Reaction to the Papers of Eran Raviv published in B.D.D nos. 22, 26, 27 and 28 versus my paper published in B.D.D 28.

Eran Raviv published a series of important papers about the Jewish calendar, and more specifically about the four gates' table and the table of 61 rows, which he succeeded to expand to 213 rows in relating to the 30 indexed *keviyot* instead of the 14 traditional *keviyot*. The present letter to the editor is in no way a form of contradiction; it is only a complement to his learned papers.

1. Eran Raviv adopted a surprising convention; which badly requires an explanation and an elucidation. I noted indeed that he wrote in the four papers that
Beharad =2*25920+5*1080+204= 57444 hal instead of 31524 hal (see my article BDD 28 page 7).
In his papers BDD22 p. 45, BDD26 pp. 53, 57 and 59, BDD27 p. 73 and in BDD28 pp. 117-121, 124, 126 and 127 he classified the limits of his four gates' table and his table of 213 rows, departing from the beginning of the week, on Saturday 6 p.m. or (1) – 0 – 0 until the end of the week, (7) – 23 – 1079. His first limit in the table of 213 rows (1) – 0 – 409 is identified with 26329. Similarly his first limit in the four gates' table, 0 – 18 – 0 is identified with 19440. This seems really surprising. In order to understand the problem, let us recall that a moment of the Jewish week is represented by a triplet D – H – h. D is the begun day, H is the number of completed hours and d is the number of completed *halakim*. Thus D – H – h designates a moment in the Dth day of the week after that H hours and h *halakim* of this day elapsed. Toomer calls this in *Ptolemy's Almagest* p. 276 note 4, the inclusive notation. Now if we want to express this moment in *halakim*, then we want to find the number of *halakim* elapsed from the beginning of the week until the considered moment. It is (D – 1)*25920 +H*1080+h. This is the exclusive notation.
In order to make the difference clear, we use the following convention:
Beharad = (2) – 5 – 204 = 31524 is the inclusive notation, the begun day being put in brackets.
Beharad = 1 – 5 – 204 = 31524 is the exclusive notation, completed days, hours and *halakim*.
At the beginning of the week: (1) – 0 – 0 the number of elapsed *halakim* is 0.
At the end of the week: (7) – 23 – 1079 = 6 – 23 – 1079 = 181439 *hal*, the number of elapsed *halakim*. Therefore in the table of 61 rows (BDD 28 pp. 25-26) the Jewish week is scanned from (1) – 0 – 0 until 6 – 23 – 1079 or from 0 *hal* until 181439 *hal* and the first line is 409 hal.
Eran Raviv, in order to make the calculations more convenient, and avoid being obliged to subtract one day when calculating the *halakim* and to add a day when going back from *halakim* to the classical notation, decided to express the moment in *halakim* by the formula D * 25920 +H * 1080 + h. That means that he chose a fictitious epoch of the Jewish week, shifted backwards by one day and placed it on Friday 6 p.m. at (7) – 0 – 0.
Now when the week begins on (1) – 0 – 0, the span of time between the epoch and the beginning of the week is indeed 25920 and when it ends on (7) – 23 – 1079 the span of time between the fictitious epoch and the end of the week is 207359 *hal*.
This explanation must imperatively be given; it will allow the reader to solve the apparent contradiction between the papers of Eran Raviv and mine. The convention of Eran Raviv is probably more convenient, as soon as one is aware of its principle, but mine corresponds to the truth and satisfies certainly more the purist. The reader will choose the best solution. In any case one must always work with the same convention, otherwise it would be catastrophic.

2. In B.D.D 26 pp. 51-52, Eran Raviv noted that, at the origin, the four gates were related to the four possible days of Rosh ha-Shanah and that Yehiel Borenstein transposed them to the

four types of years. However when we read *Sefer ha-Ibbur* of R. Abraham bar Hiya, [edition Filipowski, London 1851 pp. 63-64 (II, 9)] and *Yessod Olam* of R. Isaac ha-Yisraeli, [edition Baer Goldberg, Berlin 1858 (IV, 10) p. 19b], we see that they first introduced 4 gates according indeed to the four possible days of Rosh ha-Shanah and then, afterwards, they introduced again 4 gates corresponding to the 4 types of years. It seems thus that in fact we have two sets of 4 crossed gates. Today we follow Borenstein who put the emphasis on the four types of years. Note however, that Joffe (*Korot Heshbon ha-Ibbur* 1931 p. 171) explicitly divided the table into 4 columns corresponding to the four possible days of Rosh ha-Shanah and still put the emphasis on the days.

