The Precession of the Equinox in the Jewish Astronomy of the Middle Ages.

The phenomenon of the precession of the equinox was discovered by Hipparchus at the end of the second century before the Common Era. Ptolemy and al-Battani differed only about its quantification. Unfortunately, in this particular case, the influence of the Indian astronomy, exerted through Thabit ben Qura, ibn Zarkali and mainly the Alfonsine Tables, was negative and introduced the confusing concept of the trepidation into the astronomy of the middle age and delayed accordingly its progression. However most of Jewish scholars did not accept it.

In the present paper we show the importance of the precession in the writings, not only of Jewish astronomers, but also of Jewish scholars and thinkers. We quote different authors and elucidate their difficult quotations. Of exceptional importance is a nearly unknown quotation from Abraham Zacut, extract from *Tekhunat Zacut,* which is completely elucidated. We show how Jewish scholars were aware of the most recent developments of astronomy and were deeply involved in the discussions and argumentations around the controversial and much debated questions in the news.

The Precession of the Equinox in the Jewish Astronomy of the Middle Ages.

1. Introduction.

The secular variation of the celestial longitude of the stars is called the precession of the equinox. It was discovered during the second century before the Common Era by the Greek astronomer Hipparchus, by the comparison of the longitude of the star α Virginis (Spica) that he had measured, with the longitude that the astronomer Timocharis had found one and a half century before. The ancients could not make meridian observations, as they had no clock, but they could measure angular distances allowing them to link up a star to the sun, taking as an intermediary reference, the moon or the planet Venus. The connection of the star to the moon or to Venus was made by night or at twilight and the connection of the intermediary celestial body, was made by day, taking into account the displacement that occurred during the span of time passed. Timocharis[1] and Hipparchus used a very clever method. During a lunar eclipse, the center of the shadow is located in the ecliptic in a point diametrically opposed to the position of the sun and its longitude l_0 + 180° was known to the Greek astronomers who had a table of the sun's movements. Therefore measures of angular distances during the lunar eclipse allowed the determination of the celestial longitude of a star near to the ecliptic by linking it to the center of the shadow. Timocharis found a longitude of 172° for Spica in about 273 B.C.E. while around 129 B.C.E. Hipparchus found 174°. He concluded that the vernal point had undergone a displacement in the retrograde direction of 2° in 144 years with regard to the stars. The longitude of the same star was 203°;08' in 1950, hence a displacement of about 31° in 2222 years or 50.2'' in a year.[2] Hipparchus understood the precession as a movement of direct rotation of the eighth sphere, the sphere of the fixed stars, around the axis joining the poles of the ecliptic. The position of these two poles was fixed with regard to the stars. Copernicus formulated the modern interpretation: The direction of the rotation axis of the earth is not fixed in the space but its angle with the ecliptic is constant. The direction of the rotation axis of the earth draws a revolution cone around the axis joining the poles of the celestial ecliptic. The corresponding trajectory of the northern celestial pole is a little circle of latitude 90° - ε covered at the rate of

[1] See Bailly J.S. Histoire de l'astronomie Moderne depuis la fondation de l'école d'Alexandrie, Paris 1779, Vol 1,p. 446. See Matter, J. Histoire de l'Ecole d'Alexandrie Paris 1844, p. 175 and Matter, J. Essai Historique sur l'Ecole d'Alexandrie, Paris 1820, p. 137.

[2] Or 1° in 71.68 years. See Pannekoek, A. A History of Astronomy London 1961 and Dover USA, 1989, pp. 149 – 150.
See Danjon, A. Astronomie Generale, Paris 1958 and 1986, p. 79.

50.2'' per year. The point γ takes about 25816 years to cover the ecliptic. It is also the period of revolution of the celestial poles (of the equator) around the poles of the ecliptic. At this stage, it was considered until the first half of the 18th century that the ecliptic is a plane rigorously fixed. Furthermore the scholars were aware that the angle ε diminishes slowly and therefore the circle described by the northern pole around the pole of the ecliptic is in fact, not a circle but an open curve.

2. R. Abraham bar Hiyya ha-Nassi (? – c 1136).
    1. Sefer *Tsurat ha-Arets*.[3]

השער העשירי

במהלך כוכבי שבת ופירוש הדעות אשר החכמים נחלקים עליהם במהלכות כוכבי שבת...

בטלמיוס ואבר"כוס וכל חכמי רומי ויון היו מסכימים בכוכבי שבת שהם מתגלגלים חלק א' בכל מאה שנה וגומרים מהלך הרקיע בששה ושלושים אלף שנה והם נסמכים במהלך על קוטב אופן המזלות ועי"ז אין מרחבן מהאופן המזלות מתחלף בעינינו. וחכמי ישמעל הבאים אחריהם לא הייתה דעתם כדעת בטלמיוס וחבריו בסדר מהלך הזה וחלקו עליהם במדתו ואמרו כי הכוכבים האלו מתגלגלים בכל מאה שנה חלק א' וחצי וגומרים מהלך כל הרקיע בעשרים וארבע אלף שנה. וחכמי הודו וכלדיים היו להם בכוכבים האלה דעת אחת והם סוברים שכוכבי שבת אינם מקיפים את כל הרקיע אבל הם מתקדמין ומתאחרין ח' מעלות מאופן המזלות למזרח ולמערב והגורם למהלך הזה היא קוטב אופן המזלות שהוא סובב על עגלה קטנה אשר קטרה שמונה מעלות למזרח ולמערב והקוטב מקיף את העגלה הזאת באלף ושש מאות שנה. ומפני גלגל הקוטב על העגלה הזאת עלה בלב רבים מבני אדם על כוכבי שבת שהם מקיפים את כל הרקיע כי לא נגלה אליהם סוד בזה ולא הכירו גלגל קוטב האופן אשר הוא מיסב את כוכבי שבת ממערב למזרח שמונה מעלות למזרח בשמונה מאות שנה והוא מזיר אותו כמו כן שמונה מעלות בשמונה מאות שנה וחוזרים אל מקומם הראשון אחרי אלף ושש מאות שנה, בימים אשר הקוטב מקיף בהם את כל העגלה וחוזר אל הנקודה אשר התחיל ממנה לסבוב. ומפני שמהלכן נגבל ממהלך קוטב האופן משם ולא נמצא למרחבן מן האופן חלוף מענינו, ומהלך הקוטב נקרא קדום ואיחור וע"ז נקראו כוכבי שבת בלע"ז ובלשון יונים ורומים (בא"י ביני"ש) אשר פירושו בלשונם לך ושוב, כאלו אמרת כוכבים שובבים שהם הולכים ושבים.... ובא אלב"תאני אחריו בכדי שבע מאות ועשרים שנה ועשה כמעשיהם ומצא בין מחקרו ובין מחקר בטלמיוס כדי י"א מעלות וגזר על מהלכן שהיא מעלה אחת וחצי במאה שנה. ונמצאו הכוכבים האלה מזמן מחקר הקדמונים עד זמן אלב"תאני הלכו חמש עשרה מעלה וחומש מעלה בכדי אלף מאה וארבעים. ולדברי אנשי הודו.....והטענה הגדולה עליהם שהם אמרו הכוכבים האלה הולכים וחוזרים בכדי שמונה מעלות מוסיפין עליהם ואנו מוצאים אותם מתקדמים על המקום אשר היו בזמן מחקר הקדמונים על מחקר אלב"תאני יותר ט"ו מעלות. ומכאן יהיו דברי אנשי הודו וכל סייתם נשברים ונסתרים....

About the movement of the fixed stars and the explanation of the different opinions about which the scholars differ.

Ptolemy, Hipparchus and all the Roman and Greek scholars did all agree that the movement of the fixed stars is one degree in 100 years and they cover the whole sky in 36000 years and they turn around the pole of the ecliptic and therefore their latitude, the distance from the ecliptic, doesn't change. But the Arabic scholars had not the same opinion as Ptolemy and his colleagues about this displacement and they contradicted the formers and considered that this movement is 1.5° in 100 years so that the stars complete the rotation of the sky in 24000 years. The Indian and Chaldean scholars had another opinion in this subject. They considered that the stars don't complete a rotation around the sky but their movement is a movement of pushing ahead and turning back on 8° along

[3] Basilea 1545; with a Latin translation, Munster 1545 and with a commentary by R. Jonathan from Radunia, Lithuania, Offenbach 1720. We used this last edition.

the ecliptic to the east and then to the west. The cause of this movement lies in the pole of the ecliptic which turns on a little circle whose diameter is 8°,[4] directed to the west and the east and the pole of the ecliptic covers this little circle in 1600 years. Because of the movement of the pole on this little circle, many people still thought that the stars cover the whole sky because they did not understand the principle of the little circle which pushes the stars ahead on a distance of 8 degrees during 800 years in one direction towards the east and then turns them back in the opposite direction on a distance of 8 degrees during 800 other years while the pole of the ecliptic covers the little circle and comes back to the initial point after 1600 years. And because the movement of the stars derives from the movement of the pole of the ecliptic on this little circle and the latitude of the stars does not change, this movement is a backward and forward movement. It is for this reason that we speak of 'fixed stars' and in foreign language, Roman and Greek: 'va et vient' which means in their language forward and backward. As if we spoke of uncivilized stars which go backward and forward…. Then came al-Battani,[5] about 720 years after him (Ptolemy) and he found between his measurements and those of Ptolemy 11°[6] and he concluded that the movement of the precession is 1.5° in 100 years. It appears thus that the stars advanced from the time of the observations of the ancients until the time of al-Battani by 15°;12''[7] in 1140 years.[8] Now according to the Indian scholars… The great argument against them is that they claim that the stars move 8° backwards and then 8° in the direct direction but we find that the stars moved always in one direction and since the first observations of the ancients the stars moved by more than 15°. This destroys their claim.

2. Sefer *Mehalekhot ha-Kokhavim*: chapter 17.

שער י"ז.

מהלך כוכבי שבת נסמך על קוטבי מהלך אופן המזלות ומדת מהלכן כולן מדה אחת שווה בשיעורה לכל אחד מהם ומרחב כל אחד מהם מאופן המזלות עומד על ערך אחד, אינו לא מוסיף ולא גורע ולא נוטה מצד אל צד. על הסדר הזה מצאו אותו כל החוקרים עליו מימי קדם וכל הקדמונים מצאו סדר מהלכן (ממזרח למערב) [ממערב למזרח] כמהלכן כוכבי לכת המיושר וכולן הסכימו על המהלך שאין בו שום חילוף. והקדמונים העידו עליו שהוא מעלה אחת בכל מאה שנה ומקצת האחרונים הבאים אחריהם העידו שהוא מעלה אחת בכל שבע וששים שנה. ואנו סדרנו מהלכן בלוח שעשינו לו על מעלה אחת בכל מאה שנה כדעת הקדמונים. ועל ערך מהלך הזה מצאו מהלך גובה הרום וראשי התנינים לחמשת כוכבי הנבוכה.....

[4] This cinematic movement is problematic because, in this model, the pole of the equator is a fixed point while the pole of the ecliptic covers a little circle having a diameter of 8°. During this movement of precession the latitude of the stars is constant but the angle of 23°;35' between the equator and the ecliptic or between the pole of the equator and the pole of the ecliptic, becomes now the angle between the pole of the equator, a fixed point and the pole of the ecliptic, which covers a little circle. This angle varies between ε + 4° and ε - 4° if ε is the angle between the pole of the equator and the center of the little circle of the pole of the ecliptic. This model is not satisfactory.

[5] al-Battani (c. 858 – 929).

[6] al-Battani writes 11°;10', see Nallino, al-Battani opus Astronomicum vol 1, p. .

[7] For a span of time of 1140 years we should have a precession of 11.40 * 1.5 =17.10°. Apparently he refers to the historical values: Timocharis: 172°, Hipparchus: 174°, Ptolemy: 176°;40' and al-Battani 187°;50'. Hence a precession of 15°;50' between Al-Battani and Timocharis. Abraham bar-Hiyya must have a value of 172°;40' for the time of Timocharis. According to the figures of Abraham bar Hiyya, 15°;10' in 1140 years correspond to a precession of 1.33 ° in 100 years. Now when we compare the value of bar Hiyya in 1104: 191°;14' with the value of al-Battani in 880: 187°;50', we see that he practically adopted a precession of 3°;24' in 224 years or 1.52 ° in 100 years. In general the precession adopted by Abraham bar Hiyya is only 3°;20' in 224 years or 1.49° in 100 years.

[8] Between the observation of Timocharis (– 272 C.E.) and al-Battani (880 C.E.) we have a span of time of 1152 years. al-Battani speaks of 1140 years and considers that the experience of Timocharis was in – 260 C.E.

Chapter 17.

The movement of the fixed stars rests on the pole of the ecliptic and the rate of their movement is the same for all the stars whether on the ecliptic or with latitude. All the searchers, beginning from the antiquity, found the precession with the same properties, moving from west to east[9] [in the direct direction, toward the diurnal rotation of the sky] like the movement of the planets and all agreed on a constant movement of precession without variation. The ancients claimed that it is 1° in 100 years but other claimed that this movement is 1° in each 67 years. But we calculated the movement of the stars according to the rate of 1° in 100 years like the opinion of the ancients. We calculated also at the same rate the movement of the apogee and the perigee and the ascending nodes of the five planets…

3. *Luhot ha-Nassi*: Tables of precession.

In his tables R. Abraham bar Hiyya wrote a table of precession based on the value given by Ptolemy: a precession of 1° in 100 years. Nevertheless, at the end he wrote a certain number of tables giving the correction in order to agree with al-Battani's tables. This was also the case of the precession for which he gave the correction allowing to agree with the precession of 1° in 66 years.

4. *Luhot ha-Nassi*: Tables of stars.

Abraham bar Hiyya wrote three tables of stars, the first was devoted to the stars of the first magnitude (14 stars), the second to the main stars of second magnitude (14 stars)[10] and the third to the main other stars (48 stars).[11] The abridged catalogue of R. Abraham bar Hiyya was constructed for the beginning of the cycle 257, thus for his epoch of 21 September 1104.

The reference catalogue is the monumental catalogue of Ptolemy which contains 1022 + 3 = 1025 stars established for the date of 20 July 137 C.E.[12] These 1025 stars are divided into three categories: 360 stars belong to boreal constellations, 349 stars belong to zodiacal constellations and 316 stars belong to austral constellations. According to al-Battani, the catalogue of Ptolemy was based on a catalogue established before by Menelaüs[13] in 92 C.E, which Ptolemy adapted and expanded.[14] Although this attribution is contested, it played an important role in the interpretation and the quantification of the precession.[15]

al-Battani established a comparable catalogue for the year 880 C.E.[16]

[9] The diurnal movement is from east to west (the sun rises at the east and sets at the west). The movement of the sun on the ecliptic is in the direct direction, from west to east, toward the diurnal movement of the sky.
[10] The last star of this second list is 'khsil which is Sahil which is of the first magnitude'
[11] MS Malatestiana pp. 58b – 59b and MS Paris folio 35 a – 36a.
[12] See Toomer pp. 341 – 399.
[13] Astronomer who lived in Rome in the second half of the first century.
[14] See Nallino, al-Battani Opus Astronomicum, Milano 1903, vol. 1, p. 124 and p. 292.
[15] See Nallino vol 2, pp. 269 – 270 and Abraham Zacut, *Ha-Hibbur ha-Gadol*, chapter 9, Salamanca 1478 (still in manuscript).
[16] See Nallino vol 2, pp. 144 – 177 and 274 – 277.

The stars of the first magnitude in the catalogue of Ptolemy.

Number	Modern Name	Other name	Reference Toomer	Longitude	Latitude
1	α Boo	Arcturus	V,23	177°	+31°;30′
2	α Lyr	Waga	VI,1	257°;20′	+62°
3	α Aur	Capella	XII,3	55°	+22°;30′
4	α Tau	Aldebaran	XXIII,14	42°;40′	– 5°;10′
5	α Leo	Regulus	XXVI,8	122°;30′	+ 0°;10′
6	β Leo	Demebola	XXVI,27	144°;30′	+11°;50′
7	α Vir	Spica	XXVII,14	176°;40′	– 2°
8	α Psa	Formalhaut	XXXII,42	307°	–20°;20′
9	α Ori	Betelgeuze	XXXV,2	62°	– 17°
10	β Ori	Rigel	XXXV,35	49°;50′	–31°;30′
11	θ Eri	Eridanus	XXXVI,34	0°;10′	–53°;30′
12	α CMa	Sirius	XXXVIII,1	77°;40′	–39°;10′
13	α CMi	Procyon	XXXIX,2	89°;20′	– 16°;10′
14	α Car	Canopus	XL,44	77°;20′	– 75°
15	α Cen	Bungala	XLIV,35	218°;20′	–41°;10′

Table 1. The stars of the first magnitude according to Ptolemy in 137 C.E. The longitude of 218°;20' of α Cen seems to be a misprint in Toomer and it should be 188°;20', thus Libra 8°;20' instead of Scorpius 8°;20.

The stars of the first magnitude in the catalogue of al-Battani.

