Talmudic Metrology VI
Sabbath's Limits and the Jewish Time Reckoning.

Summary.

The question of the limits of the Sabbath is an important one in Jewish society, which is built around the concept of the Sabbath, a major element and symbol in the Jewish religion.
This problem is debated in B. Sabbath and in Y. Berakhot.
A thorough analysis of the Talmudic passages tries to get a fully understanding of the extant Talmudic opinions. We show that the divergence between Abaye and Rava seems to already forecast the discussion between R' Tam and his opponents. We examine thoroughly the opinions of the different authorities, and we examine and propose a clear exegesis of Ram's theory (R'Eliezer of Metz) of the Sabbath's limits. We try to understand the theory of R' Tam by understanding the theories of his closest pupils and of the authorities around his time.
We examine the understanding of R' Tam through history and we show how its original understanding evolved. Little by little, it reached such a point that the differences between it and the previously opposing position, today called the position of the Geonim, were reduced to very few. We reject a modern theory that denies any difference between the two positions as well as the argumentation that this was already the understanding of former authorities.
We note that most of the rabbis had an incorrect knowledge of the variation through the solar year of the length of twilight.
The first table, scientifically based, of Sabbath's limits and Jewish time, was issued in Hanover in 1756, but Sabbath's tables were still calculated on an incorrect basis at the end of the nineteenth century.

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A. About Hour Counting in the Surrounding Roman Society.

1. Antiquity.

Ancient authors repeatedly point out that the division of the day into hours and the use of time-measuring devices had not been customary since time immemorial, but that both practices belonged to a time of historical and datable memory.
Herodotus reports that the Greeks had adopted sundials and the 12 divisions of the day from the Babylonians.
The Greeks divided the day into three or four segments, which were given designations like "early afternoon" or were named for mealtimes or various activities.
For civil use, nighttime had no division at all. For military purposes, it was broken down into three or four segments,[1] the length of which varied with the seasons.
It is not clear whether the calendar day began in the evening or in the morning.
The use of 12 divisions of the day or temporal hours, and of "hora" as an hour's time is attested to only from the time of Alexander the Great.
The 12 divisions of the day became possible only with the use of time measuring devices. This deliberate division of the day, a social convention in Greek and Roman civilization, was a Babylonian legacy. The Babylonians separated the day into daytime and nighttime, dividing each period-daylight from sunrise to sunset, and nighttime from sunset to sunrise- into 12 segments, called hours, of equal length. The duration of these hours and their temporal location varied with the length of daylight, depending on the season and the geographical location. But the end of the sixth hour was always designated the midday point, when the sun is at its culmination.
Only twice a year, at the equinoxes, were the hours of the day and the night equal in length. These hours were called "temporal hours" or "unequal hours." The ratio of the longest to the shortest daylight hours in Upper Egypt and Palestine was about 1.26:1, in Athens 1.55:1, in Rome 1.73:1 and in Southern Germany 2:1. In addition, in Rome the four divisions of the day and the division of the night into four watches –"vigilia"-continued to be used, because they were sufficient for most practical purposes. In Rome these segments were publicly signaled by the officials. The quarters of the day were named after their last hours. As points in time, hour indications, with the exception of the first hour "hora prima" must always be understood in the sense of the expired hour. Therefore "nona" or "hora nona" as a space of time could designate the ninth hour of the day-the space of time between 2 and 3 p.m.- or more rarely the ninth hour of the night. It could also refer to early afternoon, encompassing the seventh, eighth and ninth hours, or as a point in time it could mean the end of the ninth hour: 3 p.m. "Hora prima" as a point of time represented the beginning of the first hour, or sunrise. "Hora prima," "mane" and "vespera" were used to describe the natural boundaries of daylight and the cock's crow. "Gallicantus" was commonly used to indicate the point in time just before dawn. The four quarters of the day were also designated by "mane," the first half of the morning, "ad meridiem," the second half of the morning, "de meridiem," the first half of the afternoon and "suprema," the end of the afternoon. Only educated people could be expected to have used differentiated hour indications, but even they had limited need for them.
Approximate times of day and night could also be described with sufficient clarity by using terms that were oriented towards natural events, which were much more highly differentiated in those days. This applies especially to transitions from light to dark as the following qualitative breakdown of the night,

which was still common in the middle Ages, shows in exemplary fashion. In the evening: “occasus solis” sunset, “crepusculum” dusk, “vesperum” appearance of the evening stars, “conticinium” silence, “concubium” sleeping time, “intempestum” complete cessation of all activities.
In the morning “gallicinium” cock’s crow, “ante lucem” the point in time just before dawn, “aurora” first flush of dawn, “diluculum” dawn, “exortus solis” or “hora prima” sunrise, “mane” end of dawn and morning.
All these moments can be classified according to the following table.

“ante lucem” just before dawn
“galicantus” cock’s crow
“aurora” first flush of dawn
“diluculum” dawn
“exortus solis” sunrise
“mane” first half of the morning
“ad meridiem” second half of the morning
“meridies” noon
“de meridie” first half of the afternoon
“suprema” second half of the afternoon
“occasus solis” sunset
“crepusculum” twilight
“vesperum” appearance of the stars
“conticinium” silence
“concubium” sleeping time
“intempestum” end of all activities[2]

2. High Middle-Ages.[3]

The Roman division of the day and the night into four parts was maintained and even completed because of the importance of the Christian prayers, which must be said at precise temporary hours. These hours had a religious character and were called “officia” or “horae canonicae.” During the night the four divisions were:

“caput vigiliarum” first watch or “conticinium” silence
“vigilia media” middle watch or “intempestum” end of all activity
“gallicinium” or “gallicantus” cock’s crow, third watch
“matutina” or “antelucanum” fourth watch.
The four divisions of the day were:
“mane” from sunrise until “hora tertia” (9 a.m.)
“ad meridiem” from “hora tertia” until “hora sexta” or “meridies” (noon)
“ de meridie” from “hora sexta” until “hora nona” (3 p.m.)
“suprema” from “hora nona” until “occasus solis” sunset.