3. Eran Raviv expanded the table of 61 rows into a table of 213 rows and this is a remarkable achievement. However, if I am not mistaken he did not calculate the frequency of the 30 indexed *keviyot*. Of course he could have done it easily with his program.

A short explanation of the program and an organization chart of it would have interested the reader. It would have removed a part of the mysterious character of the results.

I note that Eran Raviv did not exploit all the resources and information that we can draw from the expansion of the four gates' table. The four gates' table is much easier to understand and to master. It allows a deeper understanding of the calendar and its *moladot*.

I will show that the calculation of the frequency of the indexed *keviyot* is easier on this way than through the table of 213 rows. It can be performed manually and can be understood.

With the table of 213 rows, we lose any control and must rely on the results of the computer.

Because of the space required, we have divided the four gates into four different pages.

4. **Principle of the calculation of frequencies.**

The period of the calendar contains 689472 years = 19 * 36288 = 3.8 * 181440.

We have thus 36288 cycles in this period, each year of rank k repeats itself 36288 times in the great period of the Jewish calendar. But the 181440 instants (in *halakim*) of the week don't repeat themselves uniformly as a *molad* of the 689472 years. Some repeat themselves 4 times and others only three times. On average, each instant of the week is 3.8 times the *molad* of a year.

The 181440 instants of the week contribute to 689472 years.

10 consecutive instants contribute to 38 years i.e. they are the *moladot* of 38 years of the great period. Let us examine the years of the four gates.

First gate: years 2, 5, 10, 13 and16. Each group of 10 consecutive instants of the week contributes to 10 years.

The year of rank 2 through the end-digits 0 and 5
5 through the end-digits 1 and 6
10 through the end-digits 7 and 2
13 through the end-digits 8 and 3
16 through the end-digits 9 and 4

Each end-digit contributes for 1 year.

Second gate: years 1, 4, 9, 12, 15. Each group of 10 consecutive instants of the week contributes to 10 years.

The year of rank 1 through the end-digits 4 and 9
4 through the end-digits 5 and 0
9 through the end-digits 1 and 6
12 through the end-digits 2 and 7
15 through the end-digits 3 and 8

Each end-digit contributes for 1 year.

Third gate: years 3, 6, 8, 11, 14, 17, 19. Each group of 10 consecutive instants of the week contributes to 14 years.
The year of rank 3 through the end-digits 6 and 1
6 through the end-digits 7 and 2
8 through the end-digits 2 and 7
11 through the end-digits 3 and 8
14 through the end-digits 4 and 9
17 through the end-digits 5 and 0
19 through the end-digits 0 and 5
The end-digits 1, 3, 4, 6, 8, 9 contribute, each of them, for one year. The end-digits 0, 2, 5, 7 contribute, each of them, for 2 years. Hence 4 * 2 + 6 = 14 years.
Fourth gate: years 7, 18. Each group of 10 consecutive instants of the week contributes to 4 years.
The year of rank 7 through the end-digits 6 and 1
18 through the end-digits 4 and 9
Each end-digit1, 4, 6 and 9 contributes for 1 year.
Conclusion: Each group of 10 consecutive instants of the week contributes to 38 years.
This allows the calculation of the number of years corresponding to a span of time of the Jewish week, whether it contains a multiple of 10 consecutive instants or a remainder of a number of consecutive instants less than 10 *halakim*.
For example, **in the first gate**, the span of time between 113604 and 123119 (inclusive) has an extend of 9516 instants: 951 groups of ten consecutive instants corresponding to 10*951 years and a remainder of 6 instants with end-digits 4, 5, 6, 7, 8 and 9 corresponding to 6 additional years. **In the third gate**, the span of time between 90335 and 103679 (inclusive) has an extent of 13345 instants: 1334 groups of ten consecutive instants corresponding to 14*1334 years and a remainder of 5 instants with end-digits 5, 6, 7, 8 and 9 corresponding to 7 additional years. The end-digit 5 corresponds to a year of rank 17 and a year of rank 19, the end-digit 6 corresponds to a year of rank 3, the end-digit 7coresponds to a year of rank 6 and a year of rank 8. The end-digit 8 corresponds to a year of rank 11 and the end-digit 9 corresponds to a year of rank 14. In all, 7 years. **In the third gate**, the span of time between 151691 and 155519 (inclusive) has an extent of 3829 instants: 383 groups of ten consecutive instants corresponding to 14*383 years minus one instant, 155520, improperly included. The end-digit of this instant is 0; it corresponds to a year of rank 17 and to a year of rank 19. In all 2 years which were counted too much. The considered span of time contributes to 14*383 – 2 = 5360 years. **In the fourth gate**, the span of time between 139524 and 155519 (inclusive) has an extent of 15996 instants: 1600 groups of ten consecutive instants corresponding to 4*1600 years minus 4 instants that were improperly included. Their end-digits are 0, 1, 2 and 3. A year of rank 7 was thus counted too much. The contribution of this area is thus finally 4*1600 – 1= 6399. With these four examples we covered all the possible difficulties that could be met in the evaluation of the number of years corresponding to any span of time of the Jewish week, for any of the four kinds of years of the Jewish cycle of 19 years.