Number	Modern Name	Other name	Longitude	Latitude
1	α Boo	As-simak ar-ramih	188°;10′	+31°;30′
2	α Lyr	An- nasr (capra)	268°;30′	+62°
3	α Aur	Capella	66°;10′	+22°;30′

4	α Tau	Aldebaran, ad-dabaran	53°;50’	– 5°;10’
5	α Leo	Cor leonis, Qalb al-assad	134°	+ 0°;10’
6	β Leo	As-Sarfah, Dhanab al-assad	155°;40’	+11°;50’
7	α Vir	Spica, as simak al-azal	187°;50’	– 2°
8	α Psa	Fam al-hul al-garnubi	318°;10’	–20°;20’
9	α Ori	Mankib al-gawza	73°;10’	– 17°
10	β Ori	Rigl al-gawza	61°	–31°;30’
11	θ Eri	Ultima stellarum fluvii	11°;20’	–53°;30’
12	α CMa	Ash-shira al-yamaniyyah	88°;50’	–39°;10’
13	α CMi	Ash-shira ash-shamiyyah	100°;20’	–16°;10’
14	α Car= αNav	Canopus, suhayl al yamani	88°;20’	–75°
15	α Cen	Centaurus, rigl al-faras	199°;30’	–41°;10’

Table 2. The stars of the first magnitude according to al-Battani in 880. al-Battani counted an additional star of magnitude 1 in the constellation of Sagittarius, long: 274°; lat: 6°;30’. It is certainly a mistake; this star has the magnitude 5. See Nallino vol 2, p. 163, note 16.

The stars of the first magnitude in the catalogue of Abraham bar Hiyya.

Number		Modern Name	Other name	Longitude	Latitude
1	H	α Boo	**הנתמך הרומח, סימאך רמך**	191°;30’	+31°;30’
2	J	α Lyr	**נשר נופל, נתר ואקע**	271°;50’	+62°
3	G	α Aur	**מושך הרפן, אל עיוק**	69°;30’	+22°;30’
4	A	α Tau	**עין השור והוא אלדברן**	57°;10’	– 5°;10’
5	L	α Leo	**לב הארי, קלב אלאסד**	137°;02’	+ 0°;10’
6		β Leo	Denebola, Dhanab al-assad	159°	+11°;50’
7	I	α Vir	**נתמך לאכחזיו, סמאך אעזל**	191°;14’	– 2°
8	N	α Psa	**פי הדג הדרומי**	321°;30’	–20°;20’
9	D	α Ori	**צד תאומים ימיני, מנתכט גוזא אימן**	81°;30’	– 17°

10	B	β Ori	**דגל תאומים, דגל אלגיזא**	64°;20’	–31°;30’
11	C	θ Eri	**אחרית הנהר, אכר אל נהר**	14°;40’	–53°;30’
12	F	α CMa	**כלב גדול, שערי עכור**	92°;10’	–39°;10’
13	E	α CMi	**הכלב הקטן, שערי גמיעא**	103°;40’	–16°;10’
14	N	α Car	**כסיל והוא סהיל מהערך ראשון**	91°;40’	–75°
15	M	α Cen	**דגל הסוס מקדם,דגל אלפרס מקדמה**	202°;40’	–41°;10’

Table 3. The stars of the first magnitude according to bar Hiyyah in 1104 C.E. The star β Leo is not mentioned in the list of bar Hiyya; we find instead a star with long: 277°;18’ and lat: –6°;30’ which is unknown. It is certainly the result of a misprint anterior to the two manuscripts considered. In the third Ms Malatestiana, there is an additional problem of the shift of a column of figures, but the two former lists seem the most likely. We note also that Abraham bar Hiyya identifies the star כסיל with the star Canopus in the constellation of Argo.

The comparison of the list of al-Battani with that of Ptolemy shows that al-Battani adds 11°;10’ to the longitudes of Ptolemy. In fact, this is explicitly mentioned by al-Battani in Vol 1, chapter LI.[17] This figure corresponds exactly to a precession of 1.5 degrees in 100 years. Indeed 880 – 137 = 743 years and 7.43 * 1.5 = 11.15° = 11°;08’ which he rounds off to 11°;10’. Similarly the comparison of the list of Abraham bar Hiyya to that of Ptolemy shows that Abraham bar Hiyya adds 14°;30’ to the longitudes of Ptolemy. Now the span of time separating them is 1104 – 137 = 967 years. The precession considered in the construction of this list of stars is then 14.5 / 9.67 = 1.50°. Now if compare the list of Abraham bar Hiyya with that of al-Battani we note a precession of 14°;30’ – 11°;10’ = 3°;20’ in 224 years or 1.49° in 100 years. This is really surprising; Abraham bar Hiyya championed a precession of 1 degree in 100 years in both the first part of his book i.e. the canons of his tables and in the main tables of the second part of the book, *Luhot ha-Nassi*. Only at the end of these tables he proposed a correction table for those following al-Battani. He followed also Ptolemy in the last chapter of his book *Tsurat ha-Arets*. The position of Abraham bar Hiyya is difficult to understand, it is a real conundrum. Of course the strict application of his opinion would have reduced the difference with Ptolemy to 9.67° = 9°;40’ and his longitudes would have been smaller than al-Battani’s 200 years before, by 11°;10’ – 9°;40’ = 1°;30’. This could have been embarrassing! But he had to make a decision and the present situation was certainly not more comfortable.

Conclusion. R, Abraham bar Hiyya was fully aware of the phenomenon of the precession and of the different theories extant. In his different texts he champions the views of Ptolemy of a direct movement of the stars of 1° in 100 years. However his table of the position of the fixed stars at his epoch of 21 September 1104 C.E. was deduced from Ptolemy’s Catalogue of stars by the consideration of a precession of 1.5° in 100 years.

3. R. Abraham ibn Ezra (1089 – 1164).

[17] Nallino, vol 1, p. 124.

1. Sefer *ha-Ibbur*.[18]

דע כי מחלוקת גדולה בין חכמי המזלות בעלי הראיות ובין חכמי המזלות בעלי המשפטים והצורות. כי אנשי הצורות אומרים כי סדני הגלגל יעלו וירדו ואחרים אמרו שיש גלגל קטן במקום מזל טלה ומאזנים שיתגלגל לצפון ולדרום. על כן תהיינה מעלות השמש בחצי היום, אם הגלגל בצפון, יותר מאשר הם באמת והפך הדבר כאשר יהיה הגלגל בפאת דרום. והשתבשו על מספר המעלות: יש אומרים שהם שמונה ואחרים אמרו שהם עשר מעלות ושתי שלישיות מעלה. והנה תלמי המלך קרב מדרך העבור והוא האמת כנגד המזלות.

You must know that an important debate has arisen between the scholars of the zodiacal signs who rely on proofs and the scholars of the zodiacal signs who rely on [astrological] judgments and images. Those who trust images state that the poles of the sphere ascend and descend but the others assert that there is a small circle at the place of the vernal and autumnal points.[19] A point of this circle moves alternatively northwards and southwards. Therefore the degrees of the sun at noon, if the circle is turned northwards, seem more than they actually are and the opposite happens when the point of the circle is turned southwards. Many were mistaken about the number of degrees of the precession, some of them assert that they are 8 degrees and others state that they are 10 degrees and two thirds of one degree. King Ptolemy[20] however, was very close to the values obtained by the methods of the Jewish intercalation and this is true [when the sun's observation is made] on the ecliptic, with regard to the constellations.[21]

2. *Igeret ha-shabbat*.[22]

ובאה המחלוקת בעבור תנועת כוכבי גלגל הממזלות. כי הקדמונים אמרו כי התנועה מעלה אחת המאה שנה. והאחרונים אמרו, הששים ושש שנים. ויש אומרים בשבעים שנה. וכלי הנסיון עשוים בדרך הקרובה. כי לא יוכלו לחלק המעלות על שנים במעשה ידי אדם. ויש אומרים כי בשתי נקודות המחברת שתי עגולות קטנות. על כן התמונה פעם עולה פעם יורדת.

The controversy arose because of the movement of the fixed stars belonging to the eighth sphere, the sphere of the zodiac. The ancients said the motion is one degree in 100 years. The moderns said that it is 1° in 66 years. And others say that it is 1° in 70 years. And this controversy [is the result of the] lack of accuracy of the experimental tools. The craftsmen cannot divide with precision one degree into two parts. Others state that there are two small wheels in both intersecting points [of the equator and the ecliptic] and therefore the figure sometimes ascends and sometimes descends.

3. Sefer *ha-Ta'amim*.[23]

והנה מחלוקת אחרת בין הקדמונים שיש אומרים כי סדני גלגל המזלות יעלו גם ירדו ח' מעלות. ואחרים אמרו כי שתי עגולות הם בראש טלה ובראש מאזנים. וחכמי הודו לפי מלאכתם עשו דרך נכונה כי הם אומרים שכוכבי

[18] Abraham Ibn Ezra, *Sefer ha-Ibbur*, ed. By S.Z.H. Halberstam, Lyck 1874, p. 10a.
[19] This is another model trying to explain the phenomenon of trepidation. See details below in the chapter about the Alfonsine tables.
[20] Ibn Ezra confuses Ptolemy, the Greek astronomer of the second century (about 90 – 168) with the descendants of Ptolemy 1st (third century B.C.E), the Greek general of Alexander the great who became king of Egypt and whose descendants were kings of Egypt during more than three centuries.
[21] This text is unclear.
[22] Abraham Ibn Ezra, *Igeret ha-Shabbat*, in S.D Luzzato, *Keren Hemed* IV, 1839, Prague pp. 159 : 173 and in M. Friedlander, Transactions of the Jewish Historical Society of England, 2, 1894/95, pp. 61 – 75.
[23] MS Heb 1056 Bibliothèque Nationale de France, quoted from Shlomo Sela, Abraham Ibn Ezra and the rise of Medieval Hebrew Science, Brill 2003, p. 360.

גלגל המזלות אינם מתנועעים. והקדמונים אמרו, ובטלמיוס עמהם, כי יתנועעו לק' שנה מעלה אחת והמדקדקים אחריהם מצאו כי תנועת מעלה אחת לס"ו שנים והנכון שהתנועה לע' שנה.

There is another controversy between the ancients. Some of them assert that the poles of the zodiacal sphere, the eighth sphere of the fixed stars, ascend and descend 8 degrees- Others said that there are two circles at the equinoctial points.[24] The Indian scholars acknowledged that they worked hard and observed that the stars of the zodiacal sphere do not move. But the ancients asserted, and Ptolemy was among them, that the stars move one degree In 100 years and those, who after them observed with precision found that the motion is one degree in 66 years. The true value, however, is one degree in 70 years.

The value of 1° in 70 years is thus clearly a distinct opinion than 1° in 66 years, it is not as it could be believed a rounding off of the older value of 66 years. The value of 1° in 66 years had been already proposed by the Tabula Probata.[25] The value of 1° in 70.25 Egyptian Years comes from Ibn Yunus and is thus an independent value.[26]

4. Sefer *Keli ha-Nehoshet*.[27]

הנה אם רצית לדעת כלי הנחושת כמה שנים יש לך שנעשה, הסתכל מקום הכוכב העליון, שתשים מקומו החד על חצי השמים, והסתכל מעלת חשב האפודה, אז תמצא מעלת כוכב העליון במזל שהוא שם, והסתכל מקום הכוכב שכתבתי לך, ודע כמה מעלות יש בין שניהם, ודע שהכוכב יתנועה מעלה אחת בשבעים שנה.

If you want to know how many years ago your astrolabe was built, observe the place of the superior star by placing the pointer of the star in the rete over the middle of the heaven of the astrolabe and look at the corresponding zodiacal degree in the ecliptic of the rete. So you will know at which degree and in which zodiacal sign the superior star is. Look then at the place of the star that I wrote to you. Calculate the difference in degrees between the two measurements. You should ascertain that the star moves one degree in 70 years.

5. Ibn Ezra's commentary on Amos. 5: 8.[28]

עושה כימה וכסיל והפך לבקר צלמות ויום לילה החשיך הקורה למי הים וישפכם על פני הארץ השם שמו.

נאם אברהם המחבר, דע כי שנים מקומות יש בגלגל, והם הסדנים. ו'עש' שהוא העגלה הוא הנקרא ה'דוב', הוא קרוב מהסדן הצפוני. על כן יראה 'עש' לעולם בקיץ ובחורף, בכל הישוב הצפוני. ויש סמוך לסדן הדרומי כוכב גדול, 'אדום' יקרא, ובלשון ישמעאל,'סהיל'. ויש כוכבים קטנים קרובים אליו, ואלה יראו פעמים לכל מי שידור קרוב מקו השוה. והצפוניים לא יראו אחד מאלה, על כן קראם הכתוב 'חדרי תימן' (איוב ט:ט), בעבור שהם ביישוב אינן נראין. ונקרא 'עש' מגזירת 'עושו' (יואל ד: י"א), שהם שבעה שאינם מתפרדים. והנה נשאר לפרש 'כסיל' ו'כימה'. ודעת קדמונינו ש'כימה' הוא זנב טלה וראש שור והם ששה כוכבים נראים אע"פ שהם קטנים. וידוע בראיות גמורות שגלגל המזלות מתחבר עם הגלגל העליון בשנים מקומות, ושניהם יקראו נקודת ההשתוות. ומשם נוטה גלגל המזלות לצד צפון עשרים ושלש מעלות וחצי מעלה וחצי ששיתה, וככה לצד דרום.

[24] See details below in the chapter about the Alfonsine tables, about a model explaining the phenomenon of trepidation.

[25] See J. S. Bailly, Histoire de l'astronomie modern depuis la fondation de l'école d'Alexandrie, Paris 1779.

[26] See Nallino Al-Battani Opus Astronomicum vol 1, p. 293. See also J.J.A. Caussin de Perceval, Les Tables astronomiques de Ibn Yunus, Paris 1819, chapitre 8, p. 169 and p. 237.

[27] Abraham Ibn Ezra, *Sefer Keli ha-Nehoshet*, ed. Edelman, Koenigsberg 1845, quoted from Shlomo Sela, Abraham Ibn Ezra and the rise of Medieval Hebrew Science, Brill 2003, p. 360.

[28] Traditional commentary on Mikraot Gedolot. See also:
שני פירושי ר' אברהם אבן עזרא לתרי עשר, כרך ראשון, הושע , יואל אמוס. אוריאל סימון, בר אילן תשמ"ט.

והנה הם ג' נקודות. והשמש הולכת בדרך גלגל המזלות. על כן יתהפך הבקר לצלמות, ויום ללילה תחשיך בכל היישוב.

וטעם לקרוא הנקודה 'כימה', כי שם הייתה 'כימה' בימים הקדמונים. כי בכל מאה שנה יתגלגל גלגל המזלות ממערב למזרח קרוב ממעלה וחצי. והנה בימי בטלמיוס החכם, ואין לא רק אלף שנים, מצא 'לב האריה' על שתים מעלות, והנה היום על שמונה עשרה. וידוע, כי לעולם הוא 'לב האריה' על תשע מעלות כנגד הצורה, רק לא כנגד נקודת המחברת. והנה 'כסיל' הוא לנוכח 'כימה'.

He who made Khesil and Khima and turns the deepest darkness into morning and makes the day darken into night and calls for the waters of the sea and pours them upon the face of the earth, the Lord is his name.

Those are the words of Abraham, the author. You must know that there are two places in the sphere, which are the poles. 'Ash' which is the 'Wagon' and is called the 'bear' is near to the northern pole. Therefore 'Ash' is always visible, in summer as in winter, in the whole northern inhabited part of the earth. And there is a big star near to the southern pole, named 'Red' and in Arabic it is 'Suhayil'. Near to it there are little stars which are visible sometimes to all those living near to the equator. But these stars are invisible to those who live in the northern hemisphere. And therefore the Scriptures named them Hadrei Teiman, the chambers of the South, because they are invisible to those living in the inhabited parts of the earth. And 'Ash' is so called because it is derived from the word 'Ushu' which are seven inseparable stars. And we are still left with the task of explaining Kesil and Khima. The opinion of our ancients is that Khima is the tail of the constellation of Aries and the head of the constellation of Taurus and it consists of six visible stars although they are small. And it is known by irrefutable proofs that the zodiacal circle, the ecliptic crosses the superior circle, the equator, in two points, which are called the equinoctial points. Therefore the ecliptic is inclined toward the north by twenty three degree plus the half of a degree plus the half of a sixth of a degree [23° + 0.5° + 5' = 23°;35'] and likewise toward the south. And so they have three common points.[29] And the sun moves along the ecliptic. Therefore the morning will turn into deep darkness and the day will darken into night in all the inhabited part of the world. The reason for calling this point[30] 'Khima' is because Khima was located at this point in the ancient times.[31] It is because the zodiacal sphere i.e. the eighth sphere moves from west to east at the rate of about 1.5 degrees [in 100 years]. And the wise Ptolemy found that in his days, about thousand years ago, the star Lev ha-Arieh [Cor Leonis or today ***α Leo****] was located at 2° in Leo[32] and today it is 18° in Leo[33] [in the ninth sphere]. But it is known that α Leo is always at 9° [in the eighth sphere],[34] in relation to the constellation [and the stars as it is the case in*

[29] Three common points: the two equinoctial points and the same center.

[30] Of the ecliptic with a longitude of 30°.

[31] See below in the paragraphs devoted to Maimonides. At the time of the Flood, the vernal point was at the beginning of Taurus.

[32] In the catalogue of Ptolemy established in 137 C.E. the longitude of Cor Leonis is 2°;30' in Leo. Ibn Ezra rounded off to 2°.

[33] In about 1150, the longitude of Cor Leonis is (in the ninth sphere, thus with regard to the fixed vernal point) is 18°in Leo. This value is also a rounded off value. In *Keli ha-Nehoshet* he gives 17°; 31'. This value seems coherent with the value given by Abraham bar Hiyya for 1104: 17°;02'. With these value we find a precession of 17°;31' – 2°;30' = 15°;01' in 1150 – 137 = 1013 years = 1.48 ° / 100 years, about 1.5° in 100 years but more than 1° in 70 years as championed by Ibn Ezra.