The original canonical hours were the following:
“Matutina” at midnight (Matins)
“Laudes” at 3 a.m. (Lauds)
“hora prima” at 6 a.m. (Prime)
“hora tertia” at 9 a.m. (Tierce)
“hora sexta” at midday (Sext)

"hora nona" at 3 p.m. (Nones)
"Vespera" at 6 p.m. (Vespers)
"Completorium" at 9 p.m. (Compline)

During the fifth century some modifications took place in the canonical hours.
"Matutina" now began two hours earlier, during the third watch, "hora prima" was introduced and began at sunrise, "vespera" was advanced before sunset, "completorium" was shifted to the end of the day and "Laudes" was suppressed.
The canonical hours were then the following:

1. "Matutina" in the third part of the night or midnight
2. "hora prima" beginning at sunrise
3. "hora tertia" at 9 a.m.
4. "hora sexta" at noon
5. "hora nona" at 3 p.m.
6. "vespera" one hour before sunset at 5 p.m., instead of 6 p.m. before
7. "completorium" beginning at sunset, instead of 9 p.m.

3. Middle Ages.[4]

Gradually and until the thirteenth century, another modification happened. It is in fact a slide: the "hora sexta" disappeared and the" hora nona" slid to noon. The similitude between nona and noon dates back to this slide, corresponding apparently to some adaptation in the organization of the canonical prayers. The "nona" slides to noon, "vespera" slides to 3 p.m. and the "hora sexta" disappears.
Hours of equal length-"horae aequinoctiales" or "horae aequales"- were certainly known. They were used only in the context of scientific discussions, especially astronomical calculations, as we see in Ptolemy's Almagest. The clocks used by the ancients were mainly sundials and also water clocks for scientific use. Sundials with gnomons and scales were first used in Egypt and Babylonia.[5] Greek astronomy subsequently developed the Babylonian models of sundials into the different models and forms described by Vitruvius[6] in his book "De Architectura."
The problem of constructing sundials according to geographic latitude had been solved by the third century at the latest. From that time on, ancient sundials always indicated the temporal hours for a specific latitude. Localities were assigned a "climate" in accordance with their latitude; this could also be indicated as the ratio of the longest to the shortest day of the year.[7] Despite the development of mechanical clocks from the twelfth century on, the temporary hours remained in use for a long time. Today it seems to us that these temporary hours must have been very inconvenient and therefore with the development of the mechanical clocks one should have switched to equal hours. However, weight clocks existed for a long time before the temporary hours disappeared. Still in the fifteenth century,[8] every evening at sunset people would change the pendulum according to data in tables, in order that it would divide the length of the night i.e. the space of time between sunset and sunrise, into 12 equal parts. Similarly, they were obliged each morning to modify this gear in order for the clock to give the temporary hours of the day. The astronomical almanacs of Johannes Muller (Regiomontanus) (1436-1476) were still giving the ephemerides in temporary hours. Later, they stopped twice a day to change the length of the pendulum, and they adopted a mean length for the pendulum. The clock then worked in equinoctial hours. The transition from the old temporary hours to the equinoctial hours is connected with the development and improvement of the striking clocks and with a broad process of the laicization of the way of life of city-dwellers. The temporary hours as a sequence of prayer no longer played a role.

4. Modern Hour-Reckoning.[9]

What about the various forms of modern hour-reckoning, after the withdrawal of the temporary hours? It must be observed that there were no customs one could have picked up on. While evident that a decision had to be made, different solutions were found in different places. In the Italian form of hour-reckoning, the 24 hours of the day were counted from one half-hour after sundown to the evening of the following day. The twenty-fourth hour was the last hour of daytime; it ended at the complete disappearance of light with the appearance of the stars. The only linkage to daylight has to do with timing the point at which the counting begins in the evening. Thus the end of the 24th hour corresponded to the end of daylight, half an hour after sunset and sunset was thus at 23h: 30m.[10] In a variant form, the day and night hours were counted separately in 12 night and day hours, sunset was then at 11h: 30m. R' Azariah de Rossi,[11] R' Joel Sirkes,[12] Samuel Aboab[13] and Samson Morpurgo[14] used the Italian hour in their writings.

Reckoning the day from evening to evening may have been connected to the Judeo-Christian notion of the feast day. However the change of date usually occurred in the morning and therefore events that took place at night were assigned to the date of the previous day. The "Italian clock," also called "Ganze Uhr" or "Ganzer Zeiger," was already widely used in Italy by the 14th century and spread via Bohemia into Silesia and for a time it was also used in Poland.[15] This method of counting had the advantage that everyone could easily know how much time they had to finish their day's work (without artificial light).

The drawback to this method of counting was that the striking clocks had to be continuously, each day at 24 h**,**[16] moved ahead by a certain amount during the first half of the year and moved back during the second half. To avoid this, the clocks were adjusted only after changes in the length of the day had reached a certain point, and they were then set with the help of tables. The Italian way of counting was in use in Bohemia and Silesia until the seventeenth century and in Italy itself until the wars that followed the French Revolution and even until the end of the 19th century.[17]

Other systems of counting the hours were the Nuremberg Clock, also called the Great Clock, and the method of hour-reckoning familiar to us called "small clock" or "half clock."

The decisive advantage of the method of counting the hours using this last system of half clock was that the clocks had to be set only once a day at noon, without any tables, using only a sundial. It worked on the basis of the dual sequence of 12 hours, which we still use today. This is all it retained of the old system. Daylight was no longer of any consequence. It was a more abstract form of division with respect to the conditions of daylight; 7 a.m. could now be day or night according to the seasons.

The Nuremberg Clock, or Great Clock, followed the old tables of the length of the days. It kept the division into day-hours and night-hours (as in the system of the temporary hours) as well as the movable beginnings of both sequences, but it dropped the sequence of 12. According to this method, in use in the 14th century in Nuremberg and in Regensburg, in December the day had eight hours and the night 16 hours. Until June the day was lengthened by one hour every three weeks and the night shortened accordingly. After the mid-point of the year the procedure was reversed.

The advantage of having the hour reckoning conform to daylight, beginning theoretically at sunset and sunrise, was counterbalanced by the disadvantage of having to use approximate tables. This method was limited to a few southern German cities, and it died out at the end of the eighteenth century.

B. The Hours in the Talmudic Literature.

The elements of the first chapter explain that civil life in Palestine during the period of the Mishna and later in Palestine and Babylonia during the period of the Talmud was organized around the concept of

temporary hours. Nearly[18] all the medieval rabbinical authorities[19] recognized them for the simple reason that the system of temporary hours was the normal way of time-reckoning in the surrounding society and in everyday life until the fifteenth century. As was always accepted from antiquity, temporary hours were the hours read on sundials. Days were divided into day-hours representing the twelfth part of daytime reckoned from sunrise to sunset. What about the night? It was divided, as in Rome, into three or four equal parts beginning at sunset and ending at sunrise. Therefore all the hours mentioned in the Mishnah and referring to the time of prayers or to the time-schedule of the eve of Passover must be understood as temporary or seasonal hours counted from sunrise until sunset. This way of counting the temporary hours was universally accepted in the ancient world. These temporary hours correspond to the equal divisions of the day given by the sundials.