In the following tables (four gates' table expanded according to Eran Raviv's principle) I adopted the numbering of Eran Raviv for the *dehiyot,* (see BDD 22 p, 40 and BDD 27 p. 65). It is preferable to my numbering adopted in B.D.D 28 table 7 p. 19.

5. **Errata. Correction of a few misprints in my paper in BDD28.**
p. 10 last line: **689473** instead of 689743.
p. 21, 3 lines from bottom: **9516 *halakim*** instead of *moladot*.
p. 22, 3rd and 4th lines: **2651 *halakim*** instead of 9516 *moladot*.

Addenda and Corrigenda to my paper in BDD29, table p. 37.

In the first horizontal row, the indication: (1900)[1]

The exponent 1 refers to the following note: Tables of Moon and Sun, Jean Meeus, Kessel-Lo, 1962, pp.186 – 192.

(1900) indicates that in these tables, the coordinates and the corrections were calculated with regard to an epoch of 1 / 1 / 1900.

Expanded four gates' table.

First gate: years 2, 5, 10, 13 and 16.

Lower limit	Upper limit	Size	Postpone Days	*dehiya*	*keviya*	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בחג	10 * 992 + 4	9924
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	10*1599 +6	15996
(2)-0-0 25920	(2)-17-1079 45359	19440	0	0	בשה	10*1944	19440
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	10*648	6480
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	10*992 +4	9924
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	10*951 +6	9516
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	10*648	6480
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	10*2592	25920
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	10*992 +4	9924
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	10*951 +6	9516
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	10*648	6480
(6)-0-0 129600	(6)-23-1079 155519	9924	1	1	זחא	10*992 +4	9924
(6)-9-204 139524	(6)-23-1079 155519	15996	1	1	זשג	10*1599 +6	15996
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	10*1944	19440
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחג	10*648	6480
Total number		181440				5*36288	181440

Second gate; years 1, 4, 9, 12 and 15.

Lower limit	Upper limit	Size	Postpone Days	*dehiya*	*keviya*	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בחג	10 * 992 + 4	9924
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	10*1599 +6	15996
(2)-0-0 25920	(2)-15-588 42708	16789	0	0	בשה	10*1678 +9	16789
(2)-15-589 42709	(2)-17-1079 45359	2651	1	5	גכה	10*265 +1	2651
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	10*648	6480
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	10*992 +4	9924
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	10*951 +6	9516
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	10*648	6480
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	10*2592	25920
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	10*992 +4	9924
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	10*951 +6	9516
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	10*648	6480
(6)-0-0 129600	(6)-0-407 130007	408	1	1	זחא	10*40 +8	408
(6)-0-408 130008	(6)-23-1079 155519	25512	1	1	זשג	10*2551 +2	25512
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	10*1944	19440
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחג	10*648	6480
Total number		181440				5*36288	181440

Third gate: years 3, 6, 8, 11, 14, 17and 19.

