last solution was adopted by all the ancient astronomers and chronologists. This explains why the Julian days of the Julian period begin at noon. The existence of different concurrent systems for counting the hours of the day and defining the limits of the day is the origin of a great confusion when defining a moment of the day. Giving a certain moment of the day becomes a delicate operation making it necessary to give a few precisions in order to make sure that there is no doubt about this precise moment. Besides the problem of the exact day and the moment in this day there can also arise doubts about the exact considered year and the exact epoch of the chosen era.

2. The civil day and the astronomical day.

The civil day begins today at midnight. The civil day of the Greeks and the Romans began in the morning, at sunrise or perhaps at the break of the day. The civil day of the Jews and the Arabs begins at sunset. This day is not very practical for astronomical observation and report. The tradition, since Ptolemy, and probably before, was to refer to

[5] The chronology of Ancient Nations, Al- Biruni, English translation by Edward Sachau, London 1879, p. 6.
[6] Sefer ha-Ibbur, edition Filipowski,pp. 24-27.
[7] Yessod Olam II, chap 10 pp. 26 c and d.

the astronomical day beginning at noon and ending just before noon the next day. This day has a length of about 24h. The length of a day can vary with about maximum 30s with regard to its mean length of 24 hours. The general use is to consider an astronomical day n beginning at noon of the civil day n and ending just before noon of the day n+1. Thus the astronomical day "Sunday" begins at noon of Sunday and ends just before noon of Monday. This system was used apparently by the astronomers, under the influence of Ptolemy, during the two last millenaries. It is only in 1925 that the astronomers decided to adopt the civil day from midnight to midnight.
It is thus striking to read that R' Abraham Bar Hiya wrote his book and his tables under the assumption that the astronomical day n begins at noon of the civil day n – 1 and ends just before noon of the day.

3. The different eras used in astronomy and chronology.

Era	Weekday	1st day of era	Julian Date	JD	Type year
Nabonassar	Wednesday	Toth 1	-746, February 26	1448638	Egyptian
Philip	Sunday	Toth 1	-323,November12	1603398	Egyptian
Seleucid West	Monday	Tishri 1	-311, October 1	1607739	Luni-Solar
Seleucid East	Wednesday	Nissan 1	-310, April 3	1607923	Luni-Solar
Contracts[8]	Thurday	Tishri 1	-311,Sepember 6	1607714	Jewish
Spanish[9]	Sunday	Jan 1 [10]	-37, January 1	1707544	Julian
Diocletian[11]	Friday	Toth 1	284, August 29	1825030	Alex.Egypt
Hijra	Thursday[12]	Muhar.1	622, July 15[13]	1948439	Lunar
Yasdegerd[14]	Tuesday	Farward1	632, June 16	1952063	Egyptian

[8] The data are calculated according to the modern calendar's rules, although the fixed calendar was not yet instituted.
[9] This era is used by R' Isaac Israeli in his book *Yessod Olam*; he calls it תאריך אל צפר. See for example *Yessod Olam, Maamar* IV, chap p. 17, p. 31c and d.
[10] There were certainly other styles. Savasorda and, in all likelihood, Israeli began their Julian years on October 1. This explains that the birth of Jesus is given by Israeli in this chapter as 25 December of the year 39 according to the Spanish era or Saturday 9 Tevet 3761. 39 of the Spanish era is the year 1 CE; this implies that the beginning of the year was October 1. Otherwise December 25 would still belong to the year 1 BCE or 38 Spanish era.
[11] This is the era of the Copts used by Al-Battani.
[12] According to the fixed Arabic calendar of the astronomers.
[13] According to the fixed Arabic calendar of the astronomers while according to the Historians the first day of the era of the Hijra was Friday 16 July 622, see O. Neugebauer *A History of Ancient Mathematical Astronomy* part 3, p. 1066 note 4. He writes indeed that 622, July 15 is the norm adopted by the astronomers while historians use July 16.
This was already the position of Delambre in his *Histoire de l'Astronomie du Moyen-Age* p. 95 who wrote that the first day of the Muslim era was a Thursday, according to the astronomers. This fits an old Arab tradition according which Mahomet entered victorious the town of Medina on the 67th day after the first day of the era of the Hijra and met there the Jews who were fasting; it was Yom Kippur which occurred on 20 September 622. See: Memoire sur le Calendrier Arabe, par Mahmoud Effendi, astronome égyptien, Paris 1858.

The second column gives the weekday of the first day of the era.
The third column gives the date of the first day of the era.
The fourth column gives the Julian date of the first day of the era (civil day).
The fifth column gives the number of elapsed days of the Julian period at noon of the first day of the era. All the days are civil days.

4. Ptolemy and the Almagest (~100 - ~170).

Ptolemy in The Almagest uses Egyptian or vague years of 365 days. The counting of the years is made according to the era of Nabonassar. The epoch of this era is noon of 26 February – 746; at this moment 1,448,638 days of the Julian Period had elapsed. The epoch was the beginning of the first day of the era, it was 1 Toth. In the Almagest, Ptolemy constructed a table of mean conjunctions. The first mean conjunction in the table is the first conjunction after the epoch. This conjunction occurred 23 d, 44' 16" or 23d + 17h + 42m + 24s after the epoch. We can also say that the conjunction occurred on Toth 24, at 17h 42m 24s. Ptolemy used this second principle in his table of conjunctions, noting Toth 24, 44' 16". This notation corresponds to the current year, the current month, the current day and the completed hours, minutes and seconds. Toomer calls this notation the "*inclusive* notation."[15] In the first notation 23d, 44' 16" we mention the current year, the current month, the completed days and the completed hours, minutes and seconds; it corresponds to the "*exclusive* notation."[16]

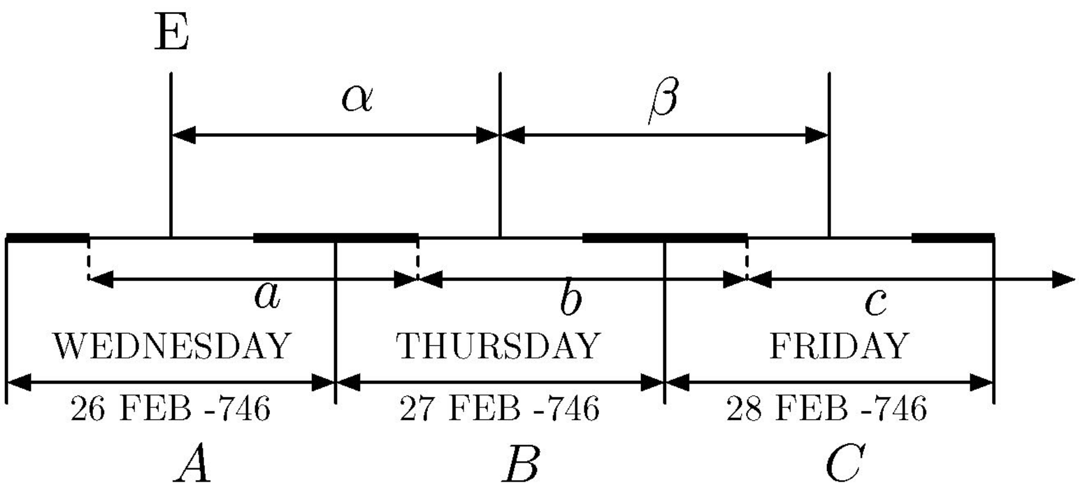

Table 1 **<u>The beginning of the era of Nabonassar according to Ptolemy.</u> The fine line represents the day while the thick line represents the night. E = Epoch = Noon of Wednesday 26 February -746 = JD 1448638. A, B and C represent the modern civil days counted from midnight to midnight; a is the Greek civil day Wednesday 1 Toth, 1 Nabonassar, b is the civil day Thursday 2 Toth, 1 Nabonassar and c is the Greek civil day Friday 3 Toth, 1 Nabonassar. α is the astronomical day Wednesday 1 Toth, 1 Nabonassar or Wednesday 26 February – 746 and β is the astronomical day Thursday 2 Toth, 1 Nabonassar or Thursday 27 February – 746.**

[14] There is an old tradition already mentioned by Al-Biruni that there are 3624 days between the era of the Hijra and the Persian era of Yasdegerd: 1948439 + 3624 = 1952063.
[15] The current day included. See Ptolemy's Almagest, G..J.Toomer p.275-276.
[16] The current day excluded. See Ptolemy's Almagest, G..J.Toomer p.275-276.

This problem exists also in the notation of the Jewish Molad. The traditional notation of the type (1) – 4 – 495 is a non homogeneous notation in which the day is the current day while the hours and halakim are completed; it corresponds to the *inclusive* notation adopted by Ptolemy. A notation 0 – 4 – 495 in which all the elements are completed elements would correspond to the *exclusive* notation. We see thus that the Jewish meabrim adopted the *inclusive* notation, probably under the influence of the Almagest.

We note thus that the epoch of the era of Nabonassar is at noon, the beginning of Toth 1 of the first year of the era of Nabonassar. Similarly the epoch of each year is at noon, the beginning of Toth 1 of this year. The astronomical days used by Ptolemy were of the type (n , n + 1).

5. Al-Battani (~858 – ~929).

The importance and the influence of Al-Battani on the astronomers who followed him was considerable. His work is a new Almagest, an improved Almagest. The dependence of Maimonides on Al-Battani had already been recognized by R' Obadia ben David, the classical commentator of Hilkhot Kiddush ha-Hodesh.[17] In a paper published in BDD,[18] I showed that the radices of Maimonides were calculated with the highest precision from the tables of Al-Battani, taking into account the problem of the equation of time. In the same paper I showed that the epoch and the radices of the tables of Abraham bar Hiya were also deduced from the tables of Al-Battani. Al-Battani had thus a considerable influence on Jewish astronomy. Therefore we must devote him a special attention and examine carefully how he calculated his astronomical days. Furthermore we must take into account the privileged tie between Al-Battani's work and R' Abraham bar Hiya. In his quality of specialist of Arab language and astronomy he was a collaborator of Plato of Tivoli in the Latin translation Al-Battani's work. It is however certain that R' Abraham bar Hiya and R' Isaac Israeli knew the extant astronomical works, whether in original Arab or in Latin translation. It is then likely that R' Abraham bar Hiya could not ignore the works of Arzachel (R' Abraham ibn Zarkali)[19] and of Al-Kwarismi.[20] Al-Battani's tables were complicated because they were based on the full theory of Ptolemy. His work was even considered as an improved Almagest. It is probably for this reason that the old, less perfect but handier tables of Al-Khwarizmi were reedited by the Spanish astronomer Maslama ibn Ahmed[21] and transferred to the meridian of Cordoba and completed by an excerpt from Al-Battani. This book, the Maslama's version, was later translated into Latin and adapted in about 1116 by the Spanish scholar Mose Sefaradi (1062-1110)

[17] See his commentary on Hilkhot Kiddush ha-Hodesh XII; 1.
[18] BDD n° 16 August 2005: The equation of time in ancient Jewish astronomy.
[19] 1029- ~1087; Abu Ishak Ibrahim ibn Yahya al-Zarkali.
[20] Second half of the 8th and beginning of the 9th century.
[21] Spanish astronomer of Cordoba; died in 1008

converted to Christianity[22] under the name Pedro Alfonso or Petrus Alfonsi in Latinized form[23] and later in 1126 it was completed by Athelhard of Bath.[24]

A. Special eras used by Al-Battani.

Era	Weekday	1st day of era	Julian Date	JD	Type year
Dhul qarnayn[25]	Friday	Toth 1	-311,November 9	1607778	Egyptian
Dhul qarnayn[26]	Saturday	September1	-311,September 1	1607709	Julian
Dhul qarnayn[27]	Friday	March 1	-310, March 1	1607890	Julian

B. The time of Al-Battani's mean conjunctions have the same significance whether calculated in Roman years or in Egyptian years.

Let us consider the following date: Toth 1 of the year 1448 of Nabonassar. At noon of this day i.e. at the beginning of this astronomical day the number of elapsed day of the Julian Period is 1448638 + 1447 * 365 = 1976793 JD. It corresponds to March 1, 700 C.E. in the Julian calendar. The two corresponding dates in the eras used by Al-Battani are 1 Toth 1012 in Al-Battani Egyptian era or vague years of the era of Dhul qarnayn[28] and 1 March 1011 in Al-Battani's era of Roman years counted from the Seleucid era of Dhul qarnayn.[29] Both eras are mentioned in the table of eras of Al-Battani. We will

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[22] In 1106 according to Millas-Vallicrosa: *Estudios Sobre la Historia de la Cienza Espanola*, 1949. He was the author of a comprehensive anti-Judaic tract: *Dialogi contra Iudaeos*, see Maimonides, Joel L. Kraemer, Doubleday 2008 p. 16.

[23] The phenomenon of conversion to Christianity is not limited to specific periods like the riots of 1391 or the expulsion period of 1492. The Jewish community was constantly harassed by the Church through intellectual confrontation and also by an everyday harassment, the dream of social ascension and the access to official functions. The contacts between Jews and new Christians must be very delicate.

[24] The exact role of each of them in the extant version is unclear.

[25] This is the Era of Nabonassar transposed to the time of the Seleucid era. The relation of equivalence is 1 Toth year 1 of Dhul qarnayn = 1 Toth 437 Nabonassar.