Ptolemy explains clearly in his Almagest how to calculate their length for a given day of the year in a given place or in other words, how to calculate their length as a function of the solar declination and the local latitude.[20]

Now what about intervals of time before sunrise and after sunset, during the period of dawn and twilight, as well as the length of dawn and twilight? In connection with the beginning and end of the Sabbath, the Talmud uses the mile, which is a unit of length as well as a unit of time. We demonstrated in a previous paper[21] that the mile represents a length of time of 18 minutes. It is also likely that the Sages of the Talmud considered that all these spans of time connected with twilight and dawn, as well as the lengths of twilight and dawn themselves, and measured in miles, are proportional to the length of the day and are measured in temporal hours. They had no way of measuring spans of times before sunrise and after sunset because the sundials and the gnomons were non-functional during those periods. Therefore, all these spans of time, which are expressed in the Talmud in miles, represent in fact temporary spans of time. Their absolute length is expressed in miles at the equinox, but they must be understood as temporary spans of time. The fixation of the time for the beginning of dawn-the beginning of the religious day- and the beginning and end of the Sabbath were in fact theoretical and set in an experimental way. This of course corresponded better with the reality. These moments are in fact correctly described by a certain solar depression, which describes a certain degree of darkness, better than they are described through temporary spans of times or fractions of the day.[22]

This way of counting the hours in the Talmud, by using temporary hours, corresponding to the hours read on the sundials i.e. temporary hours counted from sunrise until sunset, was never questioned until the twelfth century, when the French Tosafists, under the leadership of R' Tam,[23] invented, in a purely intellectual and speculative manner, a new method of counting temporary hours.[24] Their temporary hours were not calculated on the basis of a day of 12 temporary hours lasting from sunrise to sunset but of an extended day including dawn and twilight, corresponding to the religious day defined in B. Berakhot. In other words, their day of 12 temporary hours was extended to the entire time of daylight. Such a method was never used before in Talmudic and rabbinical literature and was completely unknown to the surrounding society. In other words, the Tosafists created during the twelfth century a new and completely artificial method of counting temporary hours that in fact, had been unknown by Tanaïm, Amoraim, Saboraim, Geonim and Rishonim including such authorities like R' Hananel, Rif (R' Isaac ben Jacob of Lucena), R' Abraham bar Hiya[25] and Maimonides[26] (the position of Rashi is much more problematic[27]). All of these former authorities had always believed that the hours of the day are counted from sunrise until sunset. Nevertheless, they accepted that some religious obligations could be, if not a priori, at least a posteriori, performed before sunrise, during dawn or after sunset, during twilight and before the appearance of three middle stars.[28] Thus the hours of the day were always counted from sunrise until sunset according to the definition of the astronomical day and the indications of the sundial. In other words, they accepted that the religious day did not perfectly coincide with the astronomical day. The religious day began at dawn before the astronomical day and ended at the end of twilight at the appearance of three night stars. This religious day corresponds in

fact to the natural day, corresponding to daylight and constituted by the astronomical day, counted from sunrise to sunset, enlarged by dawn and twilight, during which it was also possible to work under natural light. Nevertheless it seems completely artificial to consider such a day for the counting of the 12 temporary hours. Such temporary hours cannot be measured and their length can only be calculated once we agree on the length of dawn and twilight.

According to Babli Pesahim 94b the mean walk during a day is 40 miles, the mean walk during dawn and twilight is four miles[29] and the mean walk during the span of time between sunrise and sunset is 32 miles.[30] The Jewish mile, according to Rashi and Tosafot, is then 12*60/32 =22.5 minutes.

Furthermore at the equinox the Jewish temporary hours are 40/32 or 1.25 times the short temporary hours or sundial's temporary hours, i.e. 75 minutes, instead of 60 minutes. At the equinox the length of dawn and twilight is 4*22.5 =90 minutes.[31]

Now in contradiction with the conclusions of B. Pesahim 94b, several Tosafists have adopted a dawn and twilight of five miles.[32] The origin of this variant value of five miles for the length of dawn and twilight seems to be in the text of R' Tam's book Sefer ha Yashar, while the texts of Tosafot on Babli Pesahim and Babli Sabbath, based on the explanation of R' Tam, consider a value of four miles for dawn and twilight. It is generally accepted that R' Tam uses in Sefer ha-Yashar the value of five miles, according to the rejected opinion of Ulla, for the length of twilight in Jerusalem at the equinox, for pedagogical purposes. This allows him to address a perfect contradiction between Rabbah in B. Pesahim 94a[33] and Rabbah in B. Sabbath 34b. Nevertheless we find the value of five miles in many German Rishonim[34] and even Aharonim.[35] No satisfactory explanation has been given for this position. Although widely adopted, it remains a conundrum.

The only plausible explanation for this twilight of five miles seems to be the following: In the area of France and Germany where the Tosafists were living, corresponding to a latitude of about 48.50°, the length of the astronomical twilight, corresponding to a solar depression of 20°, is 120 minutes which corresponds exactly to five miles of 24 minutes each. Apparently, the Tosafists were not aware of the spherical shape of the earth and believed that their experimental observations were also valid for Palestine. They would then have observed experimentally that the length of twilight corresponds to about 5*24 m, corresponding then to a day of 5 + 30 + 5 = 40 miles according to Ulla². This would have convinced them that the mile is indeed 24 m and that astronomical twilight is five miles, in contradiction with the conclusions of our reading of the Talmud Babli Pesahim, according which the mile is 18 m and astronomical twilight is only four miles.

There is also another possible solution to this conundrum. If we refer to the table 1 in Talmudic Metrology II,[36] we see that besides column B, considered above, column A also offers an acceptable solution: dawn lasts five miles, the 12 hours between sunrise and sunset represent 40 miles and dusk also lasts five miles. This solution is not worse than the solution adopted by the most Rishonim: four miles for dawn, 32 miles for the 12 day hours and four miles for dusk. They could then have understood that R' Johanan,[37] in B. Pesahim 94a, who was not contradicted, does not follow Rabbi Judah blindly but has his own opinion of a day of 5 + 40 +5 = 50 miles[38] in an extended day.[39] Twilight would then last 5 *18 = 90 m or 1.5 equinoctial hours corresponding, at the equinox and at latitude of 48.5°, to a depression of 15.5°.