Lower limit	Upper limit	Size	Postpone Days	*dehiya*	*keviya*	Number years	Total
(1)-0-0 0	(1)-20-490 22090	22091	1	1	בחה	14*2209 +2	30928
(1)-20-491 22091	1-23-1079 25919	3829	1	1	בשג	14*383 – 2	5360
(2)-0-0 25920	(2)-17-1079 45359	19440	0	0	בשג	14*1944	27216
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכז	14*648	9072
(3)-0-0 51840	(3)-17-1079 71279	19440	0	0	גכז	14*1944	27216
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	החא	14*648	9072
(4)-0-0 77760	(4)-11-694 90334	12575	1	1	החא	14*1257 +7	17605
(4)-11-695 90335	(4)-23-1079 103679	13345	1	1	השג	14*1334 +7	18683
(5)-0-0 103680	(5)-17-1079 123119	19440	0	0	השג	14*1944	27216
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחג	14*648	9072
(6)-0-0 129600	(6)-20-490 151690	22091	1	1	זחג	14*2209 +2	30928
(6)-20-491 151691	(6)-23-1079 155519	3829	1	1	זשה	14*383 – 2	5360
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשה	14*1944	27216
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחה	14*648	9072
Total number		181440				7*36288	254016

Fourth gate: years 7 and 18.

Lower limit	Upper limit	Size	Postpone Days	*dehiya*	*keviya*	Number years	Total
(1)-0-0 0	(1)-9-203 9923	9924	1	1	בחג	4 * 992 +1	3969
(1)-9-204 9924	1-23-1079 25919	15996	1	1	בשה	4*1599 +3	6399
(2)-0-0 25920	(2)-15-588 42708	16789	0	0	בשה	4*1679 –1	6715
(2)-15-589 42709	(2)-17-1079 51839	2651	1	5	גכה	4*265 +1	1061
(2)-18-0 45360	(2)-23-1079 51839	6480	1	2	גכה	4*648	2592
(3)-0-0 51840	(3)-9-203 61763	9924	0	0	גכה	4*992 +1	3969
(3)-9-204 61764	(3)-17-1079 71279	9516	2	4	הכז	4*951 +3	3807
(3)-18-0 71280	(3)-23-1079 77759	6480	2	3	הכז	4*648	2592
(4)-0-0 77760	(4)-23-1079 103679	25920	1	1	הכז	4*2592	10368
(5)-0-0 103680	(5)-9-203 113603	9924	0	0	הכז	4*992 +1	3969
(5)-9-204 113604	(5)-17-1079 123119	9516	0	0	השא	4*951 +3	3807
(5)-18-0 123120	(5)-23-1079 129599	6480	2	3	זחא	4*648	2592
(6)-0-0 129600	(6)-9-203 139523	9924	1	1	זחא	4*992 +1	3969
(6)-9-204 139524	(6)-23-1079 155519	15996	1	1	זשג	4*1599 +3	6399
(7)-0-0 155520	(7)-17-1079 174959	19440	0	0	זשג	4*1944	7776
(7)-18-0 174960	(7)-23-1079 181439	6480	2	3	בחג	4*648	2592
Total number		181440				2*36288	72576

Frequencies of the indexed *keviyot*.