[26] This is a fictitious Julian calendar brought back to the era of the Two Horned, which Al-Battani places on September 1 instead of October 1. In *Heshbon Mahalekhot ha-Kokhavim* p. 50, R' Abraham bar Hiya describes a calendar based on the Julian year and beginning on Monday Tishri 26, 3450. The beginning of the era of the contracts is on Tishri 1, 3450 or September 6, – 311, therefore Tishri 26 corresponds to October 1, – 311.

[27] This is the practical era of Dhul qarnayn used by Al-Battani in all his tables. This epoch occurred 6 months after the epoch of the Era of Contracts according to Maimonides i.e. Tishri 3450 AMI (Beharad) beginning in Tishri – 311.

[28] 1 Toth year 1 of the era of Dhul qarnayn = 1 Toth year 437 of Nabonassar = 1607778 JD.
1 Toth year 1012 of Dhul qarnayn = 1 Toth 1448 of Nabonassar = 1976793 JD.
Indeed 1607890 + 253 * 366 + 757 * 365 = 1976793.

[29] 1 March year 1 of the era of Dhul qarnayn = 1 March – 310 = 1607890 JD.
1 March year 1011 of Dhul qarnayn = 1 March 700 = 1976793 JD.

further speak of vague years and Roman years. We have thus an interesting coincidence: 1 Toth 1012 and 1 March 1011 coincide. The first day of the year of both systems coincide.
Considering Al-Battani's table of conjunctions in Roman years, we find:

	Days or date			mean longitude of both bodies			Reference
999	7d	22'	9"	348°	17'	0"	p. 84
12 years	17	3	31	16	55	38	p. 86
	24d	25'	40"	5°	12'	38"	

For the vague years we use the table of conjunction pp. 29-32.

990	22d	6'	23"	8°	8	12°	p. 29
22 years	2	19	18	357	4	18	p. 31
	24d	25'	41"	5°	12'	30"	

We observe a very good coincidence. We note that the first conjunction of both years occurs on the same day and is noted on the same manner in both calendars. But we do not yet know whether Al-Battani used the inclusive or the exclusive notation.

C. Comparison of the mean conjunctions of Al-Battani with those of Ptolemy.

The first entry of Al-Battani's table of mean conjunctions calculated in Egyptian years is the year 915 of Dhul qarnayn and the first conjunction occurred on Toth 22d 14' 44". This year is 915 + 436 = 1351 of Nabonassar. 1 Toth of that year corresponds to 1448638 + 1350 * 365 = 1941388 JD which corresponds to 26 March 603 C.E. or 26 Adhar 914 SE (Al-Battani's Roman years). The first day of the era of Nabonassar is indeed JD 1448638.
Now the last year considered in the table of Ptolemy is 1101 Nabonassar. But in 250 vague years, the moment of the mean conjunction of Ptolemy shifts with – 0d 27' 51", we can establish the following table:
Al- Battani in ar-Raqqah, longitude 39.05° = 2.605555 h = 2h 36m 20s.

	Days or date			mean common longitude			Reference
Toth 915	22d	14'	44"	26°	3'	27"	p. 29

It corresponds to JD 1941409.136990740741.

Ptolemy in Alexandria, longitude 29.9° = 1.99h.

	Date			Distance to apogee			Reference
Toth 1101	22d	41'	45"	19°	11'	56"	Toomer p. 278
250 years	– 0	27	51	– 61	14	17	
Toth 1351	22d	13'	54"	317°	57'	39"	

It corresponds to JD 1941409.148611110445.
Longitude of the solar apogee: 65 30

Longitude at conjunction	23°	37’	39”

The difference between ar-Raqqah and Alexandria is 39.05 – 29.9 = 9.15° or 37m. We have thus in Alexandria: Al-Battani: 17h 53m 36s – 36m36s = 17h 17m.
Ptolemy: 17h 33m 36s.
The conjunction occurs according to Ptolemy 16m 36s later than Al-Battani but the longitude of the conjunction of Al-Battani is greater by is 2°; 36’ 13”; it corresponds to the sun’s movement in two and a half day! Anyhow we observe a good correspondence of the moments of mean conjunctions between the tables of Ptolemy and Al-Battani and we can consider Al-Battani’s table as the prolongation of that of Ptolemy. It seems likely that they have the same significance. We know that Ptolemy’s table of conjunctions gives the date in Toth of the conjunction: the notation is inclusive[30] and gives the current day and the completed parts of the day according to the principle of a date. The epoch of Ptolemy is at noon of 1 Toth of the first year of Nabonassar, the beginning of the first day of the era.
Thus in our present case, the conjunction according to Ptolemy was on Toth 22nd , at 5h 33m 36s p.m. The number of days elapsed since the beginning of Toth was 21days +5h + 33m +36s. It seems likely that Al-Battani adopted the same convention in the treatment of the Egyptian years and consecutively also in the Roman and Arab years.[31]Indeed the Almagest played a considerable role in the educated society, it was the book of reference and without any remark it is likely that Al-Battani adopted the same conventions, i.e. the use of the *inclusive* notation.[32]

D. The epochs of Al-Battani are always at noon of the day preceding the beginning of the years or the months considered i.e. 24 hours before their beginning.

1. The Roman years.

Let us consider the conjunction of 22 Toth 1351 of Nabonassar at 5h 53m 36s p.m. in ar-Raqqah or 16 April 603, which Al-Battani notes 16 Nissan 914 SE (Al-Battani’s Roman years).
Using the tables of mean conjunction in Roman years, we find:

903	18d	33’	15”	358°	24’	53”	p. 84
11 years	28	41	39	27	38	39	p. 86
914 March	47d	14’	44”	26°	3’	32”	
or April	16d	14’	44”	52°	7’	4”	

The conjunction occurred on 16 April 603 at 5h 53m 36s p.m.

[30] See Ptolemy’s Almagest, G..J.Toomer p.275-276.
[31] See the precedent paragraph where we showed that the indications given for the conjunctions have the same meaning when working with Roman or Egyptian years.
[32] This must however still be formally proved. There remains indeed the possibility that Al-Battani used astronomical days of the type (n – 1, n). His month would begin a day earlier and his notation would be the exclusive notation.

With the syzygies tables of Meeus (1963) we find the mean conjunction on April 16 at 16h 44m ET or 15h 35m UT. This confirms our assumption: Al-Battani used the inclusive notation.

If we check with Al-Battani's table the mean longitude of sun and moon at that moment, we get:

	Sun's mean long.			Moon's mean long.			Reference Al-Battani
911	340°	12'	27"	95°	22'	32"	p. 72 (extrapolation)
3 years	359	17	19	28	9	24	p. 73
Adhar	30	33	19	48	28	6	p. 74
16days	15	46	13	210	49	21	p. 75
5hours	0	12	19	2	44	42	p. 76
53m		2	11		29	6	p. 76
36s		0	2			20	p. 76
	26°	3'	49	26°	3'	31"	

We ascertain that the moment of the conjunction is 1 month + 16 days + 5 h + 53m + 36s after the epoch and not 1 month + 15 days + 5h 53m 36s as it would be the case under the assumptions of Ptolemy. Thus by contrast with Ptolemy, the span of time between the epoch and the mean conjunction contains a number of days equal to the current day of the month. Thus, according to our assumption, Al-Battani used the inclusive notation to represent any moment of the month. Similarly the astronomical days have the same meaning as in the Almagest; they are astronomical days of the type (n, n + 1). But Al-Battani chose an epoch situated at noon, 24 hours before the beginning of March 1. This gave him the great advantage that the number of days elapsed since the epoch is equal to the number representing the date. This avoids the problem met by Ptolemy that the number of days elapsed since the epoch is one day less than the number of the current day. I had already noted this point in my book.[33] The epoch of the radices given by Al-Battani for the year 880, corresponded to noon of the last day of February and not to 1 March. On this way the distance between the epoch and a moment defined as d March h hours is d days + h hours.

2. The tables of mean conjunctions arranged according to the Julian years of the era of Dhul quarnayn.

I have been objected[34] that these tables prove that, by contrast with the thesis of this paper, Al-Battani considered an astronomical day of the type (n – 1, n). The first astronomical day of the month March begins at noon of the last day of February and ends

[33] Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam, Sifriati 1996, p.125. See also The Equation of Time in Ancient Jewish Astronomy BDD 16, p. 12 note 25. Eng Yakov Löwinger was already involved in the discussion.

[34] Engineer Jacob Löwinger, "my fierce opponent", considered this argument as decisive. I think that only the argument of the four dates of eclipse observation can be considered as a decisive argument. The organization of the tables is of a completely different order. Al-Battani wanted avoiding figures greater than the lunation and also any reference to the last day of February. The best solution was that the day between the epoch and the beginning of March 1 should be called March 0.

just before noon of the civil day March 1. Indeed we find in the table of conjunctions, p. 84 for the year 1119 SE[35]: 0d 46'15" and in the table of the mean oppositions, p. 85, we find for the year 1503 SE[36]: 0d 19' 40". Although the distance of these two moments is less than one day from the epoch these moments are presented as belonging to the month of March. This is possible only if we accept that the astronomical day 1 March begins at the epoch, at noon of the last civil day of February and that the astronomical days are of the type (n – 1, n); so far this argumentation.

In fact this reasoning is unfounded and groundless. It is impossible in a table of mean conjunctions to ensure a perfect correspondence between the conjunctions and the civil months of reference.[37] For example in the conjunction tables of Al-Battani on pp. 84-87 we find: 1095 SE: 25d 42' 54"

1119 : 0d 46' 15"[38]

1143 : 5d 21' 26"

Let us find the conjunction of March 1098 SE: 25d 42' 54"+ 27d 9' 45" = 52d 52' 39".
This conjunction is not in March, it is already the conjunction of April 2d 52' 39".
Let us find the conjunction of March 1118 SE: 25d 42' 54"+ 16d 3' 9" = 41d 56' 3".
This conjunction is not in March, it is already the conjunction of April 10d 56' 3".
Let us find the conjunction of March 1119 SE: 25d 42' 54"+ 4d 35' 11" = 30d 18' 5". Al-Battani adopted 30d 18' 5" – 29d 31' 50"" = 0d 46' 15".
Let us find the conjunction of March 1143 SE: 30d 18' 5"+ 4d 35' 11" = 34d 53' 16".
This conjunction is thus the conjunction of April 3d 53' 16".

It is thus impossible to avoid shifts and lacks of concordance between the nominal conjunction of March and the civil month in which it occurs.

Al-Battani adopted the rule to proscribe any date greater than 29d 31' 50", the length of the synodic lunar month. When we get greater figures we subtract 29d 31' 50".

These rules don't prevent some little anomalies which can be easily be adapted by the subtraction of 29d 31' 30". When the figure is less than 1d it means that the conjunction occurred on the last day of the former month, after the epoch but before the beginning of the month.

E. The astronomical days of Al-Battani are the same as the pre-modern astronomical days of the type (n, n+1): the observation of four eclipses by Al-Battani.

Until now we based our reasoning on the following considerations: Al-Battani used the Egyptian calendar based on vague years of 365 days and on the era of Nabonassar on the same basis as Ptolemy, all the more Al-Battani's table of mean conjunctions is really the prolongation of Ptolemy's table. It is thus likely that the definition of the astronomical day and the notation of the moments of the day should be the same as those of Ptolemy. The only difference would then be that Al-Battani placed his epoch 24 hours before the

[35] 808 C.E.
[36] 1192 C.E.
[37] This appears also when one uses the Sygygies Tables of Jean Meeus 1963.
[38] This conjunction belongs still to February, it occurs on 29 February 808 (a leap year) at 18h 30m (counted from noon). The preceding conjunction is 0d 14' 25" and occurs on 31 January at 5h 46m counted from noon.

beginning of the period, the current year or the current month in order to make the number of elapsed days and hours since the epoch equal to the date according to Ptolemy's inclusive notation. Furthermore, we showed that any moment is described on the same manner in both the calendar in Egyptian years and the calendar in Roman years. All this is logic and likely but does not yet constitute a formal proof. In order to get such a formal proof we will examine the report of the observation of four eclipses by Al-Battani.

Al-Battani described the observation of four eclipses.[39] The description of the four eclipses allows proving that he made use of the concept of an astronomical day n, beginning at noon of the civil day n and ending just before noon of the civil day n+1. Indeed the comparison of the dates given by Al-Battani with the dates of these four eclipses according to modern astronomy removes any ambiguity. Because of the importance of each word in the enunciation of the date of the eclipses, we give a literal even if not an elegant translation. The description of the four moments is always made according to the principle of the inclusive notation i.e. the wording of a date including the current year, the current month, the current day and the completed hours and minutes. It is impossible to understand the text otherwise. Furthermore the date is expressed with the same figures as the distance of the moment from the epoch. This proves definitively that for Al-Battani, as for Ptolemy, the 1 March begins at noon of the civil day 1 March and the epoch is 24 hours before, at noon of the last day of February that he named 0 March. Al-Battani, as Ptolemy, used astronomical days of the type (n, n+1).

First eclipse, solar eclipse.

It was in the year 1202 of the era of Dhul qarnayn or 1214 of the era of Alexander's death, in the eight day of the month of Ab (August) in the town of ar-Raqqah at one temporal hour after noon. This time corresponds to Sunday 8 August at 1h true time p.m. (temporary hour).