Tosafot based themselves on B. Berakhot 4a quoting a verse in Nehemiah IV: 15 to justify that the span of time between daybreak (the beginning of dawn) and the appearance of the stars is called day; therefore they considered that the hours mentioned in the Talmud are temporary hours related to this day. Now the appearance of the stars fixing the end of Sabbath, is also mentioned in B. Sabbath 35b; they appear at about the end of Bein ha-Shemashot, 0.75 miles after sunset or slightly later. The appearance of the stars is finally mentioned in B. Pesahim 94b, 4 miles after sunset; it is here the symmetric of daybreak.

The plain explanation of these Talmudic passages is the following:[40]

1. The stars[41]of B. Pesahim 93b–94b are the symmetric of daybreak and correspond therefore to the moment of the apparition of all the stars (the end of astronomical twilight) symmetric to daybreak. Just before daybreak, all the stars are still visible. Symmetrically, at the end of astronomical twilight, all the stars are now visible.
2. The temporary hours are the natural temporary hours read on a sundial; they represent the twelve divisions of a day limited by sunrise and sunset.
3. The religious day defined in B. Berakhot 2a, corresponds to daylight, it begins at daybreak and ends with the disappearing of light, when activities end at the appearance of the first stars.
4. The end of Sabbath corresponds to the appearance of three stars, the same stars as those of B. Berakhot.
5. The stars mentioned in B. Pesahim appear four miles after sunset.

By contrast, R' Tam and the Tosafists adopted another principle, that the appearance of the stars mentioned in B. Pesahim is the same as that mentioned in B. Sabbath 34b and B. Berakhot. This allowed them to behold midday in concordance with the culmination position of the sun[42] because of the symmetry of this extended day with regard to midday, but it brought them to another difficulty i.e. to consider that the three stars that mark the beginning of the night and the end of Sabbath, as discussed in B. Sabbath and Y. Berakhot, are the last stars appearing at the lower western horizon and not the first night stars appearing near to the zenith or at the eastern horizon. Furthermore it obliged them to consider two sunsets, the first sunset or later the beginning of sunset, corresponding to the plain sunset when the sun disappears at the horizon and a second sunset or later the end of sunset, 3.25 miles later, at the beginning of Bein ha-Shemashot, 0.75 miles before the appearance of the stars. Now Tosafot consider also that the counting of the temporary hours relates to this extended day, beginning at daybreak and ending at the end of the astronomical twilight.[43] This way of considering the Jewish day and of counting its temporary hours is in contradiction with the nature itself, of the natural day as defined by Nehemiah and of the derivate Jewish day. This day begins at dawn but ends with the appearance of the evening stars around the end of civil twilight, when the lack of visibility brings an end to human activities. This seems to be the day to which Nehemiah refers and to which B. Sabbath and Y. Berakhot refer, a day extended to include the entire time of daylight, from dawn until the end of daylight, when it becomes impossible to have normal activity outside without artificial lighting. It appears that such a day is not symmetric[44] with regard to the culmination position of the sun, which must coincide with midday. This seems to prove that the division of such a day into 12 equal parts, in order to create great temporary hours, is not without difficulties. Furthermore, any system of temporary hours different from the sundial's temporary hours corresponds to a day that won't be equal to the night at the equinox. These are nevertheless two generally accepted principles in the Talmud, as well as in the surrounding society, that day and night are equal at equinox[45] and that the sun is at its culmination at the end of the sixth hour.[46]

In conclusion, it is clear that there is only one way of counting the temporary hours of the day: the manner that was used since antiquity dividing the span of time between sunrise and sunset into 12 equal segments. The Talmud and the Rabbis could not imagine anything else. Now, since the twelfth century, a significant proportion of the Rabbis have followed the principles developed by the Tosafists[47] about the notions of an extended Jewish day,[48] great temporary hours and consequently the limits of day and night and the limits of the Sabbath. Nevertheless in order to solve the difficulty caused by the late acceptance and termination of Sabbath some rabbis tried to reinterpret the classical theory of R' Tam and rewrite it according to one of the following schemes:[49]

1. The appearance of the stars in B. Pesahim is not at the end of the astronomical twilight but earlier at the beginning of the night and the symmetrical daybreak is now something clearly later than the astronomical daybreak; it is rather the moment when we can hardly

distinguish a friend at first light. The religious day remains symmetric with regard to midday. But sunrise and sunset are then no more the evident phenomenon that we know. The first and the second sunset become theoretical notions without connection with the evident notion of sunset.

2. The appearance of the stars fixing the end of Sabbath corresponds to the stars appearing at the beginning of the night. The natural day is not symmetric with regard to midday; the end of the sixth hour or simply 6 hours in this day occur before true noon, somewhere about 11h: 30m.
3. The appearance of the stars fixing the end of Sabbath corresponds to the stars appearing at the beginning of the night. The religious day lasts from daybreak until the end of astronomical twilight in order to maintain the symmetry of this day with regard to the zenithal position of the sun, i.e. that at 6 h it is true noon.
4. There is no difference between the stars of R' Tam and those of the Geonim, The stars marking the beginning of the night are the first stars appearing at the end of daylight. The natural sunset corresponds now with the second sunset and the first sunset is something intangible happening 3.25 miles before sunset. Daybreak mentioned in B. Pesahim is now something intangible occurring 0.75 miles before sunrise. The natural and religious day is now symmetric and practically identical with the astronomic day; its temporary hours are practically the same as the astronomical temporary hours.

None of these four solutions satisfies the three Talmudic passages of reference and the principle expressed by Tosafot that the appearance of the stars mentioned in the different Talmudic passages is the same. The fourth solution takes any signification away from the text of B. Pesahim. All these solutions are so farfetched that they raise more problems than they solve.
In the pursuit of the present paper we will try to understand the original theory of R' Tam, to show that the Rabbis who followed him understood him according to the classical understanding and that all the modern adaptations cannot be ascribed to R' Tam.