[34] In ancient astronomy the stars are fixed on the eighth sphere and the precession of the equinox was understood as follows: The eight star is rotating in the direct direction at a rate subject to discussion, 1 degree in 100 years (Ptolemy), in 66 years (al-Battani) or 70 years (Ibn Yunus). This rotation is around the axis ππ', where π and π' are the poles of the ecliptic. This is the only way to explain why the latitude of the stars does

the eighth sphere] but not in relation with the point of intersection of the equator and the ecliptic, [i.e. the vernal point, as it is the case in the ninth sphere].

Conclusion. Abraham ibn Ezra had the most complete view on the phenomenon of the precession. We don't know on which basis he preferred the rate of precession given by Ibn Yunus of 1° in 70 years with regard to the very similar rate of Al-Battani of 1° in 66 years. However the longitude of the star Cor Leonis which he gave was deduced from the value of Ptolemy by the application of a precession of 1° in 66 years[35] during 1000 years.[36]

4. Maimonides (1138 – 1204). *Sefer ha-Mada : Hilkhot Yessodei ha-Torah* 3 : 6 and 7.

גלגל התשיעי שהוא מקיף את הכל, חלקוהו החכמים הקדמונים לשנים עשר חלק. כל חלק וחלק העלו לו שם על שם צורה זו שתראה בו מן הכוכבים שלמטה ממנו שהם מכוונים תחתיו. והן המזלות ששמותם: טלה, שור, תאומים, סרטן, אריה, בתולה, מאזנים, עקרב, קשת, גדי, דלי, דגים. גלגל התשיעי עצמו, אין בו לא חלוקה ולא צורה מכל הצורות האלו ולא כוכב. אלא בחיבור הכוכבים שבגלגל השמיני הוא שייראה בכוכבים גדולים שבו תבנית הצורות האלו, או קרוב מהן. ואלו השתים עשרה צורות לא היו מכוונות כנגד אותן החלקים אלא בזמן המבול שאז העלו להן שמות אלו.<u>אבל בזמן הזה כבר סבבו מעט, לפי שכל הכוכבים שבגלגל שמיני כולן</u>

---

not change with the precession. The stars and the position of the vernal point at a chosen initial date, remains fixed on the eighth sphere. The ninth sphere rotates (the diurnal rotation) around the axis PP', where P and P' are the poles of the equator. But if we don't consider the diurnal rotation or better if we observe it always at the same hour, it is fixed with regard to the precession movement of the eighth sphere. On the ninth sphere the vernal point is fixed and the virtual projection of the stars moves forward at the rate of the precession. When the ancients gave the position of the stars on the ninth sphere, it means that they gave the true position of the stars with regard to the true vernal point of the date. When they gave it on the eighth sphere that means that they gave the position of the stars with regard to the position of the vernal point at the moment when the eighth sphere coincided with the ninth sphere. It is thus fundamental to know the date of reference, when both spheres coincided, in order to know at the considered moment the amount of shift between the eighth sphere and the ninth sphere, which is the amount of the precession.
Ibn Ezra mentioned the longitude of 9° in Leo or 129° for Cor Leonis in the eighth sphere. This value is certainly the rounding off of 9°; 08' of Cor Leonis in the eighth sphere indicated in the table of the eighth sphere given by Abraham Zacut in his Almanach Perpetuum (see further). It appears that during the middle ages, the scholars referred always to the same table of the eighth sphere. It was in fact the list established by ibn al-Kammad which became the standard list of the stars on the eighth sphere. But the date of reference of this table is not known with precision. Abraham Zacut (see further) calculated that its date of reference was 563 C.E. but this date has no real signification. Therefore others thought that the date of reference was 16 July 622, the date of the Hegira. Others considered a date of reference of 581, 41 years before the Hegira. Anyhow the exact date of reference of this catalogue is debated. From the text of Ibn Ezra, it seems that he thought that the date of reference of this list of stars of the eighth sphere was the time when the vernal point was at the beginning of the constellation of Aries so that the signs of the zodiac read on the ninth sphere coincided with the images of the constellations (of the stars fixed on the eighth sphere). This must be the signification of the passage of Ibn Ezra telling that <u>. וידוע, כי לעולם הוא 'לב האריה' על תשע מעלות כנגד הצורה, רק לא כנגד נקודת המחברת.</u> Cor Leonis is 9° with regard to the constellation (in the eighth sphere) but not with regard to the vernal point (in the ninth sphere, where its longitude is 17°;31' in Abraham Ibn Ezra's time). But this is difficult to understand because the date of reference must be about 9°;08' – 2°;30' = 6°; 38' / 1.48 = 448 years after Ptolemy, thus 137 + 448 = 585 C.E. We can perform the calculation otherwise: in 1150 the longitude of Cor Leonis was 17°;31'in Leo. The longitude of Cor Leonis in the eighth sphere is 9°;08'. The eighth and the ninth sphere coincided 17°;31' – 9°;08' = 8°;23'/1.48 = 566 years before, in 584 C.E. This date is not far from the date of reference given by Zacut. At this date the eighth and the ninth spheres coincided. It is much later than the accepted date – 50 C.E. of coincidence. We know indeed that the coincidence of the signs of the zodiac and the zodiacal constellations was before the Common Era somewhere between Hipparchus and his followers in about – 50, when the image of the constellations were defined.

[35] (1000 / 66.7) = 15° and 2°;30' + 15° = 17° but (1000 / 70) = 14°;17' and 2°;30' + 14°;17' = 16°;47'

[36] 1150 – 137 = 1013. He writes 1000 years. It was perhaps 1140 instead of 1150.

סובבים כמו השמש והירח, אלא שהן סובבין בכבדות. וחלק שיתהלך השמש כנגדו ביום אחד ילך כנגדו כל כוכב מהן בקירוב משבעים שנה.

The ninth sphere which encircles all the other spheres, the ancients divided it in twelve parts and for each of them they gave a name after the name of the image of the stars that you see underlying this part. They are the signs of the zodiac: Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricorn, Aquarius and Pisces. The ninth sphere itself has neither division nor images of the different forms mentioned, and it does not include stars. It is only through the configuration of the stars [in the eighth sphere] that we can see the image of these forms drawn by the greatest stars of these constellations or similar to them. And these twelve images [in the eight sphere] were not in perfect correspondence with the twelve divisions [of the ninth sphere] only at the time of the Flood, when these names were given to them. But now, in these times they have already moved [forward] slightly, because all the stars [in the eighth sphere] revolve similarly to the sun and the moon [from west to east]. But they revolve heavily and the [angular] distance covered by the sun in one day will be covered by each of the stars in about seventy years.

Maimonides' exposition is certainly the clearest possible. However Maimonides' affirmation that the zodiacal signs of the eighth sphere coincided with the zodiacal constellation at the time of the Deluge is problematic. It is generally accepted that the vernal point will, in its movement of precession, reach the constellation of Pisces in 2100 C.E. That means that with a rate of 360°in 25800 years or 30° in 2150 years, the vernal point was at the beginning of Aries in 2100 – 2150= – 50 C.E. and in 1658 A.M = – 2103 C.E.[37] the vernal point was in about 29°; 20' of Aries , בזנב טלה, at the limit of the beginning of Taurus.

5. R. Moses Cohen and R. Isaac ibn Sid in the canon of the Alfonsine tables (1276).

The theory of the trepidation was introduces by Thabit ben Qura. It was introduced and developed in Spain by Arzachel in its Toledan tables. It was adopted in the Alfonsine tables, in the Castilian Alfonsine tables and in the Parisian Alfonsine Tables, in a mature form, a compromise between the ancient theory of the trepidation and the classical theory of the precession. In the ancient theory of the trepidation, the vernal point advances at the rate of one degree in 100 years during 800 years and then comes back backward at the same rate of one degree in 100 years during 800 years. In the Alfonsine tables the precession is the sum of two functions p_1 and p_2. The first function is: p_1= 360 t/49000. p_1 is proportional to the time; it reaches 360° after 49000 years and 1° after 136.11 years. It is thus less than half the precession given by Al-Battani. The second function is the periodic function p2= arc sin (sin 9°* sin (360 * t /7000). After 7000 years, t=7000 and sin (360 * t /7000) = 0 and p2 = 0. If t = 3500 p2 = 0. If t = 1750, then sin (360 * t /7000) = sin 90° = 1 and arc sin (sin 9°* sin (360 * t /7000) = arc sin (sin 9°) = 9° and if t = 5250 then sin (360 * t /7000) = sin 270° = – 1 and arc sin (sin 9°* sin (360 * t /7000) = arc sin (– sin 9°) = – 9°. This function p2 has thus a sinusoidal form but the tabulated function does not confine itself to the simple function p2 = 9° sin (360*t/7000) but to the more complicated function p_2= arc sin (sin 9°* sin (360 * t /7000).[38] The precession evolves thus always in the same direction but at a variable rate.

[37] The date of the Flood according to the traditional Jewish chronology.

[38] See José Chabas and Bernard Goldstein, A Survey of European Astronomical Tables in the Late Middle Ages, Brill 2012, p. 51.

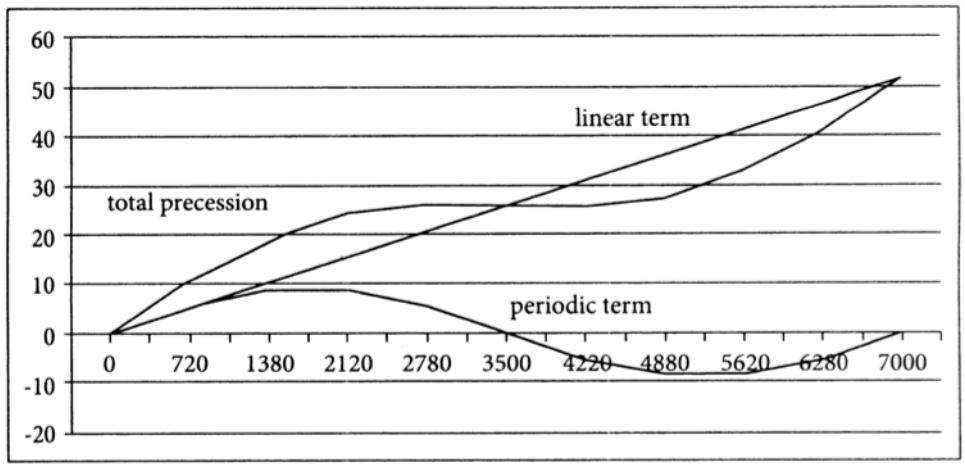


Figure 1: The precession according to the Alfonsine theory. To a linear term p_1 we must add a periodic term p_2.

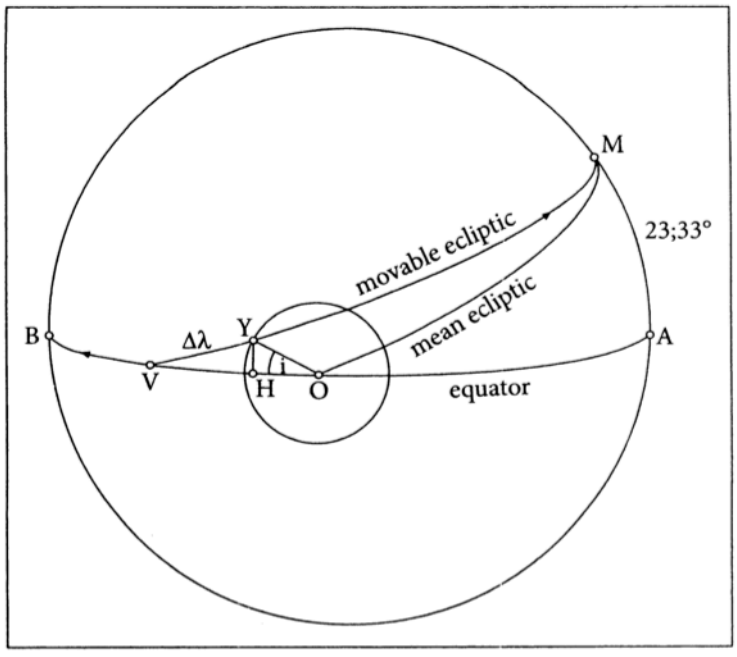


Figure 2: The model of the trepidation. This model was introduced by Thabit and Ibn Ezra mentions it in his writings (see above). O is the center of the small circle. Arcs MO and AO have 90°, angle MOA is ε (23°;35' for Al-Battani and followers but 23°;33' in the Alfonsine tables). Y is Aries 0° (sidereally fixed in the eighth sphere) and V is the vernal equinox. The amount of trepidation Δλ is the arc YV; the variable obliquity is angle YVO; the mean motion i is angle VOY and YH is called the equation of the radius of the small circle OY. The present figure and the former were borrowed to J. Chabas and B.B. Goldstein, A survey of European Astronomical Tables in the Late Middle Ages, Brill 2012.

6. R. Isaac Yisraeli (? – 1317): Sefer *Yessod Olam* (1310)[39] in Ma'amar Sheni, chapter 6.

גלגל המזלות הוא מתגלגל תמיד סביב מרכזו ועל קוטביו במתינות רבה הולך ונוסע בכוכביו כלפי פנים לעולם כמו מעלה אחת בלבד בכל מאה שנה. ואחרי בטלמיוס, כמו תש"ם שנה עמד החכם בתני וחקר על זה בעיונו והודה לבטלמיוס במה שאמר כי מנהג העתקת הגלגל הזה בכוכביו הוא כלפי מזרח לעולם אלא שהוכיח באותותיו העיונים ההם והראה כי כמו מעלה ומחצה הוא מדת ההעתקה הזאת בכל ק' שנה ....ואחרי אלה עמדו אנשים חכמים מחכמי ישמעאל כמו אברהם הזרקאלי וחבריו שהיו בעיר טולטילא ומקומות אחרים בספרד וכשראו כל המחלוקת שיש בין הראשונים והאחרונים בזה הענין והבלבול והשבוש הבא מחמת כן בחשבון מקום הכוכבים ומחניהם מהמזלות בכל עת ועת באו להם בטוב טעמם וחקרו על זה בעיון גדול מאוד. ואחר שלוו עיונם לעיוני בטלמיוס ואברכס ושאר המעיינים שהיו לפני אלה ולאחריהם, הורו ואמרו כי העיון והחישוב האמיתי העיד להם והראה כי העתקה ונסיעה יש לגלגל המזלות בכוכביו בכבידות רבה אבל אינו כלפי פנים לעולם כמו שחשב בטלמיוס וחבירו אלא פעם וזמן רב ידועה היא העתקתו כלפי פנים ועל סדר המזלות ופעם אחרת וזמן רב אחר ידוע הוא חוזר ונעתק ונוסע בכוכביו כלפי אחור ושלא על סדר המזלות....

The zodiacal sphere moves always around its center and its poles. It moves forwards, very slowly with its stars at the rate of only one degree in 100 years. About 750 years after Ptolemy, the scholar al-Battani analyzed the subject. He agreed with Ptolemy on the point that the movement of the stars is always toward the east but he proved by his examinations that the movement is 1.5 degrees in 100 years. After that came new scholars from the Arab scholars like ibn Zarkali and his colleagues from Toledo and other towns of Spain, and when they saw the disputes between the ancients and the moderns and the great resulting difficulties for determining the longitudes of the stars they examined the subject with the greatest attention and compared their observations to those of Ptolemy and Hipparchus and they concluded finally that the movement of the stars includes an access and a recess. The movement of the zodiacal sphere is very slow but it is not always forwards as Hipparchus and Ptolemy thought but during a long period it is forwards in the direct direction of the zodiacal signs but then later it is for a long period, retrograde and in the direction opposite to the order of the zodiacal signs.

R. Isaac Yisraeli devotes then two pages to describe more deeply this alternative movement known by the name of trepidation. Undoubtedly, it is this theory that Yisraeli champions. It was indeed the theory accepted by the Alfonsine tables and which brought us many years backward.

7. R. Ovadia ben David: Commentary on the Astronomical chapters of Rambam[40] (c.1340)

גלגל תשיעי שהוא מקיף הכל וכו' זהו 'אלפלך אלאטלס' והוא אין בו כוכבים כלל אבל חכמי יון חילקו הגלגל השמיני, שיש בו כל אלו הכוכבים הנראים שהוא תחת הגלגל התשיעי , לי"ב חלקים. כל חלק ל' מעלות. וקראו לכל חלק שם לקוח מקיבוץ הכוכבים שבאותו חלק. שפעמים היה קיבוצם בדמות טלה או שור וכל כיוצא בזה וחלקו הגלגל התשיעי לי"ב חלקים כנגד אלו החלקים. ועשו זה מפני הלימוד כדי שיבין הלומד אלו העניינים במהרה. כיצד כשנרצה לידע שהשמש סבבה ל' מעלות נאמר שכבר סבבה כל כל מזל טלה וכן הכל על פי זה הדרך.

The ninth sphere, which encircles all… This is the ninth sphere, it does not include any star but the Greek scholars divided the eighth sphere in which we have all the stars, into twelve parts. Each part has 30 degrees and they gave to each part a name adopted according to the form of the group of stars. Sometimes the group of stars had the shape of a goat or a bull or similar to it. They divided also the ninth sphere into twelve parts in face of the first parts. And they did so for educational reason, so

[39] R. Isaac Yisraeli, *Sefer Yessod Olam*, ed. Baer Goldberg, Berlin 1848.
[40] *Hikhot yessodei ha-Torah* 3; 6.

that the one who is learning this subject should make it out. So if we want to know that the sun has already covered 30 degrees, we will say that it covered the sign of Aries or something similar.