According to the Canon of Solar Eclipses[40] this eclipse occurred on 8 August 891 at 10h 50m 59s ET Greenwich or 2046714.953 JD.

Second eclipse, solar eclipse.

It was in the year 1212 of the era of Dhul qarnayn or 1224 of the era of Alexander's death in Antioch, at nearly 3 2/3 equinoctial hours before [true] noon of the following day XXIII of the month kanun (January). In the town of ar-Raqqah, to whose time we refer, the middle of the eclipse was thus less than 3 1/2 equinoctial hours before [true] noon.

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[39] These four eclipses observed by Al-Battani were universally known. One of these eclipses is recorded in detail by R' Isaac Israeli in *Yessod Olam Ma'amar* IV, chap 7 p. 12a. Bailly in *Histoire de l'Astronomie Moderne*, tome 1, p231, mentions also these observations: " Albategnius nous a laissé quatre observations du soleil et de la lune qui……, sont utiles pour remplir les déserts qui séparent les astronomes d'Alexandrie et les astronomes modernes ». Delambre reports also the details of these four eclipses in his Histoire de l'Astronomie du Moyen-Age pp. 57-59.

These observations were recently used by Stephenson in his book *Historical Eclipses and Earth Rotation*, Cambridge University Press 1997 in order to calculate ΔT for the period of Al-Battani.

[40] Canon of Solar Eclipses – 2003 to + 2526, Hermann Mucke and Jean Meeus. Astronomisches Buro, Wien 1983.

This time corresponds to Thursday 22 January 901 at 20h 20m true time according to the use of the astronomers or Friday 23 January at 8h 20m a.m. according to the civil use. According to the Canon of Solar Eclipses this eclipse occurred on 23 January at 7h 26m 29s ET Greenwich or 2050170.811 JD.

Third eclipse, lunar eclipse.

This eclipse was in year 1194 of the era Dhul qarnayn or 1206 of the era of Alexander's death, on the day XXIII of the month of Tammuz (July). The middle of the eclipse in ar-Raqqah was at slightly more than 8 equinoctial hours after [true] noon. This corresponds to Tuesday 23 July 883 at 8h p.m. true time.
According to the Cannon of Lunar Eclipses,[41] this eclipse occurred on 23 July 883 at 17h 41 m ET Greenwich or 2043777.24 JD.

Fourth eclipse, lunar eclipse.

The middle of this eclipse occurred in year 1212 of the era of Dhul qarnayn or 1224 of the era of Alexander's death in Antiochia at nearly 15 1/3 hours after [true] noon on the day II of the month Ab (August). In ar-Raqqah it was at 15 1/3 + 1/4 = 15h 35 m equinoctial hours after [true] noon. This corresponds to Sunday 2 August 901 at 15h 35m true time according to the use of the astronomers or Monday 3 August 901 at 3h 35m according to the civil use.
According to the Canon of Lunar Eclipses this eclipse occurred on 3 August 901 at 1h 2m ET Greenwich or 2050362.54 JD.

The comparison of the dates of these four eclipses observed by Al-Battani and the indications of the modern canons allow concluding the very good precision of Al-Battani's observations. Al-Battani's dates expressed in astronomical days correspond perfectly to modern dates. If Thursday 22 January 20h 20m corresponds to the civil day Friday 23 January at 8h 20m a.m. it is clear that the astronomical day 22 January began at noon of the civil day Thursday 22 January and it ended just before noon of the civil day Friday 23 January. Al-Battani's astronomical day is thus exactly the same as the astronomical day used by the astronomers until 1925. This astronomical day n begins at noon of the civil day n bearing the same name and it ends just before noon of the following day n + 1. Al-Battani's formulation of the dates is similar to our way of giving dates; we give the current year, the current month, the current day of the month and finally the completed hours of the day thus the inclusive notation. The formulation of the dates in the inclusive notation is the result of the precise grammatical use of the declinations, and can in no way be understood in the exclusive notation, giving the completed days and hours of the current month**.**

Thus, by means of an external proof, using a canon of eclipses, we prove that Al-Battani used an astronomical day of the type (n, n+1).[42]

[41] Canon of Lunar Eclipses – 2002 to + 2526, Hermann Mucke and Jean meeus, Astronomisches Buro, Wien.

We can also show that even in the absence of any external canon of eclipses, Bar Hiya must by the simple comparison of the mean conjunctions deduced from Al-Battani's tables with the Jewish Moladot and the data of these four eclipses, have deduced that Al-Battani used an astronomical day of the type (n, n+1) and an epoch at the beginning of March 0.

Indeed we know that in consequence of the eccentricity of the sun's and moon's orbits, the sun may be 1.9° on either side of its mean position and the moon 6.3°. Moreover there are periodic perturbations in the moon's longitude. However at a new or full moon the elongation D is near to 0° or 180° and the perturbations reduce the moon's maximum deviation from 6.3° to 5.4°. Therefore the relative positions of the two bodies may vary 1.9° + 5.4° =7.3° from their mean position near to conjunction or opposition. As the hourly motion of the mean elongation is 0.51° the maximum time interval between the mean conjunction and the true conjunction or between the mean opposition and the true opposition is 7.3 /0.51 =14.3 hours. The ancient were aware of these elements and they knew the order of size of the maximum span of time between mean conjunction and true conjunction. This must allow then to check the definition of the astronomical day of Al-Battani used in the definition of the moment of occurrence of the four eclipses.

If we consider the fourth eclipse for example:

Fourth eclipse. Solar eclipse of Sunday 2 August 901 at 15h 20m according to the astronomical use, (Monday 3 August 901 at 3h 20m a.m.).

According to the table of mean oppositions:

1191	29d	17'	44"	11°	40'	52"
21 years	7	57	16	7	47	21
4 months	118	7	21	116	25	39
	155d	21'	51"	135°	53'	52"

The mean opposition was on 155 March or Sunday 2 August 901 at 8h 44m 24s p.m. ar-Raqqah.

The Molad was	4662	4	10	541
	– 1 month –	1	12	793
	– ½ month –	14	18	396.5
		2	3	481.5

The Molad – 1.5 months was on Sunday 2 August at 9h 27m p.m. Jerusalem.
The true opposition followed the mean opposition by about 6h 36m.

[42] We mean under this abbreviated expression: an astronomical day beginning at noon of the civil day n and ending just before noon of the civil day n+1.

We can be certain that our understanding of the astronomical day mentioned by Al-Battani i.e. that the astronomical day n begins at noon of the civil day n and ends just before noon of the civil day n+1, is the only possible understanding of the text. Indeed if our understanding was false and the astronomical day was one day before then in the fourth eclipse the mean opposition would follow the true opposition by about 17h 24m. This span of time is superior to the maximum span of time between true and mean phases. This assumption is thus impossible. The astronomical days used by Al-Battani in the description of his observations must be understood according to the traditional understanding that the astronomical day n begins at noon of the civil day n and ends just before noon of the civil day n+1.

Thus without having at his disposal a canon of eclipses and without a long calculation of the true phasis with Al-Battani' tables, R' Abraham bar Hiya, using only the tables of mean conjunctions and the Hebrew method of the Molad was able to check that the dates of the four eclipses, given in the style of inclusive dates, were necessarily given according to rule that the astronomical day begins 12 hours after the civil day.[43]

At this stage we can already conclude that Al-Battani's astronomical day n, at least in the Roman and Egyptian calendars, is of the type (n, n + 1); it begins at noon of the day n and it ends just before noon of the day n + 1. But by contrast with Ptolemy, his epoch is at noon of the last day of the preceding year or month. In his astronomical tables the

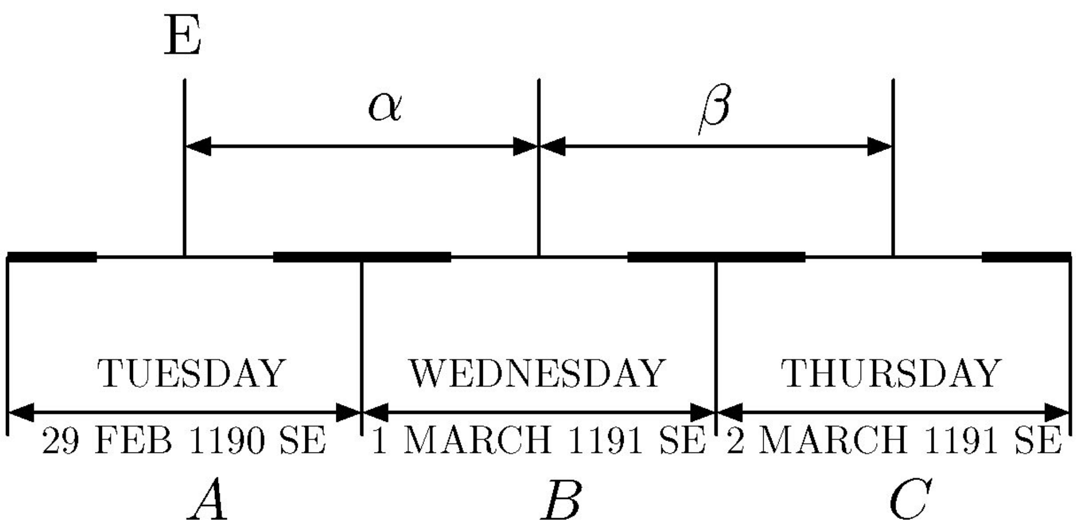

Table 2 <u>The beginning of the year 880 C.E. = 1191 SE according to Al-Battani.</u> E = Epoch = Noon of Tuesday 29 February 879 C.E. (Old Style) = JD 2042537. A, B and C are the modern civil days from midnight to midnight. α is the astronomical day Tuesday 29 February 1190 SE and β is the astronomical day Wednesday 1 March 1191 SE.

determination of the mean solar and lunar coordinates at a certain moment is achieved by the introduction of successively the current year, the completed month, the completed number of days from the epoch equal to the number of the current day of the month and the completed fraction of the current day. In the table of conjunctions the result can be

[43] If we consider that the civil day begins at midnight.

understood as the number of elapsed days and hours since the epoch or the date i.e. the current day and the completed hours.

F. The Arabic years.

In the astronomical tables arranged according to the Arabic years, Al-Battani ga ve for the main epoch of the era of the Hijra corresponding to 622 C.E or 933 Dhul qarnayn the following radices:

	Sun's mean long	Moon's mean long	Reference Al-Battani
Year 1	113° 58' 4"	119° 43' 46"	p. 19.

If we compare with the tables of mean conjunctions arranged according to the Julian calendar we find that the mean conjunction of Muharram year 1 of the Hijra = Av 4382 corresponding to July 622 C.E. or 933 S.E. was

927	23° 8' 26"	3° 9' 31"	p. 84
6	23 47 40	23 1 0	p. 86
Ayyar	88 35 30	87 19 14	p. 87 Ayar=Mai
Adhar	135d 31' 36"	113° 29' 25"	Adhar=March

This conjunction occurred thus slightly before the epoch on the astronomical day Tuesday 13 July 622 at 12h 38m 24s or the civil day Wednesday 14 July 622 at 0h 38m 24s a.m. ar-Raqqah. The Molad of Av 4382 was 4 – 7 – 112 corresponding to Wednesday 14 July at 1h 6m a.m. Jerusalem.

If we compare with the tables arranged according to the Julian years of Dul qarnayn we get the following data for the date of the epoch of the Hijra at noon:

931	340° 23' 37"	228° 58° 5"	p. 72
2	359 31 33	258 46 16	p. 73
Haziran	120 14 58	167 31 14	p. 74 Haziran=June
14 days	13 47 56	184 28 11	p. 75
Epoch	113° 58' 4"	119° 43' 46"	

We ascertain a perfect coincidence with the data of the Arabic table and we can conclude that the epoch of the Arabic year was on the civil day Wednesday 14 July at noon.

In order to understand Al-Battani's Arab calendar we dispose of his equivalence table pp. 9- 18 giving the Roman date corresponding to Muharram 1 of each year of the Hijra. According to it 1 Muharram year 1 of the Hijra = Thursday 15 July 933 S.E. or 622 C.E. but the problem is to give the correct interpretation to this table.
Nallino[44] writes about it:
"*Juxta civilem Muslimorum usum dies ab occasu solis computatur, ita ut nox ad sequens mane pertineat. Astronomi diem a meridie supputant; ergo nox diei Veneris juxta usum civilem fit apud eos nox diei Jovis. Diei discrepantia in epocha hegirae ita explicatur.*"

[44] Al-Battani vol 2 p. 200.

"According to the Muslim civil use, the day is counted from sunset and this night stretches until the next morning. Astronomers count their day from noon; therefore the night belonging to Friday according to the Muslim civil use is for these astronomers the night belonging to the astronomical day Thursday. The disagreement of one day in the epoch of the Hijra is so explained."

Thus Nallino understood in Al-Battani's table that the astronomical day Thursday 15 July 622 corresponds practically to the first day of the era of the Hijra which began on Thursday evening 15 July 622 and ended on Friday 16 July before night. This day is exactly the Jewish day Friday 3 Av 4382. It would also be 1 Muharram 1 Hijra. The astronomical day July 15, 622 overlaps indeed 75% of this civil Arab day. According to this explanation the Roman dates appearing in the table are astronomical days. The first astronomical day of Muharram would begin on Thursday at noon, 24 hours after the epoch at noon of Wednesday 14 July 622. The system would then be perfectly parallel to the system adopted with the Roman and Egyptian years.