C. The Limits of the Sabbath.

The entrance of the Sabbath is a particular problem because the rabbis believe that there is a period of incertitude between day and night. This period is called Bein ha-Shemashot (abbreviated as BHS). Sabbath must be accepted on Friday before the beginning of BHS and on Saturday evening, only after the end of BHS, is working allowed again. The exact limits of BHS are discussed in B. Sabbath 34b and in Y. Berakhot I: 1; this discussion is of great importance. One significant problem is that the departure point for the fixation of BHS is a moment called משתשקע החמה which was understood in different manners: before sunset, at sunset, after sunset or after the end of sunset (when the sun has crossed the firmament).[50]

1. Summary of the Opinions in the Talmud.

1. Babylonian Talmud: Sabbath 34b.

Rabbi Nehemiah: BHS lasts 0.5 miles after משתשקע

Rabbi Hanina says: if someone wants to know when the BHS of Rabbi Nehemiah begins, he should climb down the Carmel until the sea, when it is enlightened by the setting sun, immerse himself in the sea and get out. This is the beginning of the BHS of Rabbi Nehemiah.[51]
Rabbi Judah: Rabbah: BHS lasts 0.75 mile after משתשקע

Rav Joseph: BHS lasts 2/3 mile after the reddish color in the eastern horizon and lowest part of the celestial hemisphere disappears, and at about the same time, the eastern horizon darkens. Indeed, according to R' Joseph, after משתשקע we have a first stage when the eastern sky is reddish; it belongs still to the day. BHS begins only when the eastern horizon grays and fades. Slightly later the eastern horizon also begins to darken. BHS ends when the zenith darkens and equalizes the eastern horizon.
BHS of Rabbi Judah, according to both, Rabbah and Rav Joseph, ends when the zenith darkens. At this moment the eastern hemisphere is no longer lightened, and we must be near the end of civil twilight.
Rabbi Jose: BHS is a cast of glance (a short and undetermined span of time). It is accepted that it begins after the BHS of Rabbi Judah.

Abaye was looking toward the east to find the reddish color in the sky, while Rava was looking toward the west.

Rabbah bar Bar-Hannah in the name of Rabbi Johanan: At the entrance of Sabbath the law is in accordance with Rabbi Judah (about the beginning of BHS) while at the end of Sabbath and for other items,[52] the law is according to Rabbi Jose.
Rav Judah in the name of Samuel: 1 star = day
2 stars = BHS
3 stars = night.

2. *Jerusalem Talmud: Berakhot I: 1.*

Rabbi Pinhas in the name of R' Abba bar Papa:
1 star = day
2 stars = doubt
3 stars = night.
Rabbi Jacob Deromna (from the South):
Until the first star: day
Between first and second star: doubt
Second star: night.
Braita: As long as the eastern horizon is reddish, it is day.
When the western horizon grays and fades, הכסיפו, it is the beginning of BHS.
When the eastern sky darkens, השחיר, and the zenith equalizes the darkness of the horizon, it is night.[53]
Rabbi: it is BHS as soon as the sun's lowest limb sets and the moon's upper limb rises.
Rabbi Hanina: It is BHS as soon as the sun's upper limb sets and the moon's upper limb rises.
Rabbi Samuel bar Hiya in the name of Rabbi Hanina: If when the sun begins to set, a man standing on top of the Carmel climbs down from the head of the Carmel and immerses himself in the sea, he may later eat Teruma[54] *because we can assume that he had a valid immersion during the day.*[55]

Braita. Rabbi Nehemiah: BHS lasts 0.5 mile after משתשקע
Rabbi Jose: BHS is a cast of glance.

The Jewish religious day begins at daybreak and ends with the appearance of three middle stars. *R' Hanina, the friend of the Sages, objects that the morning boundary should be similar to the evening boundary, which is fixed by three stars, although the sun is still in the middle (*in the process*) of the crossing the firmament.* The halakhic day ends with the apparition of three stars when the sun is still in

the process of the crossing of the firmament and we nevertheless consider this crossing as belonging to the night. We should then also in the morning consider the crossing of the firmament as belonging to the night, as long as the sun did not end the crossing.
Different arguments are brought to justify the fact that the crossing of the firmament in the morning belongs to the morning. Anyhow, we see from this text that the end of the Jewish day, the appearance of three stars, was considered by the Talmud as occurring during the crossing of the firmament.[56]

Rabbi Abba and Rabbi Ba justify that the morning crossing belongs to the day from daybreak by Biblical quotations.
Rabbi Jose berabbi Bon: If you attach the morning crossing to the night like the evening crossing then the religious day won't be equal to its night.[57]

The Rabbis prove by comparison with human Royal behavior that the morning crossing belongs to the day from daybreak.

Rabbi Hazna (Hanina)[58] says: *"from daybreak until the lighting of the orient there is a span of time of 4 miles and from the lighting of the orient until sunrise there is 4 miles there is another span of time of 4 miles".*
This passage is problematic because of the existence of two spans of time of 4 miles before sunrise is not likely all the more that in a parallel passage in B. Pesahim Rabbi Hanina says about the span of time between daybreak and sunrise which Lot, according to the conclusion of the discussion, is supposed to have covered, between daybreak and sunrise, in a span of time of 4 miles, that this distance was in fact 5 miles. There is thus no reason to consider two intervals of 4 miles. In fact אילת smees השכרto correspond to עמוד השחר, to daybreak[59] and the lighting of the orient seems to be a moment following closely daybreak.[60]
Remark.
The first Braita of the Jerusalem Talmud is similar to the opinion of Rav Joseph.

3. הכסיפו, השחירו.

In the Babilonian Talmud there is only one expression and there is therefore doubt about the exact meaning of הכסיף: is it to fade or to darken? The Jerusalem Talmud uses two different verbs. At the end of the first stage the reddish color disappears at the eastern horizon, and the western horizon grays or fades. Later, the sky darkens and when the zenith reaches the same darkness as the eastern horizon, it is night. The description of this phenomenon seems to fit the eastern horizon.

4. פני מזרח מאדימים.

Towards the east we observe the apparition of reddish gleams nearly an hour before sunset (at the equinox in Jerusalem). They will remain until about 20 minutes after apparent sunset, but the reddish gleams will begin to disappear near the horizon soon after sunset. Towards the west the reddish gleams appear at sunset and they will remain about an hour after sunset.[61]

5. Towards the east or towards the west?

Why were Rava and Abaye scrutinizing the horizon, one towards the west, the second towards the east? According to the Talmudic text, B. Sabbath 35a, this was connected to the reddish gleams which can be seen at the beginning of BHS according to Rava, before BHS according to Abaye. At the

western horizon, the reddish gleams begin around sunset and carry on for a long time- about one hour in Jerusalem at the equinox. The only possible reason, for Rava, to look towards the west was then to determine the moment of the complete disappearance of these reddish gleams in order to fix the moment of the beginning of BHS of Rav Joseph or to narrow the beginning of the BHS of Rabbah. This would then correspond to the position of R' Tam.