ואלו הי"ב צורות לא היו מכוונות וכו': החכמים הקדמונים לא השיגו לכוכבים שבגלגל השמיני תנועה כלל ובאחרונה השיגו להם תנועה כבידה בכל שבעים שנה מעלה אחת בקירוב. לפיכך מזמן המבול עד הנה כבר סבבו ונשתנה מקומם שהיה מכוון כנגד החלקים שבגלגל תשיעי באותו זמן כמו שתאמר שכוכב שהיה באותו זמן בתחילת מזל שור יהיה עתה בעשרים מעלה ממזל תאומים.

And these forms were not in perfect correspondence…The ancient scholar did not distinguish a movement of the stars [of the eighth sphere]. But more recently the scholars observed a heavy[41] *movement, of one degree in about each 70 years. Therefore, since the time of the Flood until now, the stars [of the eighth sphere], whose constellations were exactly in correspondence with the divisions of the same name [in the ninth sphere], have already moved. You could then say that a star which was at the time of the Flood in the beginning of the sign of Taurus is now at 20 degrees in the sign of Gemini [of the ninth sphere].*[42]

כל הכוכבים הנראים, יש מהן וכו: אמר הנראים לפי שיש כוכבים הרבה קטנים שאינם נראין מפני קטנם. והכוכבים הנראין שיכלו למנות אותן אלף וכ"ב כוכבים וחלקו אותן לששה חלקים: החלק הראשון, הכוכבים שבו גדולים מן הכוכבים שבחלק השני וכן עד סופן.

All the visible stars, some of them...He wrote "visible" because many little stars are not visible because of the small size. There are 1033 visible stars. They were divided in six groups. The first includes the brightest stars, and then we have a second group and so on.

8. R. Abraham Zacut (1452 – c.1515) in his book תכונת זכות or החיבור הגדול

פרק תשיעי מהחיבור הגדול של רבי אברהם זכות

כשתרצה לידע מקומות הכוכבים הקיימים ורחבם, ואנו צריכים לזה לידע כי מחמתם אנו יודעים הז' כוכבי לכת. וכאשר יש להם רוחב, גם בעבור שלא יצטרך לספר אחר. וגם כן לא כתבתי בראשונה בכאן אלא ט"ו הכוכבים היותר גדולים שהם מהכבוד הראשון והב'. כי התוכנים חלקו הכוכבים לו' סדרים, והיותר מאירים שהם ט"ו כוכבים קראנו מהכבוד הראשון ומהם לב האריה, אע"פ שבשיעור אורה, היא מהכבוד השני, בעבור שהיא בחגורה קראו אותה מהכבוד הראשון, כאשר פי' עלי ראשה בד' חלקים מבטלמיוס בחלק הראשון. וכן בזה הסדר יורדים עד מעלה ו' שלא יוכלו לעמוד על פחות מזה. ובסוף הספר אכתוב לך כל אלף וכ"ב כוכבים. והמעוננים כפי מקומם בזמן הזה, שהיא שנת אתע"ג לגלגל הט'. וכאשר תרצה לידע אי זה כוכב שתרצה מאי זה כבוד הוא ובאיזה מעלה מהגלגל הח' הוא, כמה רוחבו מהחגורה ואם הוא צפוני או דרומי או מאי זה מז' כוכבי לכת, קח שם הכוכב ההוא ותכנס בלוח וכנגדו תמצא כל בקשתך וזה הלוח לקחתי מר' יהודה ן' אשר ז"ל והיא מיוסדת על הגלגל הח'.

אמנם אם תרצה לידע מקומות הכוכבים האלה כפי הט' הנה בזה השתבשו אנשים גדולים וחכמים שהטעה אותם אזרקיל ואע"פ שנסתבך בקצת מזאת הסברה המלך דון אלפונשו כפי התיקונים שהוא עושה אל דרך קרובה מהאמת. כי הוא אומר שלעולם הולך הח' לפנים אלא כי בעבור מנת ראש טלה ומאלה אמת כי פעמים הוא עושה לגלגל שהוא יהיה כ"כ הולך ולפעמים ממהרו יותר ממהלכו וגם בעבור זה עשה השרשים ואמצעיים מז' כוכבי לכת ממהלכו לגלגל הט'. כי אם הולך הגלגל הח' לאחור ולא לפנים, לא יזיקנו. וגם מצאנו בספ'

[41] Slow.

[42] Thus a precession of 50°, 30° of Taurus and 20° of Gemini. This amount of precession makes sense. Indeed the flood was in 1658 AM = – 2103 C.E. R. Ovadia lived in the fourteenth century and wrote his commentary in about 1340. The considered span of time is thus 3443 years. With a precession of 1° in 70 years we get a precession of 3443 / 70 = 49.19°.

הכוכבים הקיימ' שהוציא כפי זמנו, ארבעה שנים אחר הלוחות, שחזר בו שאמר שם שבלי ספק הח' לעולם הולך לפנים כמ"ש בטלמיוס. והספר הזה הוא הספר שהעתיק ר' יהודה ן' משה הכהן למלך וזה הספר עשה אותו החכם הנרקרא אלבוחסין וכן העתיק ר' יהודה בן (אשר) [משה] הנזכר ספר המשפטים מעלי ן' רגיל ממערב וכן ספר המשיקה בצורות המזלות למלך הנזכר. ומצאנו שם שהיה לב האריה בזמנו שהוא שנת ארנ"ו. השנה הח' שהמלך בו, יום אחרון ממינו, בי"ט מעלות ול"ח דקים לפי הגלגל הט' וכפי תקוניו הוא עתה בזמן הזה שהוא שנת אתע"ח[43] בכ"ג מעלות שלימות מאריה. וא"כ בזה הזמן הוא רחוק הח' מהט' י"ג מעלות ונ"ב דקים. שהוא בקרוב מחנה א' מכ"ח מחנות ומעלה אחת, בעבור כי זה הכוכב הוא בט' מעלות וח' דקים מאריה המצויר.[44] ולפי זה תדע כי האלף וכ"ב כוכבים שהוצאתי לך לזמן הזה, אני מוסיף על מקומות הכוכבים מאלמגסתי שעשה בטלמיוס בשנה ראשונה מאנטונינוס הראשון שהוא בשנת קל"ג להגשמה שהוא תתפ"ו לנבוכדנאצר, כ' מעלות ול' דקים, לכל ס"ו שנים מעלה א שבינו ובינינו אלף שמ"ה שנים. ולא יהיה כל כך יוצא אם אתה נותן לכל ס"ו שנים מעלה א'. אל תתמה על החפץ כי אני מונה מ"א שנה קודם בטלמיוס שאז סידר מינלאו[ס] החכם כל הכוכבים שכתב בטלמיוס בספרו כאשר כתב אלבוחסין. ובטלמיוס הוסיף כ"ה דקים בעבור המ"א שנה כפי סברתו ובק' שנה מעלה א'. אין האמת כך כי אם בס"ו שנים וא"כ נסמוך על האיש[45] שסדר מקומות בראשונה כי העיון מעיד עליו. והתימה על מי שעשה פי' להוציא הכוכבים הקיימים בלוחות אלפונשו במהלך הרום עם תיקון מנת ראש טלה. כי יהיה לפעמים מהלך הכוכבים הקיימים מעלה א' בפ' או ק' או קלו' או קנ' או ר' שנים ביותר ולא ראו שעושה זה החשבון מימי אלפונשו ועד עתה שמוסיפי' זה המהלך על שורש הכוכבים שהוציאו למלך דון אלפונשו בימיו לדעת אלבוחסין שסברתו בכל ס"ו שנים מעלה א' לעולם, כפי מספר מהשנים שהיה מימי מינלאו עד אלפונשו שהם אלף וק"ס שנים שהוא מהלך י"ז מעלות ול"ג דקים. שלב האריה בימי מינלאו ב' מעלות וה' דקים מאריה ובימי אלפונשו בי"ט ל"א מאריה. וקודם מלך אלפונשו אלף וק"ס שנים לא חששו אלא מהשינוי עשה הרום שהוא לקל"ו שנים מעלה א' עם תקון מנת ראש טלה שעושה לכוכבים מהלכם יותר מזה או פחות מזה אלא בכל ס"ו שנים מעלה א' לעולם ואחר ימי אלפונשו. ובשנת תקס"ג לנוצרים שהוא שנת ד' אלפים [של"ג] חששו לתקונים. ואלו היה להם לתקן אלו התקונים מימי בטלמיוס או מימי מינילאו, אלא האמת מה שאמרנו.

ואני ראיתי בעיר שאלמנקה אתע"ד שהחשיך הירח ליד בתולה הנקרא סימק אל עזיל, והיה זה קרוב לחצי השמים ודקדקתי כל החלופים שהיה לירח והיה בי"ז מעלות ממזל מאזנים (וז') [וי'] דקים, והנה זה הכוכב כפי הח' הוא בג' וי"ח דקים ממזל מאזנים. א"כ הוא יוצא למה שאמרנו כי בשנת אתע"ג י"ג מעלות (מ"ב) [נ"ב] דקים רחוקים זה מזה שהלך הח' לפנים. וא"כ כאשר תרצה לידע מקומם הכוכבים הקיימים כפי הגלגל הט' הוסף י"ג מעלות ונ"ב דקים על מקומות בח' יהיה אז לפי הט'. ודע כי בספר המשיקה התחילו למנות המזלות כמו [ב]זמן בטלמיוס כי במעלה ג' מאריה שם לב האריה. ועל יסוד זה עשו כל מעלות המזלות במשיקה. וכן במחנות הירח לענין הגשם ושאר משפטיו תמנה כפי הזמן של בטלמיוס, ובזה אין ספק כלל כפי מה שעיינתי בכל ספרי החכמים והבן זה כי צריך מאוד למשפטי התכונה. ובעבור זה צריך לגרוע בזמן הזה כ' מעלות וחצי ממקום הירח [בלוחות הנוכחיים] לידע מקום הלבנה [לפי זמנו של בטלמיוס]. וכן מצאתי בספרי המשפטים ששלחו מפטביא ומאיטליאה וכן נראה מהראב"ע ז"ל בס' העולם, ושם מבולבל לשונו בעבור זאת הסבה. ותדע כי זה האמת אם היה המנין מהמחנות כפי הח' היה ראוי שיהיה סומך סימק אלעזיל שהיא יד בתולה במחנה

[43] This proves that the Hebrew *Ha-Hibbur ha-Gadol* was written in 1478 although the radices of his tables were calculated for the epoch 1473 as indicated in the first page of the Almanach perpetuum, Leira 1496.

[44] The area included between 120° and 150° on the mobile eighth sphere counted from its vernal point taken as point of reference corresponds to the sign of Leo in the mobile eighth sphere. Thus 9°;08’ in the zodiacal sign of Leo = 129°;08’ on the eight sphere and it corresponds to 23° in the zodiacal sign of Leo on the ninth sphere or 143° counted from the fixed point of reference, the vernal point. The precession at that moment was 13°;52’. Now what is the meaning of אריה המצויר? The stars are on the eighth sphere and the ninth sphere is empty. On the ninth sphere we compare the fixed vernal point with the projection or the virtual representation of the stars and the vernal point of the eighth sphere in their relative position as at the time of reference. The distance between the fixed vernal point and the projection of the mobile vernal point of the eighth sphere is the precession. This virtual representation of the stars and the mobile vernal point allows defining the area included between 120° and 150° counted from the projection of the mobile vernal point. This is probably what Zacut names the ‘designed’ sign of Leo.

[45] Menelaüs.

הט"ו כי הוא לפי הח' בג' י"ח ממאזנים, וכל מסדרי המחנות שמו אותה במחנה י"ד וכן נראה מדברי הרבא"ע עצמו כשסדר המחנות. ומי [ש]יודע המחנות יבין זאת, ואין צורך להבין זאת. ואם רצית לידע מקומות הז' כוכבי לכת כפי הח' בעבור שהם מתוקנים כפי הט' גרע י"ג מעלות נ"ב דקים ותדע מקומם. וכן בכל ס"ו שנים תוסיף מעלה א' שהח' הולך לפנים ולכל שנה (י"ב שניים וחצי) [נ"ד שניים נ"ה שלשיים]. ונמצא לפי זה כי האביב א' בתחילת היצירה היה המעלה השישית מתאומים המצוייר[46] בקו השווי באביבי, וכשיצאו ישראל היה תחילת שור המצוייר[47] בקו השווי באביבי ואז היו מתחילין למנות המזלות משור תאומים כאשר אנו בזמן הזה מטלה כאשר כתב אלברסין הישמעלי בספרו. ובקרוב בו' אלף שנה פחות ס' שנה הולך הח' ג' מזלות ו[ד]שכ"ג ליצירה היה ראש טלה מהח' בראש השווי האביב ואז הח' והט' יהיו שוים וח' שנים להלן היה מחברת שבתאי וצדק שהורה דת הישמעלים שהיה בשנת תקע"ה לנוצרים, כח' ימים ממרסו והיה מחברת ש"צ אז בין ל"ט או ל"ח בסוף ז' מעלות מעקרב, תחילת גדי לנוגה. וגם דע כי תנועתם על קוטבי גלגל הח' כמ"ש בטלמיוס כי הרוחב שמצאנו באלמגסתי ליד בטולה בכ"ו מעלות ומ' דקים שהוא ב' מעלות כ' דקים רוחב דרומי מהחגורה הוא עצמו פנימי מצאנו בי"ז מעלות ממאזנים ואמרתי זה בעבור כי גם האחרונים אמרו שהיה רחבם מתחלף. הנה הקרנוה כן היא, שמענה ואתה דע לך.

ודע כי כל האורך והרוחב שאמרו ומה שתמצא כתוב באלמגסטי וכן הספר כוכבים שכתבתי לך מר' יהודה ן' אשר הם לפי הח' והכוכבים שאני הוצאתי בזמן הזה לפי הט'. הכל הוא בערך קוטבי הח' וחגורתו אבל לדעת המרחב מקו המישור בערך קוטבי הט' שהם קוטבי העולם שיועיל לך זה למה שאמרנו לידע קשת יום הכוכב, תגרענו בדרך זה שתדע בראשונה מקום הכוכב לפי הט' ורוחב הכוכב מהחגורה, ואם היה רוחב צפוני תבקש מספר מעלות ומזלות ודקים מאורך הכוכב בעליון הלוח בשטה העליונה מהב' שיטין הכתובים בעליון הלוח שהיא כתובה בצבע שחור שמתחיל זאת השיטה מ0 עד תשלום כל הגלגל שהם י"ב מזלות. ותכנס עם רוחב הכוכב בצד הלוח ומה שתמצא נגד שניהם הוא המרחב ר"ל הנטיה מקו המישור, דרומי אם כתוב עליו דרומי או צפוני אם כתוב עליו צפוני. ואם תמצא כתוב נגד שניהם יותר מצ' כמו בראש סרטן כאשר רוחב הכוכב צ' שאז הוא רחוק מקו המישור קי"ג ל"ג, תגרע אותו מק"פ והוא יהיה מרחב הכוכב . ואמנם אם היה רוחב הכוכב דרומי, אז בקש מזלות ומעלות האורך בעליון הלוח בשיטה התחתונה הכתובה בצבע אדום שמתחיל זאת השיטה מו' מזלות ומעלה ובקש רוחב הכוכב בצד הלוח. ומה שתמצא נגד שניהם הוא מרחב הכוכב מקו המישור צפוני אם היה כתוב עליו דרומי או דרומי אם היה כתוב עליו צפוני ר"ל להפך ממה שתמצא שכתוב שם וזה גם כן יהיה גובה היותר מהכוכב בהיותו באופן חצי היום, נוסף על גובה ראש טלה, אם היה צפוני או פחות מזה אם היה דרומי. וגובה ראש טלה תדע כשתגרע רוחב המדינה מצ' והנשאר הוא גובה ראש טלה. ואם מרחק הכוכב מגלגל המישור יותר ממרחב המדינה והוא צפוני, גרע מרחב המדינה ממנו והנשאר גרע מצ' והנשאר הוא יהיה היותר גובה מהכוכב לצד צפון מנוכח הראש . וכן דע שאם יהיה רוחב הכוכב בשיעור שאם תחברהו עם רוחב המדינה יעלה ליותר מצ', הכוכב ההוא לא יבא תחת הארץ אם הכוכב צפוני ואם דרומי, לעולם יהיה תחת הארץ. ואם הוא פחות מצ', יסתר הכוכב לפעמים ויראה לפעמים. וכבר אמרנו למעלה איך תדע כמה יהיה קשת יומו על הארץ וכמה קשת לילו תחת הארץ.

Chapter 9 of the Hebrew canon written by Abraham Zacut in 1478 and devoted to the problem of the precession.[48]

[46] In the sixth degree of the sign of the zodiac of Gemini on the 9th sphere, thus at the longitude 65.5°.

[47] In the beginning of the sign of the zodiac of Taurus on the 9th sphere thus about.