However Nallino's explanation does not seem likely because of different reasons:

- It is generally accepted that the first day of the Hijra is indeed the civil day Friday 16 July 622 = 3 Av 4382, according to the religious and popular calendar. Indeed the new moon was only seen on Thursday evening while it was still not visible on Wednesday evening. The longitude of Medina is slightly equal to that of ar-Raqqah, the mean conjunction was thus on the civil day Wednesday 0h 38m and the true conjunction was on the same day at 7h 55m. The moon's latitude was about – 1° and sunset was at about 18h 50m local time, the age of the moon was then less than 11 hours and the moon could not be seen on this evening. The first day of Muharram of the first year of the Hijra was thus Friday 16 July. Neugebauer[45] wrote that this date is the first day of the Hijra according to the Historians.
- It is generally accepted that the first day of the Hijra is the civil day Thursday 15 July 622 = 2 Av 4382 according to the fictitious Arab cyclic calendar adopted by the Muslim astronomers.[46]
- It is certain that the table of equivalence established by Al-Battani is related to the cyclic calendar of the astronomers. It is the only predictable calendar. He certainly followed the cyclic calendar accepted by his predecessors.
- Al-Khwarizmi[47] used also the cyclic Arab calendar. He defined the astronomical day 1 Muharram of the year 1 of the Hijra, thus the first day of the Hijra as a day called Thursday beginning at noon of Wednesday (14 July 622) and ending just

[45] HAMA, A History of Ancient Mathematical Astronomy, Neugebauer, Springer 1975.
[46] They needed indeed a predictable calendar. Delambre in *Histoire de l'Astronomie du Moyen-Age* p. 95 had already noted that the first day of the era of the Hijra is Thursday according to the Astronomers. See also Ginzel Vol I p. 258 where it writes that historians usually count the era of the Hijra as beginning with 16 July, 622 C.E.
[47] His work is known to us by a Latin translation and adaptation made in 1116 by Pedro Alfonso. There is another incomplete manuscript known as the manuscript of Corpus Christi.

before noon of Thursday (15 July). Thus his first day of the era was the civil day Thursday beginning on Wednesday evening and the astronomical day was shifted forward by six hours.

- In the English translation by Neugebauer[48] of the German translation by H. Suter[49] of the Latin translation and adaptation of Al-Khwarizmi by Pedro Alfonso and Adelard of Bath, we find on p. 213 two marginal and posterior explanatory notes:

  1. *Note that Petrus adds to this one day; consequently he begins the first October at noon of this very day, not at noon of the ,preceding day as the Arabs.*
  2. *First we are told that Petrus reckoned the day "n" of a month from noon of the calendar day n to noon of the calendar day n + 1 whereas the Arabic astronomers begin day n at noon of the calendar day n – 1 and end it at noon of the day n. For this reason the solar longitude at epoch namely 6s 7°; 42' 45" has to be augmented by one day's solar motion in order to obtain the epoch value used by Petrus Alphonsi, namely 6s 8°; 41' 56".*[50]

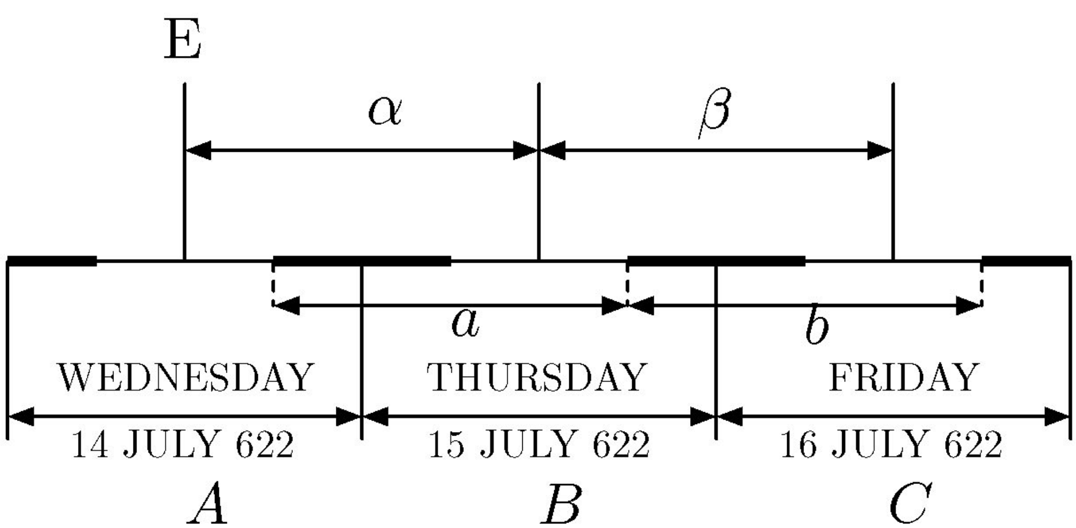

Table 3 <u>The beginning of the era of the Hijra according to Al-Khwarizmi (The Muslim cyclic calendar).</u> E = Epoch = noon of Wednesday 14 July 622 C.E. = JD 1948438; a is the Arab civil day Thursday 1 Muharram, 1 Hijra = Thursday 2 Av 4382 and b is the Arab civil day Friday 2 Muharram, 1 Hijra = Friday 3 Av 4382. α is the astronomical day Thursday 1 Muharram, 1 Hijra = the Roman astronomical day Wednesday 14 July 622 C.E. and β is the astronomical day Friday 2 Muharram, 1 Hijra = The Roman astronomical day Thursday 15 July 622 C.E.

[48] O. Neugebauer:The Astronomical tables of Al-Khwarizmi, translation with Commentaries of the Latin Version edited by H. Suter, supplemented by Corpus Christi College MS 283. Filos Skrifter, Danske Vidensk. Selsk. 4, 2, (1962).
[49] H. Suter : Die Astronomishen tafeln des Muhammad ibn Musa al-Khwarizmi in der Berarbeitung des Maslama ibn al-Madjriti und der latein. Uebersetzung des Atelhard von Bath. Danske Vidensk Selsk. Skrifter, 7. R. Hist. Og filos. Afd 3, 1 (1914).
[50] S = sign = 30°: thus 188°; 41'56".

It appears thus that the Jewish and Christian astronomers of the beginning of the twelfth century were aware and familiar with the conventions adopted by the Arab and Christian astronomers or more precisely by the astronomers working with the Roman and Greek calendar on the one hand and with the Muslim calendar from the other hand.

It appears clearly that there were two divergent practices: the Arab astronomers counted their astronomical day n from noon of the civil day n – 1 until noon of the civil day n. By contrast the Christian astronomers counted their astronomical day n from noon of the civil day n until noon of the civil day n + 1.
Therefore the meaning of Al-Battani's equivalence table seems to be the following:
The first day of the Hijra, according to the cyclic Muslim calendar of the astronomers, is the civil Arab day Thursday 1 Muharram equal to the Jewish day Thursday 2 Av 4382 beginning on Wednesday 14 July at sunset and ending Thursday 15 July at sunset; it corresponds to the civil Christian day Thursday 15 July 622.

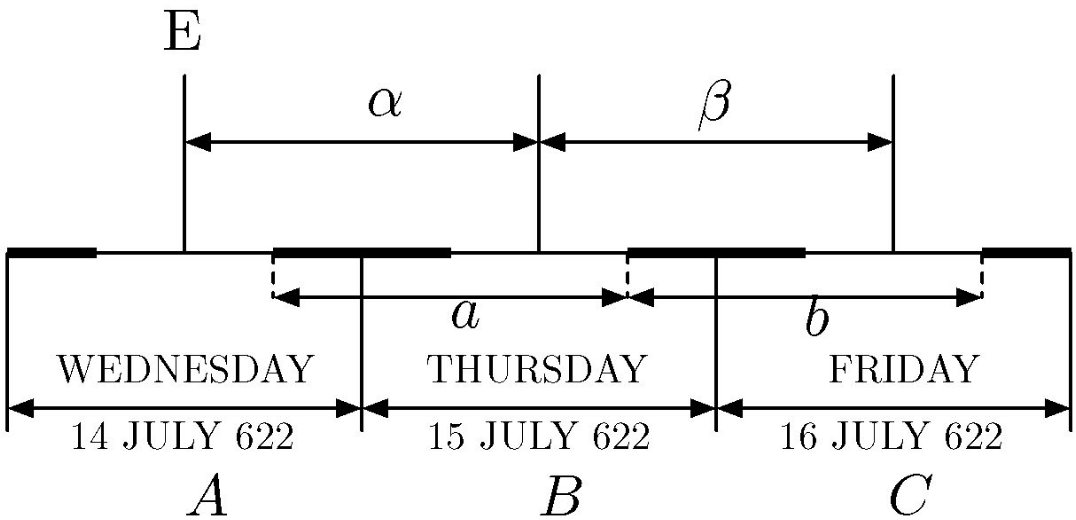

Table 4. <u>The beginning of the era of the Hijra according to Al-Battani (The Muslim cyclic calendar)</u>. E = Epoch = noon of Wednesday 14 July 622 C.E. = JD 1948438; a is the Arab civil day Thursday 1 Muharram, 1 Hijra = Thursday 2 Av 4382 and b is the Arab civil day Friday 2 Muharram, 1 Hijra = Friday 3 Av 4382. α is the astronomical day Wednesday, one day before the beginning of the Hijra = the Roman astronomical day Wednesday 14 July 622 C.E. and β is the astronomical day Thursday 1 Muharram, 1 Hijra = The Roman astronomical day Thursday 15 July 622 C.E.

The astronomical day Thursday 1 Muharram 1 Hijra begins, for Al-Battani, at noon of Thursday 15 July; it coincides with the astronomical day Thursday 15 July 622. Al-Battani is thus obliged to differ from Al-Khwarizmi about the definition of the astronomical Arab days. Al-Khwarizmi's astronomical day Thursday 1 Muharram 1 Hijra is the astronomical day Wednesday 14 July 622 and his astronomical day Friday 2 Muharram 1 Hijra is the astronomical day Thursday 15 July; the latter is also Al-Battani's astronomical day Thursday 1 Muharram 1 Hijra. Al-Battani must remain coherent with his definition of the astronomical day adopted in the Roman and Egyptian calendar. If he had followed Al-Khwarizmi the same astronomical day would be Wednesday 14 July 933 SE in the Roman calendar and Thursday 1 Muharram in the Arab calendar. This would be impossible. Al-Battani gave the priority in his work to the Roman calendar. We find the most complete set of tables in this Roman calendar: tables of sun's and moon's motion and the syzygies tables. The decisive elements making the decision and proving the difference of Al-Battani with regard to the Arab astronomers remains the description of the time of the four eclipses. It is now proved that Al-Khwarizmi used astronomical days of the type (n – 1, n) with an epoch at noon of the first day of the month while Al-Battani used astronomical days of the type (n, n+1) with an epoch at noon of the day preceding the first day of the month or at noon of the day 0 of the month.

G. The autumnal equinox determined by Al-Battani.[51]

Another dated observation by Al-Battani is the autumnal equinox observed on 19 September 882 in ar-Raqqah 4 ¾ hours before sunrise.[52] This corresponds to 19 September 882 at 1h 15m a.m. true time. Al-Battani gives in his table of ΔE (the equation of time) for the sun's true longitude of L = 180° an angle of 6°; 04' which corresponds to 24m 16s. We know further that this correction from true time to mean time is subtractive. Therefore true equinox was at 0h 50m 46s aRABMT.[53] Taking into account a longitude of 39°; 03' for ar-Raqqah we have a time difference with Greenwich of 2h 36m 12s. We have also a time difference of 16.4 m between the mean time of Al-Battani and the modern mean time. The time of this equinox in UT is then 0h 50m 46s – 2h 36m 12s + 16m 24s = 22h 30m 49s UT. If we compare with modern data, we ascertain that the apparent vernal equinox was on 18 September 882, at 23h 51m Dynamic Time in Greenwich. ΔT was about 46m and therefore the time of the equinox was 23h 05m U.T. There exist thus a difference of about 34 minutes between the equinox according to Al-Battani and the modern data; this determination of the equinox is one of the most famous and precise astronomical determinations of the Middle Ages.[54]

H. Conclusion.

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[51] This equinox was well known by rabbinical astronomers and meabrim. R' Judah Halevi mentions it in Sefer ha-Kuzari, Ma'amar revi'i, chap 29. Ha-Kuzari ha-mephorash 1969, pp. 302-303.

[52] This was not a simple observation as it was in fact at night. It was the result of many observations made during the days before and after the equinox and finally a theoretical calculation on basis of these observations. This exceptional observation was already reported in the book of Ibn Yunus (979 – 1009): Le Livre de la grande table hakémite, A. P. Caussin de Perceval, Paris 1804, p. 132.

[53] Ar-Raqqah Al-Battani mean time with the following equation:
Al-Battani mean time + 16.4 m = Modern mean time. See Jean Meeus: Astronomical Algorithms, Willmann-Bell, Richmond, Virginia, 1991, p. 71-75.