At the eastern horizon, the colored gleams appear already an hour before sunset and carry on about another 20 minutes after sunset. But very soon after sunset, the eastern horizon fades and the reddish gleams draw, raise and move off from the horizon.[62] Abaye would then have tried to determine the moment when the reddish gleams fade at the horizon and begin to draw up higher in the eastern sky. Abaye was looking at the fading of the eastern horizon in order to determine the beginning of the BHS of Rav Joseph or narrow the beginning of the BHS of Rabbah. Anyhow, it seems that the moment משתשקע corresponding to the beginning of the BHS of Rabbah is slightly later than apparent sunset. If it was at apparent sunset, followed 1.5 m later by the beginning of the BHS of Rav Joseph, then Abaye would have a much more precise determination by looking at the western horizon to see the apparent sunset.

The fact that Rava was looking towards the west raises the question of whether the divergence between Abaye and Rava was not a principle divergence about the beginning of the Sabbath similar to the later divergence known as the divergence between R' Tam and the Geonim.[63] In the time of Abaye and Rava they had forgotten the principles governing the fixing of the beginning of BHS. So, at least, did R' Tam understand it as he advanced as a basic argument for his theory the fact that Rava was looking towards the west.[64] Anyhow, the rule: הכסיף העליון והשווה לתחתון is a supplementary decisive objection against the examination of the western horizon and the late beginning of the Sabbath. Towards the west, the horizon does not darken before the zenith. The phenomena described by the Talmud and the text of Y. Berakhot, differentiating between הכסיפו and השחירו, enforce the position of Abaye, but B. Sabbath seems to follow Rava, apparently because the conclusion of their discussions is generally in favor of Rava.[65]

Until the twelfth century and R' Tam (~1100-1171), it was universally accepted that the beginning of the Sabbath was at about sunset[66] or slightly after and the end of Sabbath was at the appearance of three middle stars a little before or at the end of our modern civil twilight (solar depression of 6°). The detail of the exact position of each authority depends also on his opinions with regard to the length of the mile. Maimonides considered a mile of 24 minutes, so 0.75 mile is then 18 minutes. This allowed him to have a very simple, coherent and logical position: משתשקע is sunset, BHS begins at sunset[67] and the end of BHS, including an addition of two minutes to take into account the BHS of Rabbi Jose,[68] is in Jerusalem, at the time of equinox 20 minutes later.[69] At the end of BHS, we are thus 20 m after apparent sunset;[70] apparent sunset is at 18h:04m true time, the end of BHS is 18h:24m, the depression of the sun is about 5.1° and we are not far from civil twilight. Rabbi Abraham ben David must have had a similar position.[71] Now if we consider that the true Talmudic value of the mile is 18 minutes, then it is difficult to accept that the three night stars appear in Israel at the time of equinox, 13.5 minutes after sunset. It seems then more likely to explain that משתשקע means "after sunset," slightly after sunset so that 13.5 m later we are at a more acceptable time compatible with the appearance of the three stars. At the equinox, in Jerusalem, apparent sunset is at 18h: 04m true time and according to Maimonides, the end of BHS of Rabbi Judah is at 18h: 22m and the end of BHS of Rabbi Jose is at 18h:24. If we consider that the appearance of three middle stars is at 18h; 22m then משתשקע would be at 18h:8.5m,[72] i.e. 4.5 minutes after apparent sunset. On the other hand, if we consider that the end of the BHS of Rabbi Judah is at the moment when the zenith darkens, which corresponds to the civil twilight,[73] then in Jerusalem and at the equinox, the end of BHS of Rabbi Judah is at 18h: 28m and it would begin at 18h: 14.5m[74] i.e. 10.5 m after apparent sunset. BHS of Rabbi Judah on the day of the equinox would then begin between 18h: 8.5m and 18h: 14.5m and it

would end between 18h: 22m and 18h: 28m.This seems to be the plain explanation of the Talmud; it corresponds also to the position of such authorities like Rabad,[75] Ravan[76] and Ram[77], who placed the beginning of BHS near to sunset. This position is enforced by the attitude of Abaye, who was looking towards the eastern horizon in order to determine the moment of the graying and fading of the horizon, marking the beginning of BHS. Such a search, by Abaye, could be understood only if he considered that משתשקע is slightly after sunset. Otherwise, it would be illusory to try to determine towards the east, with acceptable precision, a moment that follows sunset by a few minutes.[78] It is also compatible with the teaching of Samuel, connecting the beginning of the night with the appearance of three stars and the BHS with the span of time between the appearance of the second and the third. It is finally in accordance with the teaching of Rabbi Hanina about the moment of משתשקע : the beginning of BHS of Rabbi Nehemiah.

2. The exact opinion of Maimonides about Sabbath's limits.

In order to have a thorough comprehension of the position of Maimonides, we must examine different quotations connected to the problem of the boundary between day and night.

1. Hilkhot Sabbath V: 4,

משתשקע החמה עד שיראו שלשה כוכבים בינוניים הוא הזמן הנקרא בין השמשות בכל מקום והוא ספק מן היום ספק מן הלילה ודנין בו להחמיר מכל מקום.

2. Hilkhot Kiddush ha-Hodesh II: 9,

ראוהו בית דין עצמן בסוף יום תשעה ועשרים אם עדיין לא יצא כוכב ליל שלשים, בית דין אומרים מקודש שעדיין יום הוא ואם ראוהו בליל שלשים אחר שיצאו שני כוכבים, למחר מושיבין שני דיינין אצל אחד מהם ויעידו השניים בפני השלשה ויקדשוהו השלשה.

3. Mishna Sabbath II: 7,

ודע כי אחר ביאת השמש עד שיראה כוכב מן הכוכבים הבינוניים בגדולה נקרא יום, ושיראה כוכב אחד עד שיראו שניים הוא גם כן יום ומשיראו שניים עד שיראו שלשה הוא זמן בין השמשות וכשיראו שלשה הוא לילה בלי ספק והעיקר אצלינו כי בין השמשות ספק.....

4. Mishna Rosh ha-Shannah III : 1,

כי הלילה אצלינו אינה אלא משעת צאת הכוכבים כמו שביארנו בשני בשבת

5. Mishna Megila II : 4,

ומה שנאמר שנחשוב יום מעת עלות השחר לאמרו בעזרא מעלות השחר עד צאת הכוכבים וקרא אותו הזמן יום והוא אמרם והיו לנו הלילה למשמר והיום למלאכה ומה שאמרו וכולם שעשו מעלות עמוד השחר כשר למי שעבר ועשה בעת הצורך.