[48] R. Abraham Zacut (Salamanca 1452 – 1515) composed his *Hibbur ha-gadol* in 1478 (still in manuscript) and his *Almanach Perpetuum* was published in 1496 in Leiria with the collaboration for the composition into Latin of Joseph Vincinho. The chapters 9 and 10 were published by Berthold Cohn under the title. *Der Almanach Perpetuum* des Abraham Zacuto. *Ein Beitrag zur Geschichte der Astonomie in Mittelaler* von Berthold Cohn, Schriften der Wissenschaftlichen Gesellschaft in Srassburg 1918. In fact this tittle is incorrect because the Hebrew *ha-Hibbur ha-Gadol* is more detailed and not identical to the later published Almanach Perpetuum. The Hebrew composition is extant in three manuscripts in the Libraries of Lyon, Munich and Vienna. A Spanish translation of these two chapters is also extant. See Cantera Burgos Fransisco, *El judio salmantino Abraham Zacut*, Madrid, Bermejo, 1931, and Fransisco Cantera Burgos, *Abraham Zacut, siglo XV*, Miranda de Ebro, 2008.

If you want to find the longitude of the stars and their latitude, note that this knowledge is necessary because through it we learn the longitude of the seven planets[49] especially if they have latitude, so that we don't need any other book. In the beginning, I wrote only the 15 [60][50] brightest stars, which belong to the first or the second magnitude. Indeed the astronomers divided the stars in six categories and the brightest which are 15 in number are called stars of the first magnitude. The star called the heart of the lion [α Leo] belongs to them; although according to its brightness it should belong to the second category. But because it lies in the ecliptic it was considered to belong to the stars of the first magnitude as Ali [ibn Rigal] explained it in the first of the four parts of Ptolemy's [book]. On this manner we go until the sixth category because it is not possible to see stars smaller than that category. At the end of this book I will write in detail all the 1022 stars [of the list of Ptolemy] classified according to their position today, which is the year 1473,[51] with regard to the ninth sphere.[52] When you wish to know about a star, what is its magnitude and what is its longitude on the sphere,[53] what is its latitude from the ecliptic, is it northern or southern or from which nature of the seven planets it is,[54] take the name of the star in the table and in face of it you will find all what you need to know. I took this table from the work of R. Judah ben Asher, of blessed memory:[55] it is based on the eighth sphere.[56] Now if you want to know the position of this [fixed] star in the ninth

---

For a comprehensive analysis of the work of Abraham Zacut and a comparison of the contents of *ha-Hibbur ha-Gadol* (Hebrew manuscript, 1478)and the Almanach Perpetuum (Latin 1496), see José Chabas and Bernard R. Goldstein: Abraham Zacut (1452 – 1515) y la astronomia en la Peninsula Iberica, Ediciones Universidad de Salamanca 2008. At about the same time (a year before the expulsion of the Jews from Portugal) a version of the Almanach Perpetuum was also published in Catalan. The title of the Latin version is: **Almanach Perpetuum Celestium Motuum** (radix 1473). Tabulae Astronomicae Raby Abraham Zacuti, Astronomi Johannis Secundi et Emanuelis Serenissimorum Regum Portugaliae, in Latinum translatae per Magistrum Josph Vizinum, discipulum autoris. The latter remained in Portugal and was compelled to convert to Christianity in 1497.

[49] A good catalogue of stars allows finding easily the longitude of a planet in conjunction with a star.

[50] Two contradictory readings between the extant manuscripts.

[51] Date of the radices of his tables, although his Hebrew canon, החיבור הגדול or תכונת זכות could have been written later, until 1478.

[52] Taking the precession into account. The ninth sphere is not affected by the movement of precession. Its rotation corresponds to the diurnal rotation that it imposes to the other 8 spheres. But it is stationary with regard to the precession. The vernal point is fixed in this ninth sphere and the longitude of the stars increases with the time. Thus this list of stars must be similar to the catalogue of Ptolemy after addition of 20.5° as explained by the author. The span of time between 1478 and 133, the date of Ptolemy's list of stars is according to Zacut (the generally accepted date is in fact 137) 1345 years and 1345 / 66 = 20.38° which Zacut rounded off to 20.5°.

[53] Later the author will explain how to calculate the coordinate of a star when it falls in conjunction with a planet, sun and moon included. Indeed, thanks to the cinematic models of their movement, we can calculate their coordinates at any moment, thus in the ninth sphere, with regard to a fixed vernal point. The star occulted by the planet has the same coordinates. Once we have a catalogue of the stars, we can use it in the opposite direction to find easily and without great calculations, the coordinates of a planet occulting a star or passing near to it.

[54] Each star is associated with a planet about its nature; it is an astrological concept.

[55] R. Judah ben Asher was the Rabbi of Toledo and the great grandson of R. Asher ben Ephraïm, the Rosh. He was the friend of R. Isaac bar Sheshet of Valencia. He died in 1391 as a martyr during the riots. Although a great talmudist (according to Zacut in his Sefer *Yuhsin*) he was a good astronomer and wrote a book about the fixed stars. See Hershman: ha-Rivash, Mossad ha-rav Kook, 1956.

[56] The eighth sphere is the sphere of the fixed stars. Its direct movement corresponds with the modern precession. In the modern precession the vernal point has a retrograde movement and the sun reaches the vernal point before reaching a fixed star. The tropical year is therefore 20 minutes shorter than the sidereal year. In the conception of the ancients, the eighth sphere rotates in the direct direction. The sun will reach the vernal point fixed on the 8th sphere before it reaches the same vernal point fixed on the stationary ninth sphere, thus the true vernal point. Thus apparently the list of R. Judah ben Asher took the precession into

sphere,[57] [you must know] that important men were mistaken by Arzachel.[58] Although King Don Alfonso accepted also partially this false view, he could, thanks to the improvements that he brought, near the truth. Indeed he stated that the eighth sphere moves always forward but, because of the movement of the head of Aries,[59] sometimes it advances slower and sometimes it advances faster. For this reason he connected the radices and the elements of the movement of the seven planets to the ninth sphere so that when the eighth sphere has a retrograde movement and no direct movement,[60] this has no detrimental consequence. We find also in the book about the fixed stars that he [the king Alfonso] issued[61] four years after the [Alfonsine] tables[62] that he retracted from his false opinion and stated that the eighth sphere, without any doubt, advances always in the direct direction as Ptolemy had stated. This book is the book that R. Judah ben Moses Hacohen translated for the king. This book was written by the wise named Abul Hussein.[63] Similarly the mentioned R. Judah ben (Asher) [Moses] translated for the king the book of the laws [of astrology] from Ali ibn Ragil from Maghreb and the book of Magica about the stellar constellations. We find there that the star, the heart of the lion, was in his time i.e. 1256 the fifth year of his [King Alfonso's] reign, on the last day of the month of Mai, in 19°; 38' according to the ninth sphere.[64] And according to his corrections it is now in this time which is the year 1478 in 23[65] complete degrees of the sign of Leo.[66] And therefore, today the eighth sphere is removed from the ninth sphere by 13°; 52', which represents nearly one of the 28 moon stations[67] and an additional degree because this star is situated in 9°; 08' in the sign of

---

consideration only until the date of reference and Zacut used it and added the precession between the date of reference of R. Judah ben Asher and 1478 in order to establish a list corresponding to the ninth sphere.

[57] The ninth sphere is the superior sphere imposing the diurnal rotation to all the internal spheres. It is not affected by the precession. We can at any moment of the history, ''photography'' on it the eighth sphere and measure the precession with regard to the fixed vernal point of the ninth sphere. Rambam had expressed it otherwise: we can draw on it the *Tsurot*, the representation of the zodiacal constellation of the eighth sphere at a particular moment of history, like the *Mabul*, the Flood, when, according to Rambam, the zodiacal signs coincided with the zodiacal constellations. In fact this statement must be corrected: at the time of the food, the vernal point was at the beginning of Taurus.

[58] Ibn Zarqali (1029 – 1087) or Arzachel was a champion of the theory of the trepidation i.e. an alternation of a slower and a faster precession. In fact Arzachel considered even an alternation of direct and retrograde movements so that in the long range there is no movement at all.

[59] See the figure explaining this alternative movement above, in the chapter of the Alfonsine tables.

[60] This text seems confuse. Apparently in the system of Arzachel, the trepidation was an alternative movement, eight hundred years in one direction and then eight hundred years in the other directions and finally no movement at all. Because of this complication, the radices and the elements of the movement of the planets were connected to the ninth sphere. King Alfonso corrected the importance of the trepidation and acknowledged that the rotation of the eighth sphere is always direct. The effect of the trepidation limited itself to slow or to accelerate this movement, thus an alternation of a slower and a faster precession.

[61] In 1256.

[62] In 1252.

[63] It is probably the book of the fixed stars written by al-Sufi also called Abul al Hussin (903 – 986) called "the wise", a Persian scholar and astronomer of the tenth century, who took the longitudes of Ptolemy, increasing them by 12°; 42' and noted the magnitudes of the stars after careful observations by himself.

[64] Apparently according to the calculations of Abul Hossein and Zacut but not of Alfonso, who used an average precession of only 1° in 136 years.

[65] 1478 – 1256 = 222 years. 222 / 66 = 3.36° = 3°; 21'. Hence: 19°; 38' + 3°; 21' = 22°; 59' rounded off to 23°.

[66] Again in the ninth sphere. The longitude of α Leo in the eighth sphere is 9° ; 08' therefore the movement of the eighth sphere is 23° - 9°; 08' = 13°; 52'.

[67] Apparently for astrologic purpose, the 360° are divided in 28 stations of 12.86° = 12°; 51'. The distance between the eighth and the ninth sphere is 13°; 52' = 12°; 51' + 1°.

Leo.[68] *You must also know about these 1022 stars that I published for this time, I have added to the longitude of the stars of the Almagest, established by Ptolemy in the first year of Antoninus I, in the year 133 of the incarnation [133 C.E*[69]*] or 886 of the era of Nebukadnezar, 20°; 30' corresponding to one degree for each 66 years. Although the distance between us and Ptolemy is 1345*[70] *years and therefore you don't reach such a difference when you count 1° for 66 years*[71]*. Therefore you should not be surprised about that because I count 41 years before Ptolemy because it was already then that the wise Menelaus*[72] *classified all the stars that Ptolemy would gather in his Almagest, as Abdul Hussein wrote it. Ptolemy added 25 minutes [of a degree] because of these 41 years according to his assumption that we add 1° in 100 years. However the truth is different and we must add 1° in 66 years. So we will rest on this man*[73] *who classified these different localizations [longitudes of the stars] because the testimony testifies in his favor. We can nevertheless be surprised that the author of the catalog of the fixed stars of the Alfonsine Tables and their longitudes wanted to [take into account the precession] through the movement of the apogee combined with the movement of the Vernal point. Indeed the precession of the fixed stars would then be 1° in 80, 100,*[74] *136,*[75] *150 or even 200 years. However (they) [he] didn't see that when one makes this calculation from King Alfonso until now, one adds this movement to the radix of that star proposed to King Alfonso according to the opinion of Abul Hossein, i.e. we add always 1° per 66 years according to the number of years elapsed between the time of Menelaüs and Alfonso which is 1160*[76] *years. It represents 17°; 33'.*[77] *Now the hearth of the Lion was at 2°; 05'*[78] *in the sign of Leo at the time of Menelaüs and at the time of Alfonso it was at 19°; 31'*[79] *in the sign of Leo. During the period of 1160 years preceding King Alfonso they considered in the calculation of the precession, only the movement of the apogee which amounts to 1° in 136 years.*[80] *This must be combined with the movement of the head of Aries it gives more or less instead of a constant movement of 1° in 66 years*[81] *after the time of Alfonso. But in the year 563*[82]

---

[68] On the eighth sphere. He doesn't explain us wherefrom he knows this figure in the ninth sphere. See below how this figure was deduced. In the Almagest (Toomer p. 367) we find for α Leo the longitude of 2.5° in the sign of Leo.

[69] We admit today that it was composed in 137 C.E.

[70] 1478 – 133 = 1345.

[71] 1345 / 66 = 20.38° = 20°; 22'<20°; 30'.

[72] Roman astronomer who observed in Roma in about 92 C.E. See See A. Pannekoek, A History of Astronomy p. 145. The span of time to consider is thus 1345 + 41 = 1386 and 1386 / 66 = 21°. But, as he states, Ptolemy had already added 25' and therefore Zacut added 21° - 25' = 20°, 35' which he rounds off to 20°; 30'. All this reasoning is based on the data given by Al-Battani (Nallino, Al-Battani Opus Astronomicum, Vol 1, p. 124).

[73] Menelaüs.

[74] The precession of Hipparchus and Prolemy.

[75] The period of the Alfonsine tables is: 49000 / 360. The signification of the different figures 80, 100, 150 and 200 is unclear.

[76] More exactly: 1256 – 92 = 1164.

[77] 1160 / 66 = 17.58° = 17°,34'.

[78] See Toomer p. 367 , the longitude of α Leo was 2°;30' in the Almagest. As Zacut explains we get the value found by Menelaüs by subtracting 25' from the longitudes of Ptolemy's catalogue. The longitude of Menelaüs was thus 2°;05'.

[79] 19°; 38' according to what he wrote above. We ascertain that 2°;05' + 1160 / 66 = 2°;05' + 17°;33' = 19°;38'.

[80] In the Alfonsine tables the precession is 360° in 49000 years or 1° in 136 years. Thus in 1160 years they considered a precession of only 8.53° = 8°;31'. Zacut doesn't explain how they justified the difference between their value of 19°;38' and the value of Ptolemy of 2°;30' or the value of 2°;05' ascribed to Menelaüs.

[81] He seems to say that the true precession is 1° in 66 years. But this is not the position adopted in the Alfonsine tables where the average precession is 1° in 136 years or 360° in 49000 years with addition (or subtraction) of a periodic term (period of 7000 years).

of the Christians, which is the year 4[323], they began to care for these corrections and they should have cared for these corrections from the time of Menelaüs or Ptolemy onwards.[83] *The truth is nevertheless as we exposed it.*[84]

Now I have observed in the town of Salamanca in year 1474 the occultation of the hand of Virgo, which is named Simal al-azal [Spica], by the moon. It happened near to the meridian. I calculated with precision all the inequalities to take into consideration for the moon, it was at 17°;(07') [10][85] *in the sign of Libra. Now this star was at 3°; 18' in the sign of Libra with regard to the eighth sphere.*[86] *This correspond with what we said, that [both spheres the eighth and the ninth] are removed, the one from the other, by 13°; (42)[52']*[87] *corresponding to the advance taken by the eighth sphere. Therefore when you want to know the positions of the fixed stars with regard to the ninth sphere, you must add 13°; 52' to the positions in the eighth sphere and you get the positions in the ninth sphere.*

Now you must know that in the book Magica, the signs of the zodiac begin like in the time of Ptolemy because it is in the third degree of the sign of Leo that, [according to it], you find the heart of the lion[88] *and it is on this basis that one has given the representation of all the stars in the Magica. The same can be said about the stations of the moon relating to the rain and other [astrologic] rules. In that field you must count as in the time of Ptolemy; there is no doubt about it as I could check in all the books of the scholars. Note this point because it is very useful for the [astrologic] rules. Therefore one must today subtract 20.5°*[89] *from the longitude of the moon [read in modern tables] to get the*

---

[82] Zacut will establish later that the year 563 of the Christians is the year of coincidence of the eighth sphere and the ninth sphere. It is the date of reference of the calculation of the coordinates of the stars in the eighth sphere, performed by ibn Al-Kammad. The difference of 13°;52' reached in 1478 between the eighth sphere and the ninth corresponds to the precession counted from 563 onwards until 1478, counted at the rate of 1° in 66 years. Indeed (1478 – 563) / 66 = 13.86° = 13°; 51'. But Zacut adds that the precession should have been taken into account, at the same rate, before 563, between Menelaüs in 92 C.E. and 563 C.E. It is not clear for me to whom this reproach is addressed. Zacut's own correct calculations will be given below.

[83] Berthold Cohn had in his German translation, understood this sentence as follows: *After the time of Alfonso, in the year 563 of the (Christians) [Muslims] which is the year (4000) [5000] [in the Jewish calendar or 1240 in the Christian calendar] they began to care for these corrections but they should have cared for these corrections from the time of Menelaüs or Ptolemy onwards.* The text is certainly corrected but his explanation is problematic. The year 563 of the Hegira does not correspond to 1240 C.E but with about 1168 C.E. This year is certainly not after Alfonso. The number of corrections necessary is important and the mistakes, for example the confusion between Christian and Muslim calendar, do not correspond to the mistakes of a copyist.

[84] That the precession is constant and its value is 1° in 66 years.

[85] In the ninth sphere. The measure of the ecliptic coordinates of the star is very difficult; it is difficult to fix the vernal point in the stellar sky or to relate the stars to the vernal point: However the occultation of a star by a planet or the moon allows the calculation of the ecliptic coordinates, in the ninth sphere, of the planet or of the moon, and by the same occasion, of the occulted star. The correction of 10' instead of 7' was introduced in order to make the calculation fit.

[86] This value is given according to a table of the eighth sphere, established for a fixed reference time, in use at his time; this date of reference is 563 C.E. Indeed the difference between the eighth and ninth spheres is 13°;52' and 13.87° / 66 = 915.2 years. The reference year was thus 1478 – 915.2 = 562.8 years. In his Latin Almanach Perpetuum, published in 1496 in Leiria we find a table of the brightest stars 'ad octave sphere' according to the eighth sphere (magnitude 1 and 2) on fols. 164v and 165r or, 320 – 321 in the reproduction fac-simile of 1915. In particular on fol. 164v we read Libra 3°;18' for Spica and Leo 9°;08' for the heart of Leo or Regulus.