[54] It is remarkable that despite the exceptional precision of the equinox determined by Al-Battani his tropical year is rather imprecise, 2m 22s too short. Indeed Al-Battani found the length of the tropical year by the comparison of the equinox of 18 September 882 at 22h 39m U.T (34 m too early) with the equinox observed by Ptolemy in Alexandria on 26 September 139 at about 5h 01m U.T. Nevertheless this observation was defective. According to modern data the equinox was on 24 September at 22h 02m E.T. If we consider ΔT = 2h 31m then the equinox was on 24 September at 19h 31m, 33.5 hours before the indications of Ptolemy. Al-Battani considered that 743 tropical years lasted 271,373.732640 days, hence a year of 365.240555 days. (See C. A. Nallino Vol.1 p. 42 and 241-42) The tropical year of Al-Battani was penalized by Ptolemy's error of 33.5 hours representing 2.7 minutes per year. If Al-Battani had neglected the observation of Ptolemy and had considered the observation of Hipparchus, he would have got a year of an exceptional precision of 365.2423 days.

Al-Battani operated coherently and similarly in the Arab and in the Roman calendar.

1. In the Roman and Greek calendar Al-Battani remained faithful to Ptolemy's system but he shifted the epoch backwards by one day. Apparently he discovered the interest of shifting the epoch by one day because it makes the number of completed days elapsed since the epoch equal to the number of the current day in the month. Thus the number of days and hours elapsed since the epoch are practically the date.

2. In the Arab calendar he adopted a symmetrical position. He followed the cyclic calendar defined by his predecessors and al-Khwarizmi but he differed from him with regard to the definition of his Arab astronomical day. It is of the type (n, n+1), it begins at noon of the civil day bearing the same name and their overlapping is only six hours. This is necessary to behold the same weekday whether we are working in the Roman calendar or in the Arab calendar. This allowed him also beholding his innovation that the epoch precedes the first astronomical day of the month of Muharram by one day. Thus the epoch of year 1 of the Hijra is at noon of the last astronomical day of the preceding year. The first astronomical day of the era begins 24 hours after the epoch and 18 hours after the first civil day of Muharram.

3. The significance of the equivalence table giving the 1 Muharram of the years of the Hijra (cyclic calendar of the astronomers pp. 9-18) in Roman years is thus the following:

- The first astronomical day of the year 1 of the Hijra is the astronomical day Thursday 15 July 622 by contrast with Al-Khwarismi who called the same astronomical day Friday 2 Muharram.
- The Arab civil day 1 Muharram of the year 1 of the Hijra was the Jewish day Thursday 2 Av 4382 corresponding to the modern civil day Thursday 15 July 622 by contrast with Nallino who considered that the Arab civil day 1 Muharram I Hijra was the Jewish day Friday 3 Av 4382 corresponding to the modern civil day Friday 16 July 622 C.E.

6. R' Abraham bar Hiya (died ~ 1136).

In the quotation mentioned in the beginning of the paper, R' Abraham bar Hiya explained that he assumed in his book *Heshbon Mahalekhot ha-Kokhavim* that the astronomical day n begins at noon of the civil day n – 1 and ends just before noon of the civil day n. Therefore the astronomical day 1 Tishri begins at noon of the civil day 29 Elul and ends just before noon of the civil day 1 Tishri. The epoch of his tables is noon of the Jewish civil day Wednesday 29 Elul 4864 A.M. At this moments begins his astronomical day "Thursday 1 Tishri, 4865 A.M"; it ends just before noon of the Jewish civil day Thursday 1 Tishri 4865 A.M." This assumption seems connected to another assumption that the epoch of the year is at noon of 29 Elul. Thus in this system the epoch is at noon of 29 Elul and however, the epoch is just at the beginning of the first astronomical day of the year. As we showed it above, it is now clear that R' Abraham bar Hiya followed the Arab system in contradiction with the system of Al-Battani in his treatment of the Arab calendar. This system was specially adapted for R' Abraham bar Hiya who presented his astronomical tables according to the Jewish calendar. It is interesting to note that R' Abraham bar Hiya, in his short description of the Arab cyclic calendar, wrote that those who count according to this system are the "calculators". They can be found in the East and the West except in Egypt where they rely on the vision of the new moon. For this reason the beginning of their months doesn't fall on the same day in their different countries.[55] In *Sefer Mehalekhot ha-Kokhavim*,[56] he wrote "this cyclical calendar is used for the dates of their contracts, of their books, and the years of their kings.[57] However they have another calculation based on the vision of the new moon; they use it for the fixing of the festivals." Bar Hiya added that in his book he is not concerned with this last calculation.[58]Savasorda wrote also that the first days of month of the Arab cyclic calendar fall always on the day of the Jewish Neomenia or on the preceding or following day.[59]

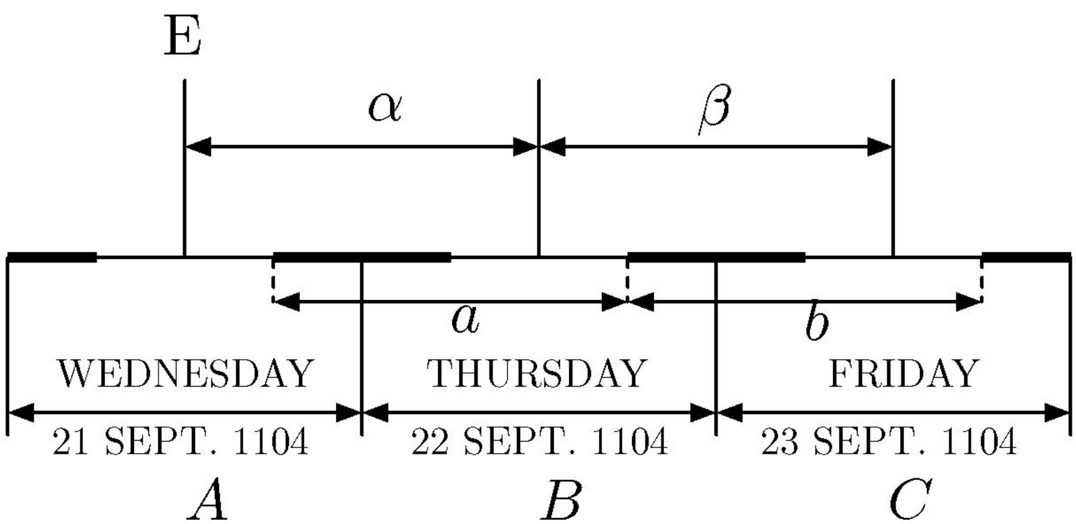

Table 5 The epoch of R' Abraham bar Hiya. E = Epoch = noon of Wednesday 21 September 1104 C.E.; a is the Jewish civil day Thursday 1Tishri 4865 and b is the Jewish civil day Friday 2 Tishri 4865. α is the astronomical day Thursday 1 Tishri 4865 = the Roman astronomical day Wednesday 21 September 1104 C.E. and β is the astronomical day Friday 2 Tishri 4865 = The Roman astronomical day Thursday 22 September 1104 C.E.

[55] *Sefer ha-Ibbur* III, chap 9 p. 100. I was surprised by the liberty of the tone and the style used by Savasorda (the Spanish appellation of bar Hiya) in his appreciation of the Arab and Christian calendars and his denomination of Mahomet, "the insane" and above all, of Jesus qualified "the hanged". This book was known by the Christian community and apparently was not censured; it is really surprising: see p. 100, 109 and 110. As already mentioned Savasorda was working with Christian and even converted scholars.
[56] *Sefer Mehalekhot ha-Kokhavim* p. 51lines 5-8.
[57] This would contradict the practical modern use of the historians who consider that the first day of the era of the Hijra was Friday 16 July 622 according to the religious and popular calendar.
[58] According to this remark of Savasorda the use of the modern historians to consider July 16 as the first day of the Hijra would not be justified; historians would also use the cyclical calendar.
[59] *Sefer Mehalekhot ha-Kokhavim* p. 52, lines 23-25.

7. R' Isaac Israeli (end of the 13th – first half of the 14th century).

R' Isaac Israeli adopted a point of view very similar to bar Hiya. He wrote even that the majority of the astronomers adopted an astronomical day (n – 1, n). He did not make the difference between Arab and Christian astronomers
Israeli wrote also that the era of the Hijra is Thursday 2 Av 4382.

However he mentioned one of the four eclipses observed by Al-Battani. In spite of the numerous mistakes in the text, the redundant information generally allows reconstituting the given dates. He mentioned thus one eclipse observed by Al-Battani; he understood it correctly and transformed it in Hebrew date.[60]

He mentioned also[61] three eclipses observed by "the Hazzan" R' Isaac ibn Sid in Toledo.

- The lunar eclipse of Thursday 24 December 1265 at 3h 20m a.m.
  The modern value is 3h 53m a.m. ET. If we subtract ΔT = 13m and 16m for Toledo's longitude of – 4°; 2' we find 3h 24 a.m.

- The lunar eclipse of Saturday 19 June 1266 at 3h 7.5m a.m.
  The modern value is 3h 47m ET. If we subtract 13m and 16m, we get 3h 18m a.m.

- The lunar eclipse of Monday 13 December 1266 at 6h 37m p. m.
  The modern value is 19h 7m. If we subtract 13m and 16m we get 6h 38m p.m.

The coincidence is perfect. However we don't know whether the times given by Israeli, were given in mean time or true time.[62] Anyhow the comparison remains excellent.

Israeli mentions also an interesting determination of an autumnal equinox by Abraham ibn Zarqali in Toledo in the year 1075 which did not fetch the attention of the scientific community.[63] The interest of this determination is its high precision, comparable to the determination of Al-Battani in 883.
Israeli mentions that the autumnal equinox of the year 454[64] of the Hijra or 4836 A.M[65] occurred in Toledo at 4 hours and 324 halakim after noon of Thursday 4 Tishri. The equinox was thus on Thursday 17 September 1075 at 16h 18m true time or 16h 10m modern local mean time.[66] The longitude of Toledo is – 4°; 02' and the mean time in Greenwich or Universal time was then 16h 16m.

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[60] Yessod Olam IV, chap 7, p 11c and d.
[61] Yessod Olam IV, chap 7, p 11c.
[62] In the Middle-Age, the correction from mean time to true time is additive, it is the equatio dierum as prescribed by Ptolemy in his Almagest or Al-Battani.
[63] *Yessod Olam* IV, chap. 15, p.29 d.
[64] In the text it writes 468.
[65] The text mentions also that it is the 10th year of the 255 the cycle.
[66] For L = 180° the equation of time of the ancients is about 6°; 04' or 24m 16s; it is subtractive from true time to mean time. Thus the equinox was at about 15h 54m ancient mean time or 15h 54m + 16.4 m = 16h 10m modern mean time.

According to the modern tables the autumnal equinox in 1075 was on Thursday 17 September 1075 at 17h 55m 16s ET. If we consider $\Delta T = 29m$[67] we find 17h 26m UT.
The error of Abraham ibn Zarkali would then be about 50m; the precision of this determination of the equinox would then be exceptional.

8. R' Abraham ibn Ezra (Tudela 1089 – Narbonne 1164).

Let us now examine the quotation of R' Abraham ibn Ezra. He wrote that the true vernal equinox of year 4907 was on Friday 11 Tishri at the seventh hour.

A. Ibn Ezra and the movement of the sun.

The understanding of Ibn Ezra's text of his *Sefer ha-ibbur* requires a good knowledge of the sun's movement according to the model of the ancients.
Referring to the figure 1, the little circle centered in C represents the sun's orbit. When the sun is in H then H_2 represents the mean position of the sun and H_1 represents the apparent or true position of the sun. The angle α = KCH is the sun's mean anomaly and the angle β = angle EHC is the quota of the sun's anomaly. The angle DCH = DCK + α = longitude of the apogee + mean anomaly = mean longitude of the sun.
H is the apparent position of the sun, as seen from the earth. The angle γEH is the apparent or true longitude of the sun; it is worth angle γEK + angle KEH = longitude of the apogee + $\alpha - \beta$ for $\alpha < 180°$ and γEK + angle KEH = longitude of the apogee + $\alpha + \beta$ for $180° < \alpha < 360°$.
When the sun is in D, N, M and G we are at the moments of the mean equinox or mean solstices. When the sun is in V, Q, W and I we are the moments of the apparent or true equinox or solstices.
We name t_1 the span of time necessary for the sun to cover the arcs VD and NQ, t_2 the span of time necessary to cover the arcs GI and WM; t_1 is the span of time between the apparent vernal equinox and the mean vernal equinox, it is also the span of time between the mean autumnal equinox and the apparent vernal equinox. Similarly t_2 is the span of time between the mean summer solstice and the apparent summer solstice; it is also the span of time between the apparent winter solstice and the mean winter solstice.
When the sun is in V at apparent vernal equinox, the apparent longitude of the sun is L = 0° and the mean longitude of the sun is $360° - t_1$; apparent equinox occurs before mean equinox. When the sun is in Q at apparent autumnal equinox, the apparent longitude of the sun is 180° and the mean longitude of the sun is $180° + t_1$; mean autumnal equinox occurs before apparent equinox. When the sun is in I at apparent summer solstice, the apparent longitude of the sun is 90° and the mean longitude of the sun is $90° + t_2$; mean summer solstice occurs before apparent summer solstice. When the sun is in W at apparent winter solstice, the apparent longitude of the sun is 270° and the mean longitude of the sun is $270° - t_2$. Apparent winter solstice occurs before mean winter solstice.