Ordinary years				**Leap years**			
keviyah	*dehiyah*	Number Of years	Percent %	*keviah*	*dehiyah*	Number of years	Percent %
בחג		**39369**	**5.7100**	**בחה**		**40000**	**5.8015**
בחג	1	23817	3.4544	בחה	1	30928	4.4858
בחג	3	15552	2.2556	בחה	3	9072	1.3158
בשה		**81335**	**11.7967**	**בשז**		**32576**	**4.7248**
בשה	0	42944	6.2285	בשז	0	27216	3.9474
בשה	1	38391	5.5682	בשז	1	5360	0.7774
גכה		**43081**	**6.2484**	**גכז**		**36288**	**5.2632**
גכה	0	23817	3.4544	גכז	0	27216	3.9474
גכה	2	15552	2.2556	גכז	2	9072	1.3158
גכה	5	3712	0.5384				
הכז		**124416**	**18.0451**				
הכז	0	23817	3.4544				
הכז	1	62208	9.0226				
הכז	3	15552	2.2556				
הכז	4	22839	3.3125				
				החא		**26677**	**3.8692**
				החא	1	17605	2.5534
				החא	3	9072	1.3158
השא		**22839**	**3.3125**	**השג**		**45899**	**6.6571**
השא	0	22839	3.3125	השג	0	27216	3.9474
				השג	1	18683	2.7098
זחא		**29853**	**4.3298**	**זחג**		**40000**	**5.8015**
זחא	1	14301	2.0742	זחג	1	30928	4.4858
זחא	3	15552	2.2556	זחג	3	9072	1.3158
זשג		**94563**	**13.7153**	**זשה**		**32576**	**4.7248**
זשג	0	46656	6.7669	זשה	0	27216	3.9474
זשג	1	47907	6.9484	זשה	1	5360	0.7774
Total		**435456**	**63.1579 = 12/19**			**254016**	**36.8421 = 7/19**

Frequency of the different *dehiyot*.

Dehiyot	Ordinary year	% ordinary years	Leap years	%leap years	Total years	%total years
0	160073	36.7599	108864	42.8571	268937	39.0062
1	186624	42.8571	108864	42.8571	295488	42.8571
2	15552	3.5714	9072	3.5714	24624	3.5714
3	46656	10.7143	27216	10.7143	73872	10.7143
4	22839	5.2449			22839	3.3125
5	3712	0.8524			3712	0.5384
Total	435456	100	254016	100	689472	100

Note: % ordinary years *12/19 + % leap years *7/19 = % total years.

6. **The theorem of Bayes and its application in the Jewish calendar.**

Let E1, E2, E3 and E4 be four different events, of which one must necessarily materialize. We assume that each of these events results from either an event C1 or an event C2 of which one must necessarily materialize. By convention we name each of the events C1 andC2 a possible cause of the event Ei. We assume to know the probability of intervening of the causes Pr (C1) and Pr (C2), Pr (Ei / C1) and Pr (Ei / C2) the probability of realization of the event Ei if the cause C1 or C2 materializes. We have thus Pr (C1) + Pr (C2) = 1
Pr (E1 /Cj) + Pr (E2 /Cj) + Pr (E3 /Cj) + Pr (E4 /Cj) = 1 where j can take the values 1 and 2.
The formula of Bayes allows calculating Pr (Cj / Ei), the probability, when Ei materializes, of the intervention of the cause Cj:

$$\Pr(Cj / Ei) = \frac{\Pr(Cj)\Pr(Ei/Cj)}{\Pr(C1)\Pr(Ei/C1)+\Pr(C2)\Pr(Ei/C2)}$$

Example. We consider two urns. Each of them contains a known number of balls of four different colors. A blindfolded man pulls a ball out of one of the urns. We ascertain that it is a white ball. We want to know the probability that this ball was pulled out of the first urn.
Now we transpose the former example. The urn 1 contains the **O**rdinary years and the urn 2 contains the **L**eap years. The balls can have four colors, white, black, red and green. They represent years beginning on a **M**onday, **Tu**esday, **Th**ursday and **S**aturday.
The events E1, E2, E3 and E4 represent a Jewish year beginning on **M**, **Tu**, **Th** and **S**.
The events (the causes) C1 and C2, pulling a ball out of urn 1 or urn 2 represent pulling an **O**rdinary year or a **L**eap year. The formula of Bayes will allow us to calculate the probability that a random year beginning on a Monday is an Ordinary year.
In the example of the urns Pr (C1) = Pr (C2) = 0.5. In the cases of the Jewish years
Pr (C1) = Pr (0) = 12/19 = 0.6316. Pr (C2) = Pr (L) = 7/19 = 0.3684.
In the box 1 of the ordinary years the probability to pull a ball of a certain color or the probability of the events E1, E2, E3 or E4 is:
Pr (E1 / C1) = Pr (M / O) = 120704 / 435456 = 0.2772
Pr (E2 / C1) = Pr (Tu / O) = 43081 / 435456 = 0.0989
Pr (E3 / C1) = Pr (Th / O) = 147255 / 435456 = 0.3382
Pr (E4 / C1) = Pr (S / O) = 124416 / 435456 = 0.2857
In the box 2 of the leap years the probability to pull a ball of a certain color or the probability of the events E1, E2, E3 or E4 is:
Pr (E1 / C2) = Pr (M / L) = 72576 / 254016 = 0.2857
Pr (E2 / C2) = Pr (Tu / L) = 36288 / 254016 = 0.1429
Pr (E3 / C2) = Pr (Th / L) = 72576 / 254016 = 0.2857
Pr (E4 / C2) = Pr (S / L) = 72576 / 254016 = 0.2857
The formula of Bayes gives now:

$$\Pr(C1 / Ei) = \frac{\Pr(C1)\Pr(Ei/C1)}{\Pr(C1)\Pr(Ei/C1)+\Pr(C2)\Pr(Ei/C2)}$$ and Pr (C2 / Ei) = 1 - Pr (C1 / Ei).

Event E1 : A Jewish year begins on Monday, E1 is realized. The probability that it is an ordinary year is Pr (C1 / E1) = Pr (O / M) = N / D
N= 0.6316 * 0.2772 = 0.1751 and D= 0.6316 * 0.2772 + 0.3684 * 0.2857 = 0.2803

N/D = 0.6245. Pr (C1 / E1) = Pr (O / M) = 0.6245 and Pr (C2 / E1) = Pr (L / M) = 0.3755.
Event E2 : A Jewish year begins onTuesday, E2 is realized. The probability that it is an ordinary year is Pr (C1 / E2) = Pr (O / Tu) = N / D
N= 0.6316 * 0.0989 = 0.0625 and D= 0.6316 * 0.0989 + 0.3684 * 0.1429 = 0.1151
N/D = 0.5428. Pr (C1 / E2) = Pr (O / Tu) = 0.5428 and Pr (C2 / E2) = Pr (L / Tu) = 0.4570.
Event E3 : A Jewish year begins on Thursday, E3 is realized. The probability that it is an ordinary year is Pr (C1 / E3) = Pr (O / Th) = N / D
N= 0.6316 * 0.3382 = 0.2136 and D= 0.6316 * 0.3382 + 0.3684 * 0.2857 = 0.3133.
N/D = 0.6699. Pr (C1 / E3) = Pr (O / Th) = 0.6699 and Pr (C2 / E3) = Pr (L / Th) = 0.3301.
Event E4 : A Jewish year begins on Saturday, E4 is realized. The probability that it is an ordinary year is Pr (C1 / E4) = Pr (O / S) = N / D
N= 0.6316 * 0.2857 = 0.1805 and D= 0.6316 * 0.2857 + 0.3684 * 0.2857 = 0.2857.
N/D = 0.6316. Pr (C1 / E4) = Pr (O / S) = 0.6316 and Pr (C2 / E4) = Pr (L / S) = 0.3684.

Let us consider the event E3.
Pr (C1 / E3) = Pr (O / Th) = 0.6699 according to the Formula of Bayes.
The probablity that a random year is E3 and belongs to C1, i.e. that it is an ordinary year beginning on Thursday is given, according to the principle of the composed probabilities, by:
Pr (C1.E3) = Pr (E3) * Pr (C1 / E3) = (219831 / 689472) * 0.6699=0.318 * 0.6699.= 0.2136.
Pr (C1.E3) = Pr (C1) * Pr (E3 / C1) = (12 / 19) *0.3382 = 0.2136.

Direct verification.
Pr (C1 / E3) = Pr (O / Th) = 147255 / (147255 + 72576) = 0.6699.
Pr (C1.E3) = Pr (O.Th) = 147255 / 689472 = 0.2136.

The theorem of Bayes is fundamental in the cases where it is not possible to make a direct calculation of Pr (Cj / Ei). This is not the case with the Jewish calendar. It was however important to show that our table of the different frequencies fulfils the theorem of Bayes.