6. Mishna Berakhot I : 1,

ומה שהצריכו לומר משעה שהכוהנים נכנסין ולא אמר מצאת הכוכבים ואע"פ שהעת אחד להועילנו תועלת מתוך דבריו והוא שהכהן טמא, כשיטבול, מותר לו לאכול בתרומה אחר ביאת השמש וצאת הכוכבים.[79]

In the third quotation Maimonides follows the opinion of Rav Judah in Samuel's name and seems to consider this point of view as the theoretical rule defining the limits between day, BHS and night.
In the first quotation, related to the beginning of Sabbath, Maimonides follows the ruling of Rabbi Johanan that for the beginning of Sabbath we follow Rabbi Judah, who begins the BHS at the moment משתשקע, which Maimonides equalizes with apparent sunset. For other items and also for the end of

Sabbath, we follow Rabbi Jose. The appearance of three stars corresponds then with the end of the BHS of Rabbi Jose.
In the second quotation, which has raised many questions in the rabbinical literature,[80] we must understand that כוכב יום שלשים represents the first star of the 30th day i.e. the second star. Indeed the appearance of the first star does not change the status of day; it is still the 29th day. With the appearance of the second star and the beginning of BHS, we enter into the night belonging to the 30th day; this second star is called the star of the 30th day. Before its appearance, we are still in the 29th day and the sanctification of the new moon can be performed immediately. After its appearance we are already in BHS and the procedure is delayed to the next day.
The main difficulty of this passage is the special terminology used by Maimonides in the first part of the passage i.e: אם עדיין לא יצא כוכב יום שלושים. Besides, Maimonides' ruling is certainly according the rule of Rabbi Johanan[81] that except for the entrance of Sabbath where the rule is according to Rabbi Judah, the rule is according to Rabbi Jose.[82]
It is thus normal that in the case of sanctification of the new moon the BHS does not begin at sunset but is restricted to the span of time between the second and third star.
In the fourth quotation, Maimonides confirms that night begins with the appearance of the third middle star.
In the fifth quotation, Maimonides ascertains that the Jewish religious day begins at daybreak and lasts until the appearance of three stars. Nevertheless, temporary hours are counted from sunrise until sunset and there is no contradiction between these two different realities. The counting of the hours is an astronomical problem and not a religious problem.
In the sixth quotation, Maimonides uses the word ביאת השמש in a difference sense than in quotation three. In the third quotation, he means sunset but in the sixth he refers to the verse ובא השמש וטהר and designates the beginning of the night.[83] With the reading of R' Kafih, that we followed, things are still more evident.

3. The Talmudic Cosmographical Model: the Crossing of the Firmament.

Talmudic passages related to the fixing of the beginning and end of the Sabbath are, in both the Babylonian and the Jerusalem Talmud, connected to a cosmographical model[84] different from the physical reality that we know today, which Greek astronomy had already discovered. The Talmud itself was aware of the contradiction between this traditional model and the Greek model and recognized the superiority and truth of the Greek model. The use of this old model in the problem of the Sabbath's limits is principally a problem of terminology and is without consequence for the physical phenomena. It is nevertheless important to understand correctly the old Jewish model in order to understand completely the Talmudic passages and the Jewish medieval Rabbis (French and German Rabbis) who were not aware of Greek astronomy. Indeed, although the Sages of Israel are said to have recognized the superiority of Greek astronomy, the description of the phenomena related to the beginning and the end of the day is performed according to the terminology of the old Jewish model. It is also interesting to note that even if the Talmudic model is not true, it is nevertheless an excellent device to explain the process of sunset, the disappearance of light and the consequent darkening.[85] We find some details about the Jewish model in different Talmudic passages but the primary source of information is in B. Pesahim 94b where the Talmud explains that, according to the Sages of Israel, the sun moves above the celestial vault during the night and under the same celestial vault during the day. According to the Sages of the nation, the celestial vault revolves around the earth in a continuous motion, while the sun, the moon and the planets have their own direct movement. In B. Pesahim 94b, the contradiction between the Sages of Israel and those of the nations is expressed by the following obscure dictum:[86]

תנו רבנן, חכמי ישראל אומרים, גלגל קבוע ומזלות חוזרין וחכמי אומות העולם אומרים גלגל חוזר ומזלות קבועין

This dictum[87] seems to be connected with the following passage:

חכמי ישראל אומרים ביום חמה מהלכת למטה מהרקיע ובלילה למעלה מן הרקיע וחכמי אומות העולם אומרים ביום חמה מהלכת למטה מן הרקיע ובלילה למטה מן הקרקע

This Talmudic passage would then confirm that the Sages of the nations follow the Ptolemaic model, according to which the spherical earth is surrounded by a spherical vault to which the stars are fixed and which revolves around the earth in a diurnal rotation. In this diurnal rotation, sun, moon and planets are also fixed on the celestial sphere. Furthermore sun, moon and planets have their annual movements. On the contrary, the model of the Sages of Israel seems to consider a plane earth surrounded by a semi-spherical vault fixed to the earth. In this fixed universe, the sun, moon, planets and stars move and revolve according to a complex movement, which must account for the diurnal and the annual movement.

The diurnal movement, as explained in the second passage above, is a strange movement, above the opaque[88]celestial vault during the night from west to east and under the translucent celestial vault during the day from east to west. There are several mentions or allusions to the model of the Sages of Israel in the Talmud or in the Talmudic literature.[89]

The model of the Sages of Israel describing the transition between day and night has been expounded by R' Hananel in his commentary on B. Pesahim 94b, by R' Eliezer of Metz[90] in his Sefer Yereim[91] and by R' Moses Cases[92] in his Novellae on the Talmud.