When Ibn Ezra writes in *Sefer ha-Ibbur* p. 6b bottom:

[67] It represents the difference between the uniform time ET and the terrestrial time UT, the universal time. $UT = ET - \Delta T$

כי תקופת רב אדא איננה כאשר חשבו כי היא כנגד גלגל המזלות, איננה רק כנגד גלגל השמש שמוצקו רחוק ממוצק השמש..........

He means that the four equidistant symmetric points D, N, M and G do not belong to the circle of the ecliptic, as the points of longitude 0°, 90°, 180°, 270° which are symmetric but not equidistant. These four points belong to the circle of the sun's orbit and correspond to the mean equinox and solstices, and not to the apparent equinox and solstices. Of course, these points have shifted with the times with regard to the astronomical mean tekufot because the year of R' Adda, slightly equal to the year of Ptolemy, is too long.

B. The parameters of the model according to Ptolemy.[68]

The length of the tropical year is $Y = 365.25 - 1/300$ days = 365.24666666 days and
$\omega = 0.985635278444$ °/d.
Ptolemy measured that apparent spring lasts 94d 12h and apparent summer lasts 92d 12h.[69]
Thus $Y/4 + t_1 + t_2 = 94.5$
$Y/4 + t_1 - t_2 = 92.5$
Hence $t_1 = 2.1883334$d and $t_2 = 1$d.

All the long and intricate calculations of Ptolemy can be summarized by the following precise formulas:

$\text{Sin } t_1 = e * \sin L_{apogee}$
$\text{Sin } t_2 = e * \cos L_{apogee}$
$\text{Sin } t_1 / \text{Sin } t_2 = \text{tg } L_{apogee}$.
Hence $t_1 = 2.1568986°$ and $t_2 = 0.9856352784°$
$\text{Tang } L_{apogee} = 2.1879245$ and $L_{apogee} = 65.4370°$ to compare with 65.5° adopted by Ptolemy.
$e = \text{Sin } t_1 / \text{Sin } L_{apogee} = 0.4138$ to compare with 2; 30 p = 2.5 /60 = 0.416 adopted by Ptolemy.[70]

A. The parameters of the model according to Al-Battani.

The tropical year of Al-Battani is Y = 365d 5h 46m 24s = 365.240555555d.
$Y/4 = 91.31014$ d and $\omega = 0.98565177$ °/d.
Al-Battani measured that the length of apparent spring is 93d 14h and the length of apparent summer is 93d 0.75h.[71]
Thus $Y/4 + t_1 + t_2$ = 93d 14h = 93.5833
$Y/4 + t_1 + t_2$ = 93d 0.75h = 93.0313

[68] See Toomer pp. 153-156.
[69] See however Toomer p. 154 note 47.
[70] The considered angles are little and the assimilation of the sinus to the arcs would not corrupt significantly the results.
[71] See Nallino Vol I, p. 43 and p. 212.

Hence $t_1 = 1.9972 \sim 2d$
$t_2 = 0.2760d$

Converting t_1 and t_2 in degrees we find:
$t_1 = 1.971303°$ and $t_2 = 0.2718756°$.
Using again the short procedure proposed above instead of the long and intricate procedure of Al-Battani, similar to that of Ptolemy, we get:

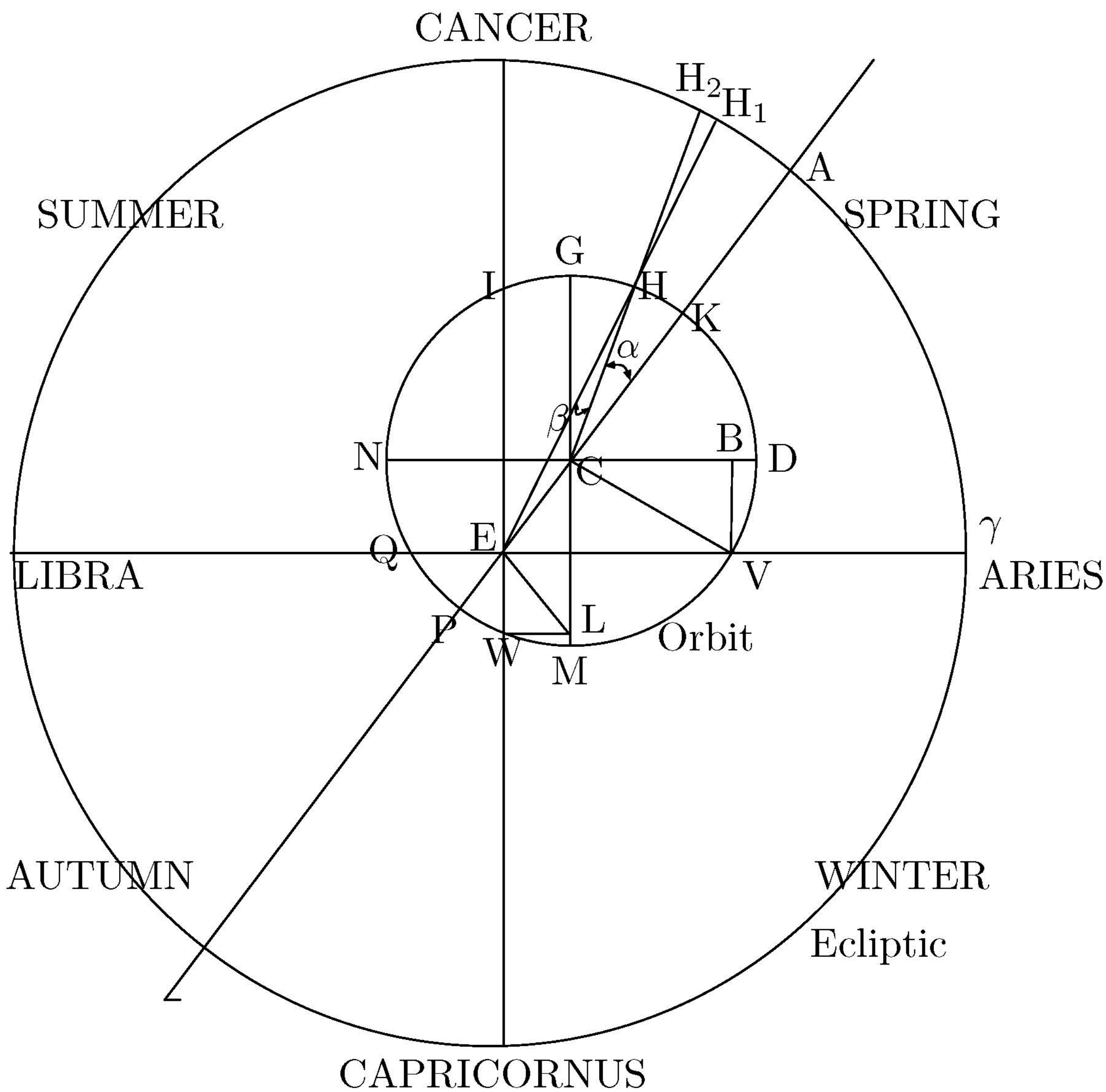

Figure 1 The model of the movement of the sun according to the ancients. A = angle KCH is the anomaly of the sun and angle β = angle EHC is the quota of the anomaly.

Tang $L_{apogee} = Sint_1 / Sin\ t_2 = 7.2493529$ and $L_{apogee} = 82.14599447$ to compare with 82°; 15' adopted by Al=Battani.

$e = Sin\ t_1 / Sin\ L_{apogee} = 0.03472467$ to compare with $e = 0.0347$ adopted by Al-Battani.

This proves the skillfulness of the ancients in the presence of angles very near to 0° or 90°.

B. Tekufah of Adda and true vernal equinox in 1147 C.E. at the time of ibn Ezra.

1. Tekufah of Adda.

In the year 4907 the Molad Tishri is	1 – 3 – 331;	Rosh Hashanah falls on Monday 9
September 1146.	2 – 4 – 438	
The Molad Nissan, is then	3 – 7 – 769	Nissan 1 is Tuesday 4 March 1147.
Tekufat Adda	13– 14 – 131 – 40	
	16 –21 – 900 – 40	Monday 17 March 1147
		Monday 14 Nissan 4907

The Tekufah of Adda was on Monday 14 Nissan or 17 March at 21h 900hal Jewish time corresponding to Monday 17 March at about 3h 50m p.m. Jerusalem mean time,[72] a few hours before the beginning of Nissan 15, the first day of Passover. In Verona the tekufah of Adda would then be on Monday 17 March at about 1h 58m p.m. The statement of ibn Ezra that the tekufah of Adda falls on the first day of Passover is effectively surprising.[73] The only possible justification of this statement would then be that ibn Ezra considered an astronomical day Tuesday 15 Nissan 4907, beginning six hours before the Jewish civil day Tuesday 15 Nissan 4907.

2. True equinox.

We know that the tables of Al-Battani were still in use in the twelfth century. Maimonides used them as late as the end of the twelfth century and R’ Abraham bar Hiya used them during the first half of the twelfth century. Let us compare the true vernal equinox of R’ Abraham ibn Ezra with that deduced from the tables of Al-Battani. Al-Battani fixed the longitude of the apogee to 82°; 15’ on 0 March 880 C.E. = JD 2042537. We want to know this longitude on 14 March 1147 = JD 2140072.
The longitude of the apogee increases by 1° in 66 Julian years = 2416.5 days. In 97535 days the longitude of the apogee increases with 4.046004° or 4°; 02’46” and becomes 86° 17’46”.

At true equinox L = 360° and l = 358°; 1’ 10” under the assumption that the quota of the anomaly is 1°; 58’50”. The anomaly is then 358°; 1’ 10” – 86°; 17’ 46” = 271°; 43’ 24”. By interpolation in the tables of Al-Battani between 271° and 272°[74]we find a quota of the anomaly of 1°; 58’ 52” leading to the final mean longitude of the sun of 358°; 1’ 8”. From the tables of Al-Battani we find, taking into account that 1147 C.E. = 1458 SE.

[72] Probably the ancient Al-Battani mean time, corresponding to 16h 06m Jerusalem modern mean time.
[73] Just before submitting this paper I ascertained that Akabia had already noted this problem and considered it as a mistake: see Tarbits 26 1956/57 p. 313: ספר העיבור לר' אברהם אבן עזרא .
[74] See Nallino, Vol I, p. 80, bottom.

1451	345°	14’	3”	Nallino p. 72
7	359	19	33	Nallino p. 73
14days	13	47	56	Nallino p. 75

14 March at noon		358°	21’	32”	
At true equinox	l =	358°	1’	8”	this was thus 20’ 24” before noon.
8 hours before the mean longitude was					19’ 43” less Nallino p. 76
16.42m before the mean longitude was					41” less Nallino p. 76

The true equinox was thus on the astronomical day 13 March at 15h 43.38m ar-Raqqah al-Battani mean time.
The distance ar-Raqqah – Greenwich is 2h 36m 12s
The distance Verona - Greenwich is 44m
The distance ar-Raqqah – Verona is thus 1h 52m 12s.
The true equinox was in Verona on 13 March at 13h 51m p.m. corresponding to the modern civil day Friday 14 March at 1h 51m a.m. local mean time (of Al-Battani) or on the Jewish civil day Friday 11 Nissan at 7h 51m Jewish local mean time. The seven hours mentioned by ibn Ezra[75] would then be Jewish hours counted from nightfall.[76]
The distance between the apparent equinox and the tekufah of Adda would then be 3 days 12h and 7 minutes instead of the four days mentioned by ibn Ezra.

According to modern astronomical data, the apparent vernal equinox in 1147 was on Friday 14 March at 14h 35m 21s dynamic time. For this year $\Delta T = 1111.95$ s = 18m 32s. The apparent equinox was thus at 14h 17m UT or 2h 17m p.m. UT. In Verona it was at about 13h 01m or 1h 01m p.m. local modern mean time or 0h 45m p.m. Al-Battani local mean time. The tables of Al-Battani are thus no more accurate, the error is about 11 hours. However the text of ibn Ezra points clearly out that the moment of this equinox was near to noon when the objects have no shadow. We must then conclude that ibn Ezra spoke of 7 hours of the day,[77] that he did not use the tables of Al-Battani and that he reached here, probably by chance, an exceptional precision.[78]

We are thus facing the following problems:

- The tekufah of Adda occurs on Nissan 14 or Friday March 17 at 3h 50m p.m. Jerusalem or 2h 13m Verona mean time, on the day preceding the first day of Passover.
- The true equinox according to Al-Battani was on Nissan 11 or March 14 at 2h 07m a.m.Verona modern mean time.

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[75] On top of the page 18. The text of Sefer ha-Ibbur is the following:
כי תקופת האמת תהיה בשעת ז' מיום ששי שהוא י"א בניסן והעד כלי הנחושת למראה ענינים גם הצל כי עז יהיה בוירונה
בלומברדיאה שאני דר בה היום, בחצות היום צל כל דבר כמוהו בעבור שרוחב מדינה זו מ"ה מעלות...........