[87] 17°;10' – 3°;18' = 13°;52'

[88] In the Almagest, (XXV, 8) the longitude of α Leo is 2°;30'. Thus *Magica* seems to count the longitudes of the eighth sphere and reproduces exactly the table of the Almagest.

[89] 1478 – 133 =1345. 1345 / 66 = 20.38° = 20° ;22' which he rounds off to 20°;30'.

longitude of the moon [according to the situation prevailing at the time of Ptolemy as consigned in his catalogue]. So did I also find in the books of [astrologic] rules that were sent to me from Padua and from Italy and this seems also to result from the book Sefer ha-Olam[90] of R. Abraham ibn Ezra but his language is confused concerning this point. You must know that this is the truth if the numbering of the moon stations is done according to the eighth sphere; then Simak-al-azal [Spica] which belongs to the hand of Virgo, must be in the 15th station, because it is at 3°; 18' in Libra according to the eighth sphere. But all those who classify the stations place it in the 14th station; in the same way did ibn Ezra in his classification of the stations. Who knows the stations, will understand it, but it is not essential to understand it.[91]

When you want to know the position of the seven planets according to the eighth sphere, then because they are tabulated with regard to the ninth sphere,[92] you must subtract 13°; 52' and you will get their position. For each [additional] 66 years, add one degree [to the preceding amount] because the eighth sphere moves forward by (12.5") [54.55"per year].[93] Therefore we ascertain that at the beginning of the creation the sixth degree of the image of Gemini was the position of the vernal point of the equator[94] and at the time of the Exodus the beginning of the image of Taurus was the position

[90] Astrologic book composed in Hebrew by ibn Ezra: version 1 written in Béziers in 1148 and a version 2 written later in France in about 1148- 1149. See Shelomo Sela: Abraham ibn Ezra and the Rise of Medieval Hebrew Science, Brill 2003.

[91] There are 28 stations of 360° / 28 = 12.86°. The 14th station begins at 167,14° and the 15th station begins at 180°. If Spica is at 3°,18' in Libra = 183°;18' it is in the 15th station. The whole passage is problematic because the longitude of 3°;18' in Libra of Spica is calculated in the eighth sphere and corresponds to the situation in the year 563. We don't see why ibn Ezra is concerned with the year 563. It seems more logic to consider that ibn Ezra considers the situation in the ninth sphere in his time or in the time of Ptolemy as some astrologers prescribed. If we consider the year 1125, we have then 1478 – 1125 = 353.
The shift is 353 / 66 = 5.35 = 5°;21'. The position of Spica in the ninth sphere in 1125 was: 17°;10' – 5°;21' = 11°;49' in Libra or 191°;49' in the 15th station and with the same interrogation as that of Zacut. If, by contrast, we refer to the value of Ptolemy, (XXVII, 14) we find Spica in Virgo 26°;40' or 176°;40' and now we are indeed in the 14th station.
The division of the 360° into 28 stations for the study of the moon is already mentioned in the commentary of R. Ovadia ben David on Rambam, *Sefer ha-Madah, Hilkhot Yessodei ha-Torah* 3: 6.

[92] The coordinates of the planets, sun and moon included, are calculated through the theory of their movement with regard to the vernal point in the ninth sphere. 'Today', in 1478 the longitude of a star in the ninth sphere is 13°; 52' greater than in the eighth sphere, which represented the situation in 563, i.e. 915 years before.

[93] 1° / 66 = 54.55''/ year.

[94] The gap between the 8th and 9th sphere is 13°; 52' = 13.87° in 1478 C.E. = 5238 A.M. Now 5238 / 66 = 79.36° = 79°;21'. The position of the vernal point on the ninth sphere was thus at the era of the creation: – 13.87° + 79.36° = + 65.49°. The distance between the ninth and the eighth sphere, which was – 13.87° in 1478 was at the creation 65.49°. The vernal point was thus in the ninth sphere in the sixth degree of the sign of the zodiac of Gemini.

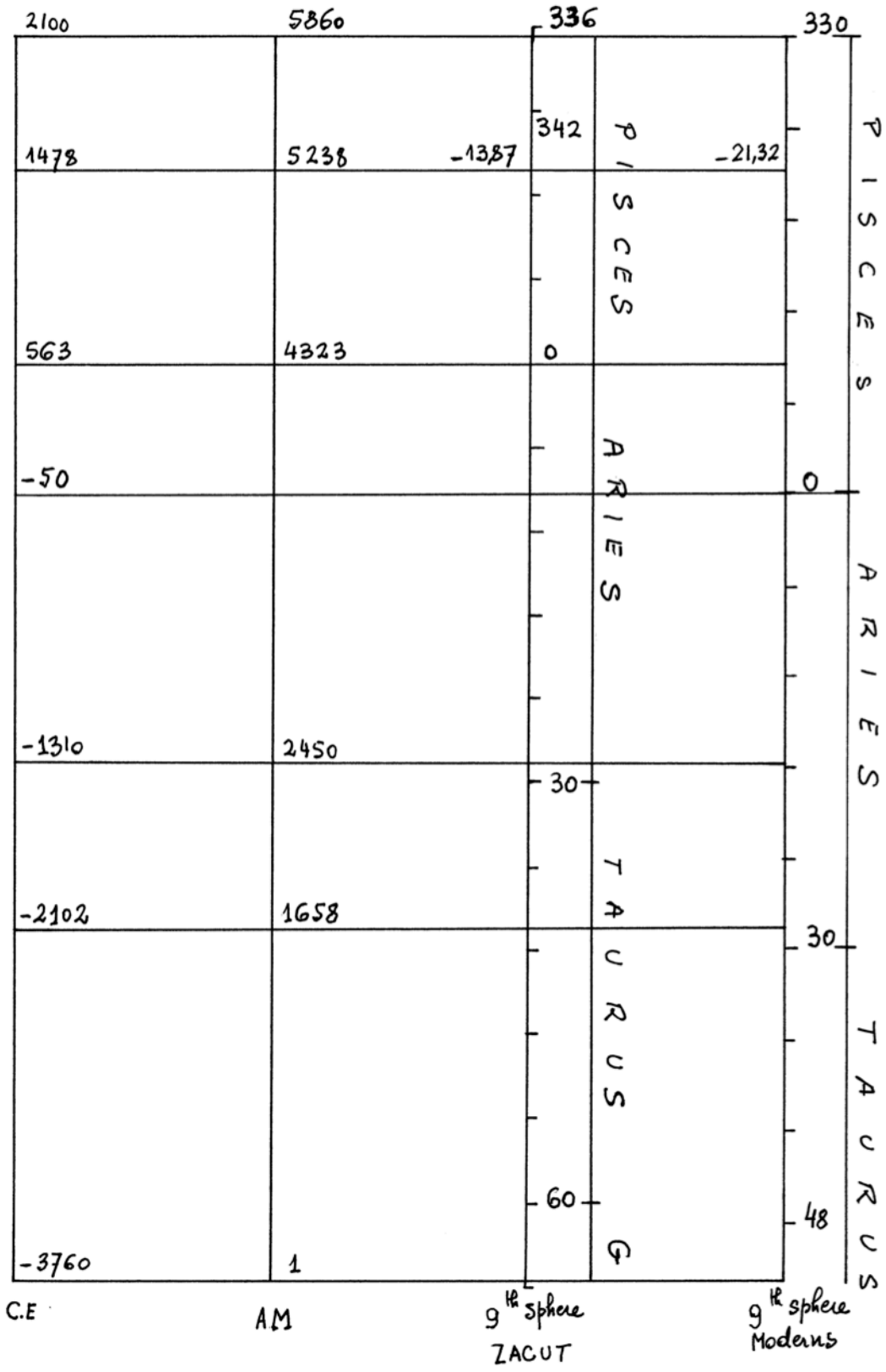


Figure 3: The position of the vernal Point in the ninth sphere: Zacut and the Moderns.

The present figure summarizes the calculations of Zacut. It is based on the following assumptions. - 1. The ancients considered that the stars are fixed in the eighth sphere which moves forward. By contrast the vernal point is fixed in the ninth sphere. The longitude of the projections of the stars of the eighth sphere on the ninth sphere increases with the time. Here, for easiness, Zacut considered fixed stars and a vernal point moving backward as we do in modern astronomy; hence the denomination: precession of the equinox. 2. The precession is of 1° in 66 years and 3. Zacut calculated that the date of reference of the eighth sphere is 563 C.E. That means that in 563 the eight and the ninth sphere coincided. Moreover Zacut admitted (as did already Ibn Ezra) that in 563 the zodiacal signs (ninth sphere) coincided with the zodiacal constellations (eighth sphere). Therefore the vernal point was at the beginning of the constellation of Aries. On the figure the left scale is the time in A.M: Anno Mundi of Beharad, the second scale is the time in C.E: Common Era. The third scale gives the position of the vernal point with regard to the zodiacal constellations according to Zacuto's assumptions. The fourth scale gives the position of the vernal point with regard of the zodiacal stellar constellations according to modern assumptions: a precession of 360° in 25800 years and a coincidence of the zodiacal signs and zodiacal constellations in -50 C.E. We did not consider the secular variations of the precession.

of the vernal point of the equator.[95] At that time they were beginning to count the signs of the zodiac respectively from Gemini and from Taurus in the same way as we today count them from Aries as Abul Hossein the Arab, wrote in his work. In about 6000 less 60 years, the eighth sphere advances by three signs.[96] Therefore in 4323 of the creation, the head of Aries of the eighth sphere coincided with the position of the vernal point on the equator and [at this time] the eighth and the ninth spheres coincided.[97] Furthermore when eight years later there was a conjunction of Jupiter and Saturn and [at that time] he [Muhamad[98]] taught the Law to the Arabs; this was on March 28, (575) [571].[99] The conjunction between Jupiter and Saturn occurred between 39 and 38 minutes at the end of the seventh degree of Scorpio while Venus was in the beginning of the sign of Capricornus [Pisces].[100] Know also that their movement is related to the poles of the eighth sphere,[101] as Ptolemy wrote it. The latitude that we found in the Almagest for the hand of Virgo in 26°, 40' in the sign of Virgo,[102] amounts to 2°; 20' southern to the ecliptic. We found for it 17°;[10][103] in the sign of Libra[104] [but we

[95] The year of the Exodus is 2448 according to Seder Olam. It means that Adam was 2448 years old. The event was in 2449 AMII (Weyad) corresponding to 2450 A.MI (Beharad) = – 1310 C.E. thus 1478 –(– 1310) = 2788. The shift of the ninth sphere with regard to the eight sphere, during this span of time is 2788 / 66 = 42.24°. In 1478 the distance between the ninth and the eight sphere was –13.87°, in – 1310, it was –13.87 +42.24 = 28.37°. The vernal point was thus at the very end of the sign of the zodiac of Aries, nearly at the head of the sign of the zodiac of Taurus.

[96] 5940 / 66 = 90°

[97] In 1478 the difference between the two spheres is 13° ;52'= 13.87°. Now 13.87 * 66 = 915.2 years. The two spheres coincided,(their point γ coincided) 915 years before in 1478 – 915 = 563 C.E. = 4323 A.M.

[98] He was born in 570 C.E.

[99] 4323 + 8 = 4331 A.M. = 571 C.E.

[100] I checked these data with the tables of B.Tuckerman: Planetary, Lunar and Solar Positions A.D. 2 to A.D. 1649, The American Philosophical Society, Philadelphia, 1964. We note that Saturn and Jupiter were near to each other during winter and spring 571. There was not a true conjunction. On March 28, the difference of Longitude was about 1.5° and in latitude 1.3° and Venus was in Aries. The two planets were the nearest on March 6, the difference of longitude was about 0.5°and the difference in latitude about 1.2°; Venus was in Pisces.

[101] The poles of the ecliptic.

[102] For the star Spica.

[103] We must add 20.5° to the values of Ptolemy to get the modern values on the ninth sphere. Thus 26°;40' + 20°;30' = 47°;10' Virgo = 17°;10' Libra or about 17° in Libra.

[104] On the ninth sphere.

didn't note any change in latitude]. I told this because the most recent scholars thought that the latitude is also changing. Look, we have examined it and this is [our conclusion]. Hear it and note it [Job 5: 27]. Know also that all the longitudes and latitudes about which they [the astronomers] speak and what you read also about in the Almagest and in the book of the Stars which I copied from R. Judah ben Asher refer to the eighth sphere. But the [list] of the stars which I issued for the present time, refers to the ninth sphere.[105] *But all of that refers to the poles of the eighth sphere and its ecliptic. Nevertheless, imagine that you want to know the declination with regard to the equator and the poles of the ninth sphere,*[106] *which are the poles of the world.*[107] *This will be useful to you when you want to know the diurnal arc of a star. You will know it then on the following manner: You must first know the position of the star on the ninth sphere and its latitude from the ecliptic. If it is northern, then you must take the number of signs [of the zodiac], degrees and minutes of the longitude of the star in the upper part of the table,*[108] *in the first of the two rows written with black ink. This row begins with 0 and extends over the whole circle representing the 12 signs. Now, with the latitude of the star enter the table on the vertical side of the table and the number that you find [at the crossing of the vertical column and horizontal row] is the declination of the star from the equator. It is southern if it is written southern and it is northern if it is written northern. If you find that the number [at the crossing of the vertical column and horizontal row] is greater than 90° as this is the case at the beginning of Cancer, when the latitude of the stars amounts to 90°, then it is removed from the equator by 113°; 33'.*[109] *Subtract this figure from 180° and you find the declination [66°; 27']. Now if the latitude of the star is southern, then you must look for the sign, the degrees and [minutes] of the longitude in the upper part of the table in the inferior row, printed in red. It begins with 6 signs and more and for the latitude in the [vertical] lateral column, and what you find corresponding to both [at the crossing of the vertical column and horizontal row] is the distance of the star from the equator [the declination]. It is northern when it is written southern and it is southern when you read northern, thus the contrary of what you will find written.*[110] *The greatest altitude of the star, at the moment of its transit will represent it [the declination] increased with the altitude of the head of Aries if it [the declination] is northern or subtracted by the altitude of the head of Aries if it [the declination] is southern.*[111] *You can find the altitude of the head of Aries by subtracting the geographical latitude from 90°, the difference represents the altitude of the head of Aries. When the distance of the star from the equator [the declination] is greater than the geographical latitude*[112] *and*

---

[105] In fact the list of stars published in his Almanach Perpetuum refers to the eighth sphere.

[106] The movement of the ninth sphere corresponds to the diurnal movements. The poles are the poles of the equator.

[107] The poles of the equator.

[108] Zacut refers to an unpublished table of transformation of ecliptic coordinates to equatorial coordinates.

[109] If λ = 90° and β = 90°then α = 90° and the distances of the star to the equator, measured on a circle of declination are 113°;33 and 66°;27. The declination is of course the smallest distance, it is 66.67°. Zacut considers that the angle ε between the ecliptic and the equator is 23°;33'

[110] The table is unknown to us. It allows transforming ecliptic coordinates into equatorial coordinates.

[111] If z is the zenithal distance at the moment of the transit, then: z = $\varphi - \delta$ and h = 90° - z = 90° – $\varphi + \delta$.
If we are on the equator δ = 0 and h = 90° – φ.

[112] **1. If δ > φ**: The culmination happens south to the zenith, the zenithal distance z and the altitude are respectively: z = $\varphi - \delta$ and h = 90° - z = 90° – $\varphi + \delta$.

2. If δ = φ: z = $\varphi - \delta = 0$ and h = 90° - z = 90° – $\varphi + \delta. = 90°$ The culmination happens at the zenith.

3. If δ < φ: The superior transit happens north to the zenith. The zenithal distance and the altitude are respectively: z = $\delta - \varphi$ and h = 90° - z = 90° + $\varphi - \delta$.

it is northern then subtract the geographical latitude from the declination and subtract the difference from 90°, the remnant is the greatest altitude of the star toward the northern side of the head [the northern altitude]. Know also that if the declination of the star has such a value that if added to the geographical latitude, we get more than 90°, this star does not go under the horizon its declination is northern and when it is southern then it remains always under the horizon. If its addition [of the declination of the star] with the geographical latitude is less than 90° then the star will be sometimes invisible and sometimes visible. We saw already above how long is the diurnal arc above the horizon and the nocturnal under the horizon.[113]

Additional remarks about the conception of the ancients.

The 8th sphere is the sphere of the fixed stars. It is affected by the precession and it moves forward at the rate 360° in 25800 years or 1° in 71.67 years, according to the modern astronomy. Ptolemy had fixed its value at 1° in 100 years and Al-Battani at 1° in 66 years. Ibn Ezra preferred the value of ibn Yunus of 1° in 70 years. If, at a fixed date, the reference date we consider that the eighth sphere and the ninth sphere coincide. The vernal point is the point of intersection of the equator and the ecliptic: The eighth star is sphere of the fixed stars. The vernal point and stars are fixed on this sphere. It moves slowly around the axis passing through the poles of the ecliptic in the direct direction at the rate of the precession. We can note the longitude of all the stars with regard to this vernal point as the origin of the longitudes and the zodiacal signs of the eighth sphere. The longitude of these stars will remain immutable on this eighth sphere. The rotation of the eighth sphere during a span of time of Y years will be (Y / 71.67) in degrees.