[76] As championed very early by Ir. Yakov Löwinger. However the Hebrew text seems to refer to seven hours in the day, near to noon when the objects have no shadow.

[77] Akabia, in a paper about ibn Ezra’s Sefer ha-Ibbur understood also seven hours of the day: see Tarbits 26, 1956/57 pp. 309-310.

[78] In my paper: The equation of time in ancient Jewish astronomy published in B.D.D 16 (August 2005), I understood also that the equinox occurred on Friday March 14, 1147 C.E. at 13h true time of Verona. I had added erroneously that this equinox was lacking in precision.

- The distance between the apparent equinox and the tekufah of Adda is 3.5 days and not 4 days. It is only 3 days according to modern astronomical data;
- Similarly the other distances mentioned by ibn Ezra, between the other apparent tekufot and the tekufot of Adda are also inaccurate.
- He writes that the equinox of Tishri coincides with the tekufah of Adda. In fact the apparent equinox follows the mean equinox by two days. Because of the shift of the tekufah of Adda of 1.5 days, the span of time between the apparent equinox and the tekufah of Adda is reduced to one day according to modern astronomical data and to half a day according to the data of Al-Battani.
- The apparent summer solstice follows the mean solstice by about six[79] hours. Because of the shift of the tekufah of Adda, the span of time between the summer solstice and the tekufah of Adda is increased to about 0.75 days according to modern astronomical data and to 1.25 days according to Al-Battani.[80]
- The apparent winter solstice precedes the mean solstice by about six hours. The span of time between the winter solstice and the tekufah of Adda is thus increased to about 1.25 days according to modern astronomical data and about 1.75 days according to Al-Battani.[81] Ibn Ezra speaks of about two days in the two last cases.

A solution to the first problem could be that ibn Ezra used also the concept of astronomical days, according to the Arabic use. The astronomical day Nissan 15 would begin at noon of the civil day Nissan 14 and it would end at noon of the civil day Nissan15. Therefore the moment of the tekufah of Adda would indeed be on the same astronomical day as the greatest part of the first day of Passover.
For the problem of the distance between the apparent equinox and the tekufah of Adda, there is no satisfactory explanation.

- A true equinox near to that of Al-Battani requires to understanding that the seven hours are Jewish hours and it gives different spans of time between true and mean tekufah of Adda.
- A true equinox at 7hours counted from noon of the astronomical day March 13 would give a difference between the true vernal equinox and the tekufah of Adda of 3.75 days which ibn Ezra would have rounded off to 4 days. But the imprecision of this true equinox would then reach about 17 hours[82].
- A true equinox at 7 hours in the day, as the text of ibn Ezra suggests, would give a difference of only three days between the true vernal equinox and the tekufah of Adda. This would lead us to contemplate the unthinkable possibility that ibn Ezra could have through a careless mistake placed the tekufah of Adda on Nissan 15 instead of Nissan 14 increasing the distance between the true vernal equinox and the tekufah of Adda from 3 to 4 days. This mistake would have repercussions on the three other spans of time.

Of course none of these solutions is satisfactory.

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[79] This number of hours correspond to the time t_2 which depends on the longitude of the apogee. The value adopted by ibn Ezra could differ significantly from the value adopted correctly by al-Battani.
[80] And not two days and hours as championed by ibn Ezra.
[81] And not two days and hours as championed by ibn Ezra.
[82] The apparent equinox according to Al-Battani is about 11 hours in advance with regard to the apparent equinox according to modern calculations; this assumption would increase the inaccuracy to about 17 hours!

Anyhow it not certain at all that ibn Ezra rested on the Tables of Al-Battani. There are different elements which plead in favor of the independence of Ibn Ezra from Al-Battani.

- By contrast with Al-Battani, Ibn Ezra followed the theory of the trepidations.[83]
- According to José Millas-Vallicrosa, Ibn Ezra considered a longitude of the apogee of 77° 54’ in 1154,[84] very different than the value of Al-Battani.
- Delambre[85] wrote that Ibn Ezra placed Arzachel above all the astronomers of his time. The theory of the trepidation adopted by Ibn Ezra followed the theory of Arzachel.
- Bailly[86] refers to the book *Initium Sapientis* of Ibn Ezra[87] and confirms the admiration and dependence of Ibn Ezra on Arzachel.
- Jose Maria Millas Vallicrosa edited Ibn Ezra’s book *El libro de los fundamentos de las tablas astronomicas*. It confirms that Ibn Ezra constructed his own tables and did rest on different sources.

Anyhow the two quotations of ibn Ezra, the first that the apparent equinox was on Friday, Nissan 11, 4907 at seven hours and the second that the distance between this true equinox and the tekufah of Adda was 4 days remain contradictory and problematic, all the more, ibn Ezra had a very good knowledge of the length of the tropical year which he fixed to 365.25 – 1/130.[88] It must allow him to know with great precision the rate of shifting of the vernal tekufah of Adda with regard to the mean vernal equinox. The excess of the Jewish year was thus, for Ibn Ezra, 6.50 minutes per year,[89] amounting to 1 day in 221.5 years. Similarly the quotation about the distance between the four true tekufot and the tekufot of R’ Adda is problematic.

9. R’ Judah ha-Levi and the Tekufah of Adda.

In Sefer ha-Kuzari, ma’amar 4, chap 29, it writes:

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[83] *Sefer ha-Ibbur* p.10a bottom. According to this theory the movement of precession of the vernal point γ is not uniform but of the form a +b sine. The discussion whether there is an oscillation of 8° or 10° 45’ amplitude refers clearly to the theory of the trepidation. 10° 45’ represents the amplitude of the variation of the longitude of the apogee proposed by Thabit ben Quorrah (835-900). See L’astronomie, G. Bigourdan, Paris, Flammarion 1916, p. 303.

[84] Estudios Sobre Historia de la Cienza Espanola, Jose Millas-Vallicrosa, Barcelona 1949, p. 298.

[85] Delambre, Astronomie du Moyen-Age, p. 176. His appreciation of Arzachel’sToledan tables is negative, he writes indeed : “Ces tables n’inspirèrent cependant pas une grande confiance; il paraît même qu’on leur préfère toujours celles d’Albategni”.

[86] Bailly, Histoire de l’Astronomie Moderne p. 599-600. His appreciation of Arzachel is positive ; he writes: “Arzachel perfectionna la méthode de déterminer les éléments de la théorie du soleil ». He considers that the Toledan tables were justified because the tables of Al-Battani were becoming obsolete. However Abraham bar Hiya, Maimonides and R’ Judah ha-Levi still rested on them.

[87] ראשית חכמה Which is a part of the better known tittle: חקות השמים, see Benjacob, Otsar ha-Sefarim, Vilnius 1880, p ; 199b, n° 794.

[88] Thus 365.242308days. See *Sefer ha-Ibbur*, p. 9b line 8. This value is very good, much better than that of Al-Battani.

[89] Instead of the exact value of 6.66 minutes per year.

אך זמן התקופה הידוע בציבור אינו זמן התקופה המדויקת. התקופה הידועה היא חלוקת השנה בצורה בלתי מדויקת לארבעה חלקים (של צ"א יום ושבע וחצי שעות) ולפי חשבון זה פעמים שיחול פסח בעונת החורף. (על סמך חשבון זה) טענו על היהודים שנשכחו מהם ידיעת עיקרי תורתם ולא דנים לפי הכלל הנקבע בתלמוד, לבלתי עשות פסח רק בתקופת ניסן, כי הרי חל אצלם פסח קודם כניסת תקופת האביב. אמנם דבר זה נכון רק בהתאם לחשבונם, והוא החשבון המפורסם והידוע. אך לא שמו לב לחשבון המדוייק של תקופת החמה אשר לא ידוע ומפורסם. ולפי חשבון זה לא יחול פסח בשום אופן עד שיהיה השמש בראש טלה, ולו רק יום אחד. ולא קרה בדבר שום טעות ושבוש במשך אלפי שנים. **רגע זה שבו תחול תקופת רב אדא חל בקרוב עם הזמן שמצא האסטרונום הערבי אלבתאני והיא הקביעה הברורה והאמיתית ביותר.**

תרגום חדש "הכוזרי המפורש" הרב מרדכי גניזי.

R' Judah ha-Levi explains that the Christian surroundings accuse the Jews of having forgotten the principles governing their calendar, they don't respect the principle given in the Talmud[90] and they accept that Passover occurs more and more early, before the vernal equinox. With the time Passover will be celebrated in the winter. Ha-Levi explains that it is the outcome of the error of the length of the Julian year, but the Jewish calendar is still correct with regard to the less known tekufah of Adda. Pessah does never begin before the sun reaches the beginning of Aries. The tekufah of Adda is in complete agreement with the observation of Al-Battani.

We saw above that Al-Battani "observed" an apparent equinox on Wednesday 19 September 882 at 0h 51m a.m. aRABMT[91] or 0h 36m a.m. JerABMT.[92] We showed also that the span of time t_1 between the apparent vernal equinox and the mean equinox is equal to two days; we conclude that the mean equinox was on Monday 17 September at 0h 36m JerABMT. The molad of Tishri 4643 was 1 – 17 – 1026
The molad of Elul 4642 was 0 – 5 – 233
29 – 21 – 853 – 74 (7th year of a cycle)
The tekufah of Adda 30 – 3 – 6 – 74 was on Monday at 3h 6hal 74 reg. J.T. or Sunday 16 September 882 at about 21h Jerusalem civil time.
The tekufah of Adda occurred thus 3h 36m before the experimental mean equinox of Al-Battani. The span of time is 4h 10m according to modern data. This is certainly the meaning of R' Judah ha-Levi's quotation. However, he must have been aware that the year of Adda is too long and that it raised problems:

Tropical year	365d	5h	48m	45.97s	= 365.2421987268 d	
Ptolemy	365	5	55	12	= 365.2466666666 d	
Adda	365	5	55	25.44	= 365.2468222222 d	
Al-Battani	365	5	46	24	= 365.2405555555 d	Nallino Vol I p. 42, 212.
Ibn Ezra	365	5	48	55	= 365.2423032407 d	365.25 – 1/130

With regard to the year of Al-Battani: the year of Adda is 9.024m too long.
In 882 the tekufah of Adda preceded the mean equinox by 216m. The coincidence occurred 216/9.024 = 23.94 years later in 906.

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90 The rule of shitsar, B. Rosh Hashanah 21a.
91 Ar-Raqqah Al-Battani mean time.
92 Jerusalem Al-Battani mean time.

In 1130, at the time of R' Judah ha-Levi: (1130 – 906) * 9.024 = 2021.38m = 1.40 days.
In 1147 at the time of Ibn Ezra: (1147 – 906) * 9.024 = 2174.78m = 1.51 days.[93]
In 1178 at the epoch of Maimonides: (1178 – 906) * 9.024 = 2454.53m = 1.70 days.[94]
With regard to modern data: the year of Adda is 6.6 m too long.
In 882 the tekufah of Adda preceded the mean equinox by 250m. The coincidence occurred 250/6.66 = 37.54 years later in 920. The delay of the tekufah of Adda with regard to the modern mean equinox in 1147 (date of the calculation of Ibn Ezra in Verona) was (1147 – 920) * 6.66 = 1511.82m = 1.05 days.[95]

10. The Alfonsine tables.[96]

The Alfonsine tables, the work of the Castilian King Alfonso X,[97] surnamed "el Sabio", the Wise, must be considered as a last offshoot of Arabian astronomy in Spain. He assembled around him a number of Arab, Christian and Jewish astronomers under the leadership of the Jewish astronomer, Isaac ben Sa'id, the Hazzan[98] of Toledo, to construct new astronomical tables. These tables rested to a large extend on the former tables and on the old principles of the astronomy of Ptolemy; they represented at the very most a bringing up to date. The tables take into account the erroneous theory of the trepidation which Al-Battani, R' Abraham bar Hiya and Yessod Olam had not considered. These tables had a tremendous influence; they were printed in Venice in 1483 and later in 1488, 1492, 1517 and in Paris in 1545 and 1553. They were thus in use during three centuries before they lost their authority because of their increasing lack of precision.
The first day of the era of Alfonso is Saturday1 June 1252, coinciding with the beginning of his reign. The epoch is Friday 31 May 1252 at noon. Similarly the first day of the Christian era (the era of incarnation)[99] is Saturday 1 January 1 C.E.[100] and the epoch of the tables is Friday 31 December 1 B.C.E.[101] The astronomical days used in the Alfonsine tables are astronomical days of the type (n – 1, n). This can be demonstrated by the comparison of the data of the Alfonsine tables and modern data.