The ninth sphere is the sphere of the diurnal movement around the axis passing through the poles of the equator. But it is fixed with regard to the precession. We must only look at it once a day at the same hour. On this fixed sphere the vernal point is fixed. The stars of the eighth sphere move forward at the rate of the precession and can be projected on the ninth sphere. Their longitude on the ninth sphere measured with regard to the fixed vernal point, increase proportionally to the time. Thus for the moderns, the star are fixed and the vernal point moves backward, hence the expression: precession. For the ancients, the vernal point is fixed and the stars move forward with the eighth sphere. Practically we depart from a reference date. At this moment both spheres, the eighth and the ninth coincide. The eighth sphere begins then to turn and the stars and the vernal point remain fixed on this eighth sphere and turn with it. The ninth sphere and the vernal point are fixed with regard to the precession in such a way that the projection of the stars on it, move forward at the rate of the precession and their longitude increases at the same rate.

---

4. At the moment of the inferior transit: $z = 180° - \delta - \varphi$ and $h = \delta - (90° - \varphi) = \varphi + \delta - 90°$.

[113] If $|\delta| < 90° - |\varphi|$, then the star rises and sets. If it is at the equator, $\delta = 0$ it rises exactly at the east and sets exactly at the west. If $\delta > 0$ it rises at the north-east and it sets at the north-west. If $\delta < 0$ it rises at the south-east and it sets at the south-west.

If $|\delta| >= 90° - |\varphi|$, its celestial parallel doesn't cross the celestial horizon. If $\delta > 0$ the star will always be above the horizon. If $\delta < 0$ the star will always be under the horizon and necessarily invisible.

Thus after the date of reference the coordinates of the stars on the ninth sphere are always greater than on the eighth sphere, but before this date of reference, the coordinates of the stars are always smaller than on the eighth sphere. It is also important to make the difference between:

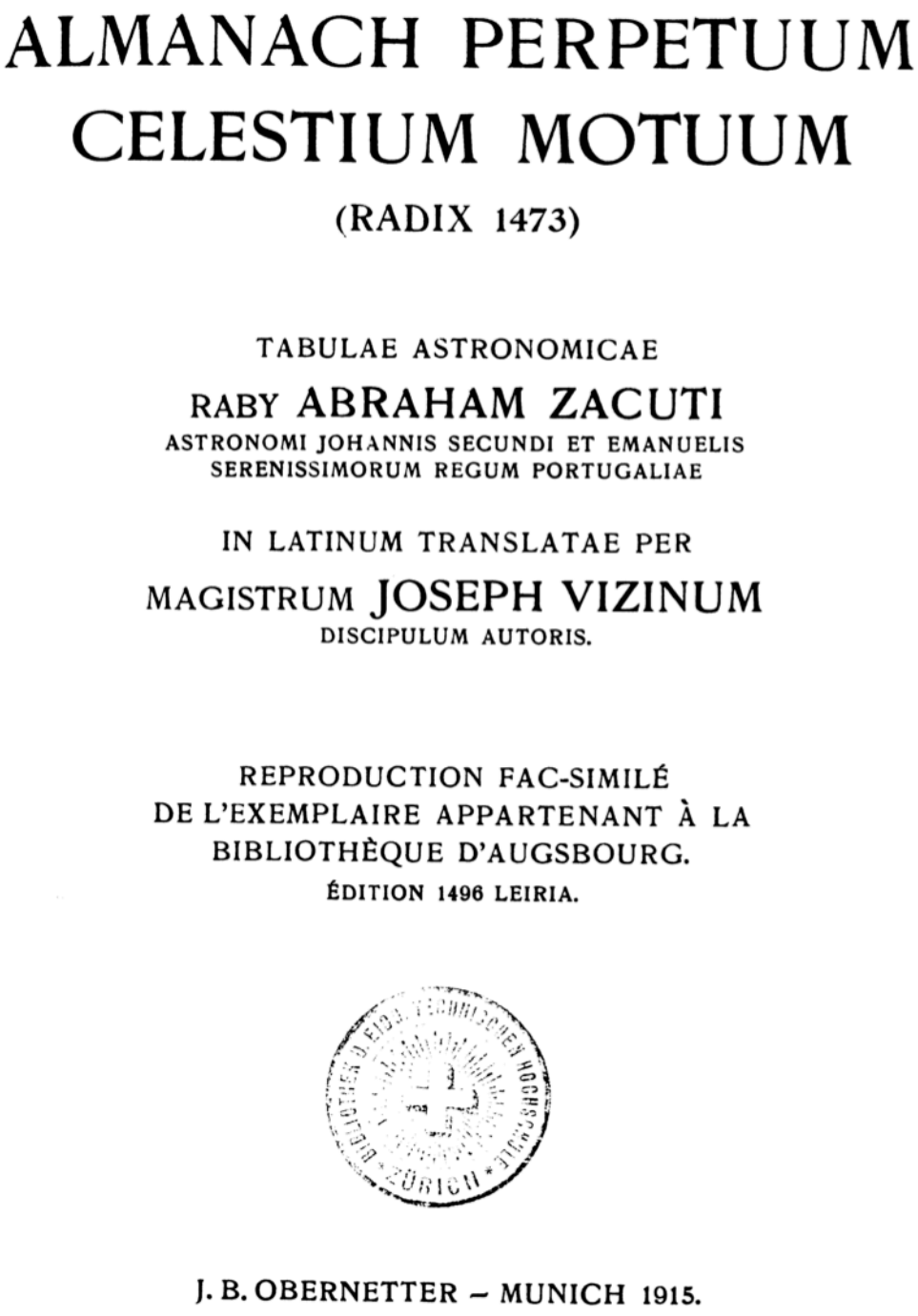
- The signs of the zodiac used to measure the longitude of the stars from the vernal point on the eighth sphere. If, for example, a star has the longitude 129°;08', the ancients said 9°;08' in Leo.
- The image of the constellations of the stars on the eighth sphere, which have the same names as the sign of the zodiac.

Now the coordinates of the star on the eighth sphere don't change with the time. Thus any catalogue of the stars, with their longitudes could be used for the eighth sphere on the condition that we know the date of this catalogue. Nevertheless, it would make sense to adopt as reference time, i.e. the time when on the eighth sphere and the ninth stars coincided, the moment when the vernal point coincided with the beginning of the stellar constellation of Aries. On this way, at the reference time, the twelve stellar constellations of the eighth sphere will coincide with the twelve zodiacal signs of the ninth sphere. The sky and the stars at the moment of reference will thus represent the situation of the sky at the end of the Hellenistic period, when the concept of the Zodiac and it signs were conceived, when the stellar constellations coincided with the zodiacal signs. As the coordinates of the stars in the catalogue of Ptolemy were established for 137 C.E these coordinates corresponding to the ninth sphere in 137 C.E. must not differ substantially from the constellations and the signs of the eighth sphere. If we consider according to modern data that in 2100 the vernal point will reach the beginning of Aquarius, then the vernal point was at the beginning of Aries 25800 * (30 / 360) = 2150 earlier, in – 50, thus 187 years before the date of Ptolemy's catalogue. In The Tables of Moon and Sun, Meeus 1962, it gives for Regulus in 1950: 149°;08'. Going back to the year – 50, we find 149.13 – (2000 / 25800)*360 = 149.13 – 27.91 = 121.23° or Leo 1°;13' and in 137 we would have 1.23 + 2.62 = 3.85° =3°;51' instead of 2°;30' in Almagest. Such a difference is perfectly likely when we consider that the table of Ptolemy was probably derived from a former table based on observation by the addition of a precession of 1° in 100 years instead of 1.52°.

Let us now come back to the text of Zacut. He writes notably[114] that

- Today, in 1478, one must add 20.5° to the values of Ptolemy's catalogue to get the longitude in the ninth sphere.[115]
- In 1478 the shift between the eighth and the ninth spheres is 13°;52'. That means that the longitude of a star in the ninth sphere is 13°;52' greater than in the eighth sphere.
- The position of α Leo in the eighth sphere is 9°;08' in Leo and in the ninth sphere, in 1478 it was 23°.[116]
- The position of Spica in the eighth sphere is 3°;18' in Libra and in the ninth sphere, in 1478, it was 17°;10' in Libra.[117]

[114] After several corrections of the corrupted text.
[115] Ptolemy gave for Regulus 2° ;30'. Zacut gave for its position in the ninth sphere 23°.
[116] In the Almagest, the longitude of α Leo is 2°;30'. The position in 1478 in the ninth sphere is 2°;30' + 20°;30' =23° and the position in the eighth sphere is 23° - 13°;52' = 9°;08'.



ALMANACH PERPETUUM
CELESTIUM MOTUUM

(RADIX 1473)

TABULAE ASTRONOMICAE
RABY ABRAHAM ZACUTI
ASTRONOMI JOHANNIS SECUNDI ET EMANUELIS
SERENISSIMORUM REGUM PORTUGALIAE

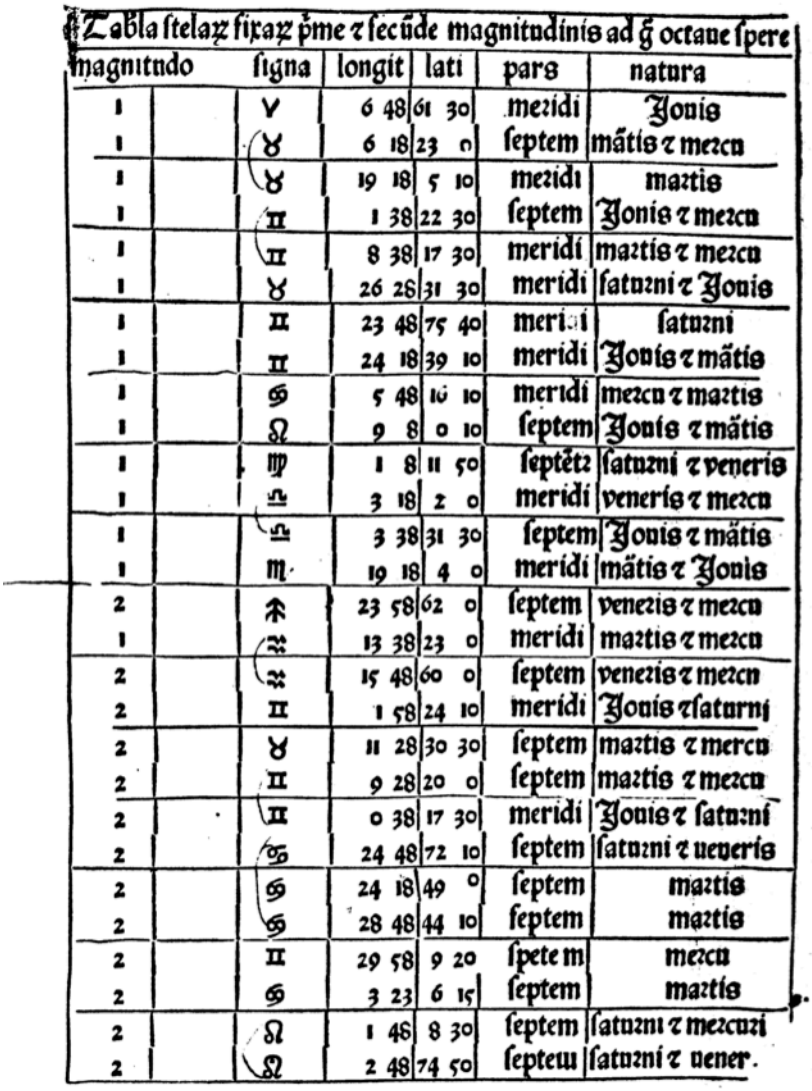
IN LATINUM TRANSLATAE PER
MAGISTRUM JOSEPH VIZINUM
DISCIPULUM AUTORIS.

REPRODUCTION FAC-SIMILÉ
DE L'EXEMPLAIRE APPARTENANT À LA
BIBLIOTHÈQUE D'AUGSBOURG.
ÉDITION 1496 LEIRIA.

J. B. OBERNETTER – MUNICH 1915.

Figure 4: The first page of the Almanach Perpetuum printed in Leira (Portugal) in 1496. Fac-simile edition of 1915 by the Portuguese Government. Xerox copy of the exemplar belonging to the Library of ETH Zurich.

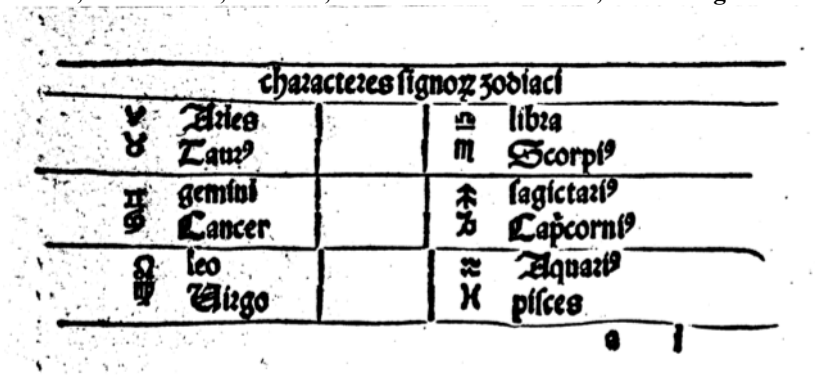
[117] In the Almagest, Toomer p. 369, Constellation XXVI 14, the longitude of Spica is 26°;40' in Virgo. The position in 1478 in the ninth sphere is 26°;40' + 20°;30' =47°;10' Virgo or 17°;10' Libra and the position in the eighth sphere is 17°;10' - 13°;52' = 3°;18'.



¶ Tabla ſtelaꝝ fixaꝝ ꝑme ⁊ ſecũde magnitudinis ad ☾ octaue ſpere

magnitudo	ſigna	longit	lati	pars	natura
1	♈	6 48	61 30	meridi	Jouis
1	♉	6 18	23 0	ſeptem	mãtis ⁊ mercu
1	♉	19 18	5 10	meridi	martis
1	♊	1 38	22 30	ſeptem	Jouis ⁊ mercu
1	♊	8 38	17 30	meridi	martis ⁊ mercu
1	♉	26 28	31 30	meridi	ſaturni ⁊ Jouis
1	♊	23 48	75 40	meridi	ſaturni
1	♊	24 18	39 10	meridi	Jouis ⁊ mãtis
1	♋	5 48	16 10	meridi	mercu ⁊ martis
1	♌	9 8	0 10	ſeptem	Jouis ⁊ mãtis
1	♍	1 8	11 50	ſeptẽtr	ſaturni ⁊ veneris
1	♎	3 18	2 0	meridi	veneris ⁊ mercu
1	♎	3 38	31 30	ſeptem	Jouis ⁊ mãtis
1	♏	19 18	4 0	meridi	mãtis ⁊ Jouis
2	♐	23 58	62 0	ſeptem	veneris ⁊ mercu
1	♒	13 38	23 0	meridi	martis ⁊ mercu
2	♒	15 48	60 0	ſeptem	veneris ⁊ mercu
2	♊	1 58	24 10	meridi	Jouis ⁊ ſaturni
2	♉	11 28	30 30	ſeptem	martis ⁊ mercu
2	♊	9 28	20 0	ſeptem	martis ⁊ mercu
2	♊	0 38	17 30	meridi	Jouis ⁊ ſaturni
2	♌	24 48	72 10	ſeptem	ſaturni ⁊ ueneris
2	♋	24 18	49 0	ſeptem	martis
2	♋	28 48	44 10	ſeptem	martis
2	♊	29 58	9 20	ſeptem	mercu
2	♋	3 23	6 15	ſeptem	martis
2	♌	1 48	8 30	ſeptem	ſaturni ⁊ mercurii
2	♌	2 48	74 50	ſeptem	ſaturni ⁊ uener.

Figure 5: First part of the table of the stars in the eighth sphere in the Almanach Perpetuum. The first column from left gives the magnitude, the second the zodiacal sign, the third, the longitude in the sign, the fourth, the latitude, the fifth, latitude north or south, the sixth gives the associated planet (astrology).



characteres ſignoꝝ zodiaci

♈	Aries	♎	libra
♉	Tauꝰ	♏	Scorpiꝰ
♊	gemini	♐	ſagictariꝰ
♋	Cancer	♑	Capcorniꝰ
♌	leo	♒	Aquariꝰ
♍	Virgo	♓	piſces

Figure 6: The symbols of the zodiacal signs. Leo 9°;08' means λ = 129°;08', the longitude of Regulus in the eighth sphere.

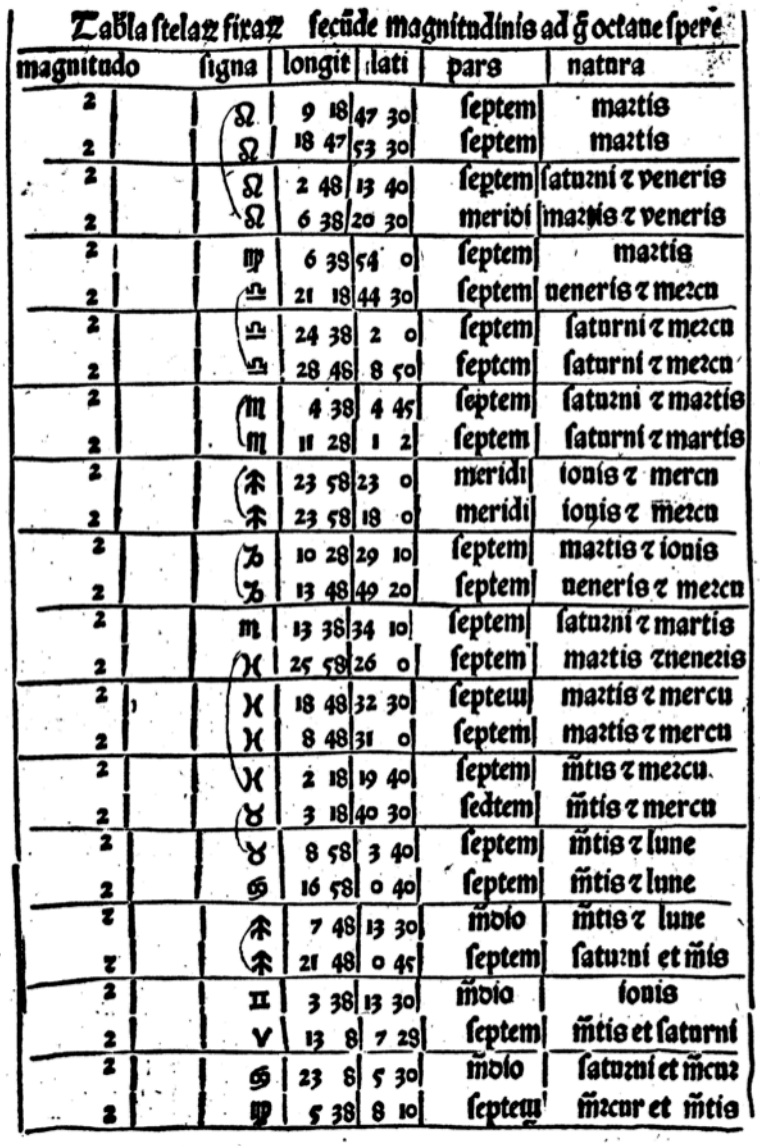
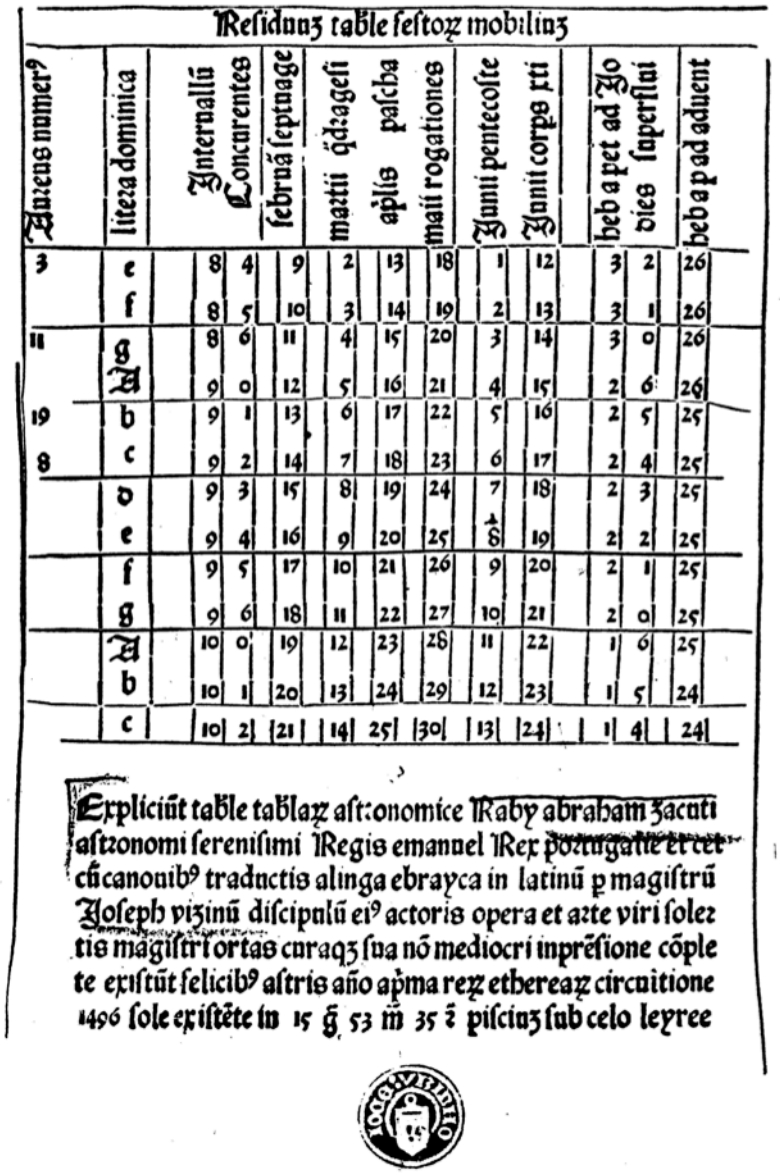


Tabla ſtelaꝝ fixaꝝ ſecũde magnitudinis ad ḡ octane ſperę

magnitudo	signa	longit	lati	pars	natura
2	♌	9 18	47 30	ſeptem	martis
2	♌	18 47	53 30	ſeptem	martis
2	♌	2 48	13 40	ſeptem	ſaturni ꝛ veneris
2	♌	6 38	20 30	meridi	martis ꝛ veneris
2	♍	6 38	54 0	ſeptem	martis
2	♎	21 18	44 30	ſeptem	veneris ꝛ mercu
2	♎	24 38	2 0	ſeptem	ſaturni ꝛ mercu
2	♎	28 48	8 50	ſeptem	ſaturni ꝛ mercu
2	♏	4 38	4 45	ſeptem	ſaturni ꝛ martis
2	♏	11 28	1 2	ſeptem	ſaturni ꝛ martis
2	♐	23 58	23 0	meridi	ionis ꝛ mercu
2	♐	23 58	18 0	meridi	ionis ꝛ mercu
2	♑	10 28	29 10	ſeptem	martis ꝛ ionis
2	♑	13 48	49 20	ſeptem	veneris ꝛ mercu
2	♒	13 38	34 10	ſeptem	ſaturni ꝛ martis
2	♓	25 58	26 0	ſeptem	martis ꝛ veneris
2	♓	18 48	32 30	ſepteꝫ	martis ꝛ mercu
2	♓	8 48	31 0	ſeptem	martis ꝛ mercu
2	♈	2 18	19 40	ſeptem	mtis ꝛ mercu.
2	♉	3 18	40 30	ſedtem	mtis ꝛ mercu
2	♉	8 58	3 40	ſeptem	mtis ꝛ lune
2	♋	16 58	0 40	ſeptem	mtis ꝛ lune
ꝛ	♐	7 48	13 30	mdio	mtis ꝛ lune
ꝛ	♐	21 48	0 45	ſeptem	ſaturni et mtis
2	♊	3 38	13 30	mdio	ionis
2	♈	13 8	7 28	ſeptem	mtis et ſaturni
2	♋	23 8	5 30	mdio	ſaturni et mcur
2	♍	5 38	8 10	ſepteꝫ	mcur et mtis

Figure 7: second part of the table of the stars in the eighth sphere in the Almanach Perpetuum. We note on figure 5 the stars of first magnitude Regulus: Leo 9°;08' = 129°;08' and Spica: Libra 3°;18' = 183°;18' considered by Zacut in his Hebrew canon.



Residuus table festoz mobilius

Aureus numer?	litera dominica	Interuallū Concurrentes		februā septuage	martii q̄dragesi	aplis pascha	maii rogationes	Junii pentecoste	Junii corpe xti		heb a pet ad Jo dies superflui		heb a pad aduent
3	e	8	4	9	2	13	18	1	12		3	2	26
	f	8	5	10	3	14	19	2	13		3	1	26
11	g	8	6	11	4	15	20	3	14		3	0	26
	A	9	0	12	5	16	21	4	15		2	6	26
19	b	9	1	13	6	17	22	5	16		2	5	25
8	c	9	2	14	7	18	23	6	17		2	4	25
	d	9	3	15	8	19	24	7	18		2	3	25
	e	9	4	16	9	20	25	8	19		2	2	25
	f	9	5	17	10	21	26	9	20		2	1	25
	g	9	6	18	11	22	27	10	21		2	0	25
	A	10	0	19	12	23	28	11	22		1	6	25
	b	10	1	20	13	24	29	12	23		1	5	24
	c	10	2	21	14	25	30	13	24		1	4	24

Expliciūt table tablaz astronomice Raby abraham zacuti astronomi serenissimi Regis emanuel Rex portugalie et cer cū canonib⁹ traductis alinga ebrayca in latinū p magistrū Joseph vizinū discipulū ei⁹ actoris opera et arte viri solez tis magistri ortas curaqz sua nō mediocri inprēsione cōple te existūt felicib⁹ astris año a p̄ma rez ethereaz circuitione 1496 sole existēte in 15 g̃ 53 m̃ 35 s̃ piscium sub celo leyree

Figure 8: Last page of the Almanach Perpetuum: a table about the mobile Christians festivals with the colophon mentioning Raby Abraham Zacut, 'Astronomer Royal' of Portugal. This title did not exist at that time; I borrowed it to the title created in 1675 by the king of England for the director of the new observatory of Greenwich. Flamsteed was the first to be called by this title.

The checking of these figures in the preceding notes shows that they are coherent. The only question is: how and why did Zacut choose his eighth sphere in the year 563. Apparently it does not correspond to anything; no star catalogue corresponds to this period. His eighth sphere, with reference to the year 563 C.E. was probably deduced from the catalogue ascribed to Menelaus and deduced from that of Ptolemy. The procedure was probably the following:

Span of time between 563 C.E and Menelaus 92 C.E.: 563 – 92 = 471 years. Shift: 471 / 66 = 7.14 = 7°;08'. But in fact Ptolemy had already added 25', and rounded it later off to 30' for the period 92

C.E. to 133 C.E.[118] therefore 7°;08' should be reduced to 6°; 38' if we refer to the values of Ptolemy. Apparently the astronomers of the Middle age lost sight of this point.

Spica. Almagest: 26°;40' Virgo. Ascribed to Menelaus by Zacut: 26°;10'. In 563: 26°;10' + 7°;08' = 33°;18' Virgo = 3°;18' Libra

Regulus or Heart of Lion. Almagest: 2°;30' Leo. Ascribed to Menelaus by Zacut: 2°;00'. In 563: 2°;00' + 7°;08' = 9°;08' Leo.

The theory of Zacut is thus perfectly coherent but the reason of the choice of the year 563 for his eighth sphere remains a conundrum. An explanation could however be as follows: Zacut noted that the precession in 1478 with regard to the catalogue of Ptolemy is 20.5° and he noted also that the difference between the ninth and the eighth sphere is 13°;52'. Hence the difference between the ninth sphere of Zacut (1478) and the catalogue of Ptolemy is 20°;30' and the difference between the eight sphere (563) and the catalogue of Ptolemy is 20°;30' – 13°;52' = 6°;38'. This can be checked in the table of the 56 brightest stars given by Zacut in his *Almanach Perpetuum* according the eighth sphere Spica is given as 3°;18' Libra and Regulus is given as 9°;08' Leo. In the catalogue of Ptolemy we find Spica: 26°;40' Virgo and Regulus: 2°;30' . The difference is indeed 6°;38'. Now It appears that the Andalusian astronomer ibn al- Kammad[119] constructed a table of 30 stars which differs from the catalogue of Ptolemy by the same difference of 6°;38'. The table of Zacut would then be an expansion of that of ibn al-Kammad. As for the date of 563 for the table of ibn al-Kammad constructed in the second half of the twelfth century, Goldstein and Chabas[120] have made the assumption that the author, using a slightly different value of the precession, could have constructed this table for the date of the Hegira, in 620. This does not yet explain on which basis this astronomer, ibn al-Kammad, established that at this epoch the vernal point was at the beginning of the astronomical constellation of Aries i.e. that at this epoch the zodiacal signs of the ninth sphere coincided with the zodiacal constellation of the eighth sphere. It is not excluded that this coincidence had also a religious and cosmic signification.[121] What a symbol if at the day of the Hegira, the vernal point was at the beginning of the constellation of Aries and the stellar constellations of the eighth sphere coincided with the zodiacal signs of the ninth sphere!

The several mistakes in the Hebrew manuscript and the complications due to the conception of the ancients complicated the good understanding. We see that while other important astronomers of that period and even later, like the great Copernicus were still mixed up in the theory of the trepidation; Zacut had definitively broken with the theory of the trepidation and he came back to the theory of Al-Battani, near to the modern values. This was a century before Tycho Brahé, but it was only the latter who determined the exact value of the precession. The notion of the nutation would only become known much later.

---

[118] Zacut considers that the date of Ptolemy's catalogue is 133 C.E.

[119] Ibn al-Kammad, died 1195. Andalusian astronomer born in Seville, al-Andalus and educated in Cordoba by the students of Al-Zarqali. He composed different zys (Arabic astronomical tables), which influenced later astronomers. See Comes, Mercè (2007): ibn al-Kammad in Thomas Hockey. The Biographical encyclopedia of Astronomers, New York, Springer, pp. 559 – 560.

[120] Goldstein and Chabbas (1996), ibn al.Kammad's Star List, Centaurus 38: 317 - 334. Carvalho, J de (1927), Dois ineditos de Abraham Zacuto, Revista de Estudios Hebraicos 1 : 7 – 54.

[121] The exact appreciation of this coincidence was very difficult to observe and had probably a subjective character. The mstake is about 9°.

9. The word of the end.

The problem of the precession was a fundamental problem in the middle age astronomy. We have seen in this paper how important Jewish personalities were involved in its elucidation and participated to the general scientific debate which it arose. Some streams of the Jewish elite opposed the study of philosophy but apparently there was never any objection against the study of natural philosophy (today we say physics) and especially astronomy. It was certainly considered as the study of the natural divine rules, parallel to the religious divine rules. Even such an *Halakhist* as Rambam could spend time to the study, besides the medical sciences for professional reasons, for making a livelihood, of natural sciences for purely speculative interest. As I suggest such a study could also have a religious signification[122] and therefore it did not arise any objection.[123] The same can be said of personalities like R. Levi ben Gershom who had a life of a professional astronomer parallel to that of a Bible commentator and a Talmudic scholar or Abraham Zacut, whose book was published a year before the expulsion of the Jews from Portugal with the mention of 'Raby Abraham Zacuti'. This spirit will survive in the sixteenth century in the schools of Maharal in Prague and of R. Moses Isserles in Cracow. But in general this spirit will decline because of the conjunction of two concordant factors. First the disappearing of the liberal spirit of the Renaissance, which was of nature to suppress the material boundaries between the cultures and the religions, because of the Counter-Reformation, a movement of counter-attack after the successes of the Reformation introduced by the Protestants. This Counter-Reformation was marked by the Council of Trent (1545), the reorganization of the inquisition (1542)[124] and the watching of the impression of books (1543)[125] with publication of an Index of forbidden books and finally the interdiction of the stay of the Jews in the Pontifical states. The result of this new situation was the greater difficulty to gather scientific knowledge and the resulting greater danger for Jewish students of external influence and pressure for conversion and apostasy.[126] Even if we had still in the seventeenth and the first half of the eighteenth century, a number of rabbis[127] graduated from the University of Padua or later some Jewish doctors graduated from Leiden and Utrecht universities, where Jews were tolerated, the exercise was not without danger. The graduated doctors exerting in the European towns were often among the most liberal and enlighten spirits, the most removed and the weakest member of the community, the firsts to break away with it and to convert to the state religion. The second factor, a consequence of the first,

[122] These principles of astronomy are expounded in *Hilkhot Yessodei ha-Torah*, chapter 3.

[123] Nevertheless, in a wonderful letter addressed to R. Jonathan ha-Cohen from Lunel and to his friends, Rambam presents these different branches of science, as concurrent to the Torah. He devoted much time to them at the expense of the Torah study. See Isaac Shilat, *Iggerot ha-Rambam*, Jerusalem 5747, vol 2, p. 502. It is possible that astronomy has, for Maimonides, a special connection with the knowledge of the divine and has therefore a special status because it reveals the infinity of the universe.

[124] Giordano Bruno and Galileo must have been well informed about it.

[125] Jewish books must now be controlled by a censor having knowledge of Hebrew.

[126] Rabbi Joseph Solomon Delmedigo (student in Padua in 1609 – 1610) in his book *Elim* (Amsterdam 1629), warned parents against sending their sons to Padua before 'the light of the Torah has shined upon them…. in order that they not turn away from it'. Similarly, Tuvia Katz (student in Padua in 1678 – 1679) wrote: ' No one in all the lands of Italy, Poland, Germany and France should consider studying medicine without first filling his belly with the written and oral Torah and other subjects'. See Ruderman D.B. Jewish Though and scientific Discovery in early Modern Europe, New Haven, Yale University Press pp. 239 – 242.

[127] Among them R. Samson Morpurgo (1681 – 1740), R. Isaac Lampronti (1679 – 1756) who are still considered as *Gedolim* (rabbis whose halakhic responsa are universally recognized), R. Joseph Solomon Delmedigo (1591 – 1655) and R. David Nieto (1654 – 1728). The two latters also appear in *Shem ha-Gedolim* (Hida, Livorno 1774) through their book.

was then a new attitude of the Jewish community which became frankly negative to the study of sciences. Still today the rightest streams of the Jewish community behold this position and raise it into an ideological and religious principle. The existence today of scientific faculties in Israel, allowing acquiring knowledge, at the highest level, without the former problems and dangers, without the new political hostility against the Jews, should remove this traditional hostility, which has lost any justification. The 'religious signification' of scientific studies and research should assert itself again in the image of our greatest scholars and at their head, Maimonides.