11. Conclusion.

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[93] See above R' Abraham ibn Ezra. It is thus normal the apparent vernal equinox precedes the tekufah of Adda by 2 + 1.5 = 3.5 days.
[94] See J. Ajdler, Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam p. 180: the span of time from Al-Battani mean equinox until tekufah of Adda was 41 hours instead of 40.91 h according to the present simplified calculation.
[95] See above the chapter about ibn Ezra.
[96] See Les Tables alphonsines avec Les canons de Jean de Saxe, édition, traduction et commentaire par Emmanuel Poulle. Paris 1984. See also a short analysis by J. B. Delambre : Histoire de l'astronomie du Moyen-âge.
[97] Alfonso X, King of Leon and Castile (Toledo 1221-Sevilia 1284).
[98] Yessod Olam IV, chap 7, p 11c and Delambre, Histoire de l'astronomie du moyen-âge p. 248.
[99] Christian fundamental dogma according which The G'd makes himself man. The Jewish texts spoke of תאריך ההגשמה.
[100] At noon of this day: 1 721 424 JD.
[101] This is explicitly mentioned in the canon of Jean de Saxe, an astronomer living in Paris in the beginning of the fourteenth century who wrote canons for the Alfonsine tables, see Poulle, p. 47. This moment, expressed in modern notations, is:1 721 423 JD.

The origin of the present paper is a challenging sentence found in *Sefer Heshbon Mahalekhot ha-Kokhavim* p. 62, line 12, which had bothered me for a long time.[102] It appears that the subject was well known by the ancient scholars, it was even so evident for them that we don't find any information on this subject in their books. It is really, by chance, that R' Abraham bar Hiya and Isaac Israeli, writing for lay minds, felt it necessary to give this precision which allowed reconstructing the way on which Christian Arab and Jewish astronomers counted the days.
Without it I would never have imagined the complexity of the problem.
It was not easy to clarify this involved subject.
It appears that R' Abraham bar Hiya and Isaac Israeli followed indeed the majority of the astronomers of their time.
As all the other historical researches of this type, the achievements of this paper are thus limited and remain in the scope of "שכחו וחזרו ויסדום"[103], we succeeded clarifying the conventions of each author and elucidating them.

12. Annex.

A. The epoch and radices of Al-Battani.

The main epoch of Al-Battani is 29 February or 0 March 880 (modern style) or 29 February 879 (old style) at noon (ancient local mean time of Al-Battani corresponding to 12h 16m 24s modern local mean time) in ar-Raqqah: 39°; 03' (east of Greenwich).
ΔT = 36 m.[104] The epoch was thus 12h – 2h 36m 12s + 16m 24s = 9h 40m 12s UT = 10h 16m 12s ET.[105]

The radices are L_0 = 342°; 48' 50"
$L_{(}$ = 165°; 40' 17"
Modern values calculated with Meeus' Tables of moon and Sun.[106]

	Sun			Moon		
800	283°	33'	32.70"	288°	25'	28.43"
80		36	54.50	174	18	23.45
20 Feb.	49	16	56.52	298	49	11.37
9 d	8	52	14.97	118	35	15.25
10h		24	38.47	5	29	24.59
16m 12s			39.92		8	53.64
	342°;	44'	56.98"	165°	46'	36.73"

[102] It was the origin of the discussion with eng. Yakov Löwinger.
[103] שבת ק"ד א', יומא פ' א', סוכה מ"ד א', מגילה ב' א'
[104] ΔT, the difference between the Dynamic time (uniform time) and the universal time (earth clock). TD = UT + ΔT. For its evaluation see Astronomical Algorithms, J. Meeus, Willmann-Bell, Virginia, chapter 9, pp. 71-75.
[105] Ephemeris time. Today it speaks of TD, Dynamical time. These denominations represent the uniform time by contrast with the universal time which depends on the terrestrial clock.
[106] These tables are already out of date. However the didactical power of tables is invaluable.

B. The epoch and the radices of the Toledo Tables.[107]

These tables were constructed on the meridian of Toledo by Abu Ishaq al-Naqqas ibn Zarqali around 1080. The epoch of the tables was shifted to the beginning of the era of the Hijra. The epoch[108] is thus at noon July 14th, 622 C.E. The radices are the mean position of heavenly bodies at the epoch. According to these tables the radices are:
$L_0 = 3^S$ 23°; 41’ 11” = 113°; 41’ 11”
$L_{(} = 4^S$ 0°; 58’ 18” = 120°; 58’ 18”
Modern values calculated with Meeus’ Tables of moon and Sun.
The epoch expressed in dynamic time is 12h + 16.44m[109] + 25m[110] + 13.41m[111] ~ 12h 55m.

	Sun	Moon
600	282° 01’ 16.48”	32° 39’ 29.81”
22	– 19 25.18	32 20 45.93
9 July .	187 16 22.78	343 30 55.22
5 d	4 55 41.65	65 52 55.14
12h	29 34.17	6 35 17.51
55m	2 15.53	30 11.76
	114°; 25’ 45.43”	121° 29’ 35.37”

C. The epoch and the radices of R’ Abraham bar Hiya.

The epoch of the tables of R’ Abraham bar Hiya was on Wednesday 21 September 1104 C.E. at noon (Al-Battani) mean time in Jerusalem.[112] The longitude of Jerusalem is 35.2°, it corresponds to a difference of 2h 20m 48s. The difference ΔT = TD – UT = 20m 49s. Therefore the epoch was 12h – 2h 20m 48s + 16m26 + 20m 49s = 10h 16m 27s

The radices of Bar Hiya were L_0 = 187° and $L_{(}$ = 187° 6’ 48”
Modern values calculated with Meeus’ Tables of moon and Sun.
The epoch expressed in dynamic time is thus 9h 37m a.m.

	Sun	Moon
1100	285° 51’ 57.13”	132° 04’ 26.36”
04	1 50.73	170 42 55.17
17 Sept	256 16 5.91	185 51 47.15
4d	3 56 33.32	52 42 20.11
10h	24 38.47	5 29 24.59
	186°; 31’ 5.56”	186° 50’ 53.38”

[107] See A Survey of the Toledan Tables, G. J. Toomer, Osiris, Vol 15, 1968 pp. 5-174.
[108] Artificially shifted to the beginning of the Hijra despite their redaction in about 1080 C.E.
[109] The ancient mean time + 16.44m = modern mean time.
[110] The time difference between Toledo and Greenwich.
[111] ΔT, the difference between the Dynamic time (uniform time) and the universal time (earth clock). TD = UT + ΔT.
[112] Although bar Hiyya used the tables of Al-Battani without making the translation ar-Raqqah – Jerusalem.

D. The epoch and the radices of the Alfonsine Tables.

The epoch of Alfonso Friday 31 May 1252 at noon (mean time, according to the use of the astronomers of the middle age: the equation of time is additive from mean time to true time). The tables are calculated for the longitude of Toledo: 4.03° W.

The radices are[113] L_0 = 76°; 37’ 12.65” and $L_($ = 336°; 5’ 21.20”

Modern values calculated with Meeus’ Tables of moon and Sun.[114]
The epoch expressed in dynamic time is 12h + 16.44m[115] + 16.12m[116] + 13.41m[117] ~ 46m

	Sun			Moon		
1200	286°	38’	05.26”	79°	57’	25.67”
52		23	59.43	59	17	57.24
30 May	147	50	49.56	176	27	34.12
1 d		59	08.33	13	10	35.03
12h		29	34.17	6	35	17.51
46m		1	53.34		25	15.29
	76°;	23’	24.83”	335°	54’	4.86”

Example 1.
The mean longitude of the moon at the beginning of the astronomical day Friday 3 July 1327 according to the Alfonsine tables is: $L_($ = 258°; 42’ 48.95”
Modern value calculated with Meeus’ Tables of moon and Sun.[118]
The considered moment expressed in dynamic time is 2 July 1327 at 12h + 16.44m[119] + 16.12m[120] + 10.26m[121] = 42.82m

1300	27°	50’	24.98”
27	332	26	46.14
29 June	211	45	04.95

[113] See Poulle, E. Les Tables Alphonsines avec les Canons de Jean de Jean de Saxe, Paris 1984, p. 124.
[114] In order to appreciate the exceptional precision of these astronomers’ observations, it is interesting to note that the increase in mean longitude of the sun and moon during one hour is 2’ 27.85” for the sun and 32’ 56.46” for the moon.
[115] The ancient mean time + 16.44m = modern mean time.
[116] The time difference between Toledo and Greenwich.
[117] ΔT, the difference between the Dynamic time (uniform time) and the universal time (terrestrial time). TD = UT + ΔT.
[118] In order to appreciate the exceptional precision of these astronomers’ observations, it is interesting to note that the increase in mean longitude of the sun and moon during one hour is 2’ 27.85” for the sun and 32’ 56.46” for the moon.
[119] The ancient mean time + 16.44m = modern mean time.
[120] The time difference between Toledo and Greenwich.
[121] ΔT, the difference between the Dynamic time (uniform time) and the universal time (terrestrial time). TD = UT + ΔT.

3 d	39	31	45.08
12h	6	35	17.51
42.82m		23	30.53
	258°; 32’ 49.19”		

Example 2.

The mean conjunction of July 1327 was according to the Alfonsine tables on 20 July 3h 58m ancient mean time of Toledo or 3h 58m + 16.44m + 16.12m = 4h 30.56m p.m. Greenwich modern mean time. Indeed this astronomical day began at noon of the civil day 19 July. Therefore this moment was on the civil day 19 July 16h 30.56m UT.

Modern value calculated with Meeus’ Syzygies Tables.

1300	21d	07h	05m	
27	2	05	12	
June	26	04	24	
June	49	16	41	
July	19	16	41	TD
			– 10.26	
July	19 d	16h	30.34	UT

The concordance of the longitudes is good; the coincidence of the times of conjunction is perfect. The two former examples prove that the astronomical days used in the Alfonsine tables are of the type (n – 1, n).[122]

E. The epoch and the radices of the Tables of King Pedro IV of Aragon and III of Catalonia (1319 – 1387) so-called “el Ceremonioso”.[123]

These tables were established by the Jewish scholar Jacob Cirsano.[124] Their epoch is Saturday 29 February 1320 (old style) or1321 (new style) at noon. The first day of the era of King Pedro is Sunday 1 March 1321.[125] The tables were calculated for the longitude of Barcelona: 2.2° E.

The radices are $L_0 = 11^S$ 5° 48’ = 335° 48’

L($= 11^S$ 13° 59’ = 343° 59’

Modern values calculated with Meeus’ Tables of moon and Sun.

[122] This important point must have escaped the attention of the great Delambre. Indeed in his Histoire de l’Astronomie du Moyen-âge, pp 251-252, He calculated the mean longitude of the sun on 20 September 1476 according to the Alfonsine tables and his modern tables and found a difference of 4’. In fact he made the calculation according to his modern tables on 20 September at noon instead of 19 September at noon and therefore he found an error of 4’ instead of about 1°. However Delambre could not ignore that the astronomers at the end of the fifteenth century noted the increasing imprecision of the Alfonsine tables.

[123] *Las tablas astronomicas del Rey Don Pedro el Ceremonioso*. Jose Maria Millas Vallicrosa. Madrid 1962.

[124] See the Hebrew text of the table’s Canons p. 103. See also A. Pannekoek: A History of Astronomy, Dover Edition p. 184 who mentions Jacob Corsono.

[125] The Latin canon p. 145 and the Hebrew canon p. 105 state clearly that year 1 of the era of King Pedro = year 1321 of the era of the incarnation (Christian era). Therefore the statement pp. 23 and 47 that the first year of the era was 1320 is erroneous. It is only in 1321 that 1 March = Sunday.

The considered moment expressed in dynamic time is 28 February 1321, 12h + 16.44m[126] – 8.8m[127] + 10.53m[128] = 18.17m

	Sun			Moon		
1300	287°	24'	13.39"	27°	50'	24.98"
21		– 5	05.78	262	57	40.90
19 Feb.	49	16	56.52	298	49	11.37
9 d	8	52	14.97	118	35	15.25
12h		29	34.17	6	35	17.51
18.17m			44.77		9	58.54
	345°;	58'	38.04"	354°	57'	48.55"

We must conclude that the manuscript used by Millas was corrupt, both radices are incorrect.

F. The epoch and radices of R' Levi ben Gershom – Ralbag (Bagnoles-sur-Sèze, Gard 1288 – Perpignan 1344).

Ralbag considered astronomical days of the type (n, n + 1) like the Christian astronomers. His year begins on 1 March 1302. His epoch is at noon, ancient mean time or 12h 16m modern mean time of 28 February 1302 (new style) or 28 February 1301 (old style) in Orange 4° east of Greenwich. It corresponds exactly to 12h UT.
ΔT = 675.66 s = 11.26 m. The epoch was thus at 12h UT or 12h 11.26m TD.

The radices are[129] $L_0 = 11^S 15°; 31' 28" = 345°; 31' 28"$
L(= $11^S 20°; 36' 58" = 350°; 36' 58"$
Modern values calculated with Meeus' Tables of moon and Sun.

	Sun			Moon		
1300	287°	24'	13.39"	27°	50'	24.98"
2		– 28	38.80	258	46	10.07
19 Feb.	49	16	56.52	298	49	11.37
9 d	8	52	14.97	118	35	15.25
12h		29	34.17	6	35	17.51
11m 26s			28.17		6	16.63
	345°;	34'	48.42"	350°	42'	35.81"

The correspondence is excellent. Ralbag, by contrast with other tables, rested on observation.

[126] The ancient mean time + 16.44m = modern mean time.
[127] The time difference between Barcelona and Greenwich.
[128] ΔT, the difference between the Dynamic time (uniform time) and the universal time (earth clock). TD = UT + ΔT.
[129] The Astronomical Tables of Levi ben Gershom by Bernard Goldstein, 1974, p. 170 and p. 172.