According to the description of R' Hananel, the sun, at sunset, disappears because it enters an opaque[93] tube and crosses the firmament during four (or five miles),[94] according to the opinions.[95] During the progression through this tube of the sun's crossing of the firmament, darkness is increasing and after four or five miles, when the sun has completed the crossing, the sun passes above the celestial vault. Complete darkness has been reached and the stars, which became visible little by little during the crossing of the firmament, are now all visible; we are at the end of astronomical twilight. During the night the sun will move from west to east above the opaque celestial vault. At the break of day, the sun reaches the eastern opening of the opaque vault and the first break of light becomes visible. The sun now crosses the firmament through the opaque tube and approaches the earth while the light increases on the earth, but because of the opacity of the tube, the sun is still invisible. Finally, after crossing the firmament, the sun reaches the inner extremity of the tube and finally comes out of the tube. The sun becomes visible and we arrive at sunrise. After sunrise, the sun moves from east to west under the celestial vault according to its visible movement. The whole description of this model is clearly based on the principle of a fixed and flat earth, the ignorance of the circular movement of the stars and celestial bodies which brings them under the horizon during a part of the rotation and the necessity of an opaque tube which is required in order to explain the increase in the morning and the decrease of light in the evening during a relatively long span of time (the astronomical dawn or twilight) after the physical disappearance of the sun. R' Hananel notes the contradiction between this model and the model taught by contemporary astronomers. This description is completely similar to that of R' Moses Cases and very similar to that of R' Eliezer of Metz. The latter adds an additional element, namely that the stars that are above the vault during the day descend and cross the vault during the process of the crossing of the firmament and become visible. The vault seems to be opaque in the model of R' Eliezer of Metz. R' Hananel does not explain how the stars become visible. Probably they are fixed to the celestial vault and the stars become visible when the intensity of the solar light diminishes. Another difference in the model of R' Eliezer of Metz is that the crossing of the firmament begins between the

apparition of the first and third star and not at sunset. When the first star appears it is still day but when the second star appears the sun might already have started setting and crossing the firmament. At the apparition of the third star the sun has already certainly entered the thickness of the firmament. In other words, the crossing of the firmament begins after sunset, between the apparition of the first and third star. In order to make things simple we will simply say that the BHS of Ram ends at the appearance of three middle stars, which he calls sunset. It is at this moment, or slightly before, that the sun begins its crossing of the firmament. In fact, the astronomical or apparent sunset is completely absent in the description of R' Eliezer of Metz. For him, sunset means the appearance of the first three night stars and the end of the sun lightened day. It is likely that this was also the conception of R' Tam. The "beginning of sunset" would then be, as for R' Eliezer of Metz, the time of the first appearance of three night stars and the end of natural light,ביאת אורו, while the "end of sunset" five miles later, would correspond to the complete sunset, ביאת שמשו, and the end of the day of R' Tam. Nevertheless, under the influence of all the Spanish authorities, who did not like the above mentioned model, considering the beginning of sunset between the disappearance of the first and third star but who will practically take over the theory of R' Tam, the "beginning of sunset" will shift and advance and be redefined as apparent sunset. It is indeed at apparent sunset that according to R' Hananel the process of crossing of the firmament begins and it makes sense to begin at this moment the crossing of the firmament.

4. The Cosmographical Model of R' Tam (c1100-1171).

We have a very fragmentary picture of R' Tam's model through the data found in Sefer ha-Yashar and in Tosafot on B. Sabbath and B. Pesahim. From them we learn that the physical description of the celestial phenomena[96] accompanying sunset and marking the entry of the Sabbath, which are connected to the passage משתשקע החמה, follow the end of sunset. The beginning of sunset is when the sun enters the firmament and the end of sunset is when the sun has crossed the firmament, 4.25 miles later.[97] At this moment, the sun is at the end of the thickness of the firmament but has not yet crossed the opaque vault. The crossing of the window in the opaque vault takes a time of 0.75 miles. This span of time of 0.75 miles, corresponding to the crossing of the thickness of the opaque vault, is precisely Bein ha-Shemashot, the time of uncertainty between day and night. It is only after five miles that the sun is finally above the opaque vault; only then has the process of darkening ended, and all the stars are now visible.[98] In Tosafot on B. Sabbath and B. Pesahim, the argumentation leading to these conclusions is based on the contradiction between B. Sabbath and B. Pesahim in relation to the length of twilight, 0.75 miles in Sabbath but four miles in Pesahim.[99]
In Sefer ha-Yashar, the beginning of the argumentation is that פני מזרח is according to Rava: מערב. There is no question of the reddish color appearing in the eastern horizon immediately after the sun enters the firmament (sunset). We speak actually about the red color appearing in the western horizon after the sun has crossed the firmament but before the sun has crossed the opaque vault. This crossing takes 0.75 miles and constitutes Bein ha-Shemashot. In the presentation of Sefer ha-Yashar, contrary to Tosafot, the contradiction between B. Sabbath and B. Pesahim about the length of twilight does not appear as the main and only point. It is likely that R' Tam saw the discussion between Rava and Abaye as a discussion in the Talmud between those beginning BHS at the beginning of the crossing of the firmament (Abaye) and those beginning BHS at the end of the crossing of the firmament (Rava). He understood that the conclusion of the Talmud is according to Rava. If this understanding is correct, then R' Tam would accept that there is an opinion in the Talmud of beginning BHS early, at the beginning of the crossing.
The great difficulty of this model[100] is that in the morning, we should have something similar corresponding to the crossing of the vault, which should also take a span of time of 0.75 miles. But there is nothing similar in Talmudic literature in the morning; Bein ha-Shemashot exists only during

evening twilight and not during dawn. This seems a totally new invention that the sun finishes its crossing of the firmament after less than 4 miles,[101] namely 3.25 miles while 0.75 miles is still necessary to cross a mysterious window; there is no element in the Talmud sustaining this theory and we don't find something similar in the morning when the sun passes under the vault at daybreak. In fact the Talmudic sources speak only of a firmament presenting a thickness of 4 miles.[102]
Another important difficulty is that R' Tam believes that day lingers until the end of the astronomical twilight. His Bein ha-Shemashot is the span of time of 0.75 miles before the appearance of all the stars. This seems, at the first glance, very difficult; this is the reason why many scholars have tried to find another explanation for the theory of R' Tam.[103] The origin of the position of R' Tam is the following: according to Nehemiah IV: 14 the day lasts from daybreak until the appearance of the stars. If one wants to behold midday in concordance with the zenithal position of the sun, as this is required by the passage of B. Pesahim 94b and different other Talmudic references,[104] then the moment of the appearance of the stars must be symmetrical to daybreak, the beginning of dawn. We have already written above that this extended day of R' Tam, lasting until the appearance of all the stars, does not seem to correspond to the natural day, which corresponds to daylight and lasts from daybreak until the appearance of the first night stars, which Nehemiah was considering. The main difficulty of this theory is thus to consider that the stars corresponding to the end of the daylight and of Sabbath, which are mentioned in B. Sabbath, are the same as those mentioned in B. Pesahim, which are symmetrical about daybreak.

5. The Cosmographical Model of R' Eliezer of Metz (c1115-c1198).

The model of R' Eliezer of Metz in his Sefer Yereim is of the greatest importance, because he was R' Tam's disciple and because he refers explicitly to R' Tam and explains the nature of their difference. The text of R' Eliezer of Metz is difficult and probably corrupt. His commentary has been mentioned by R' Moses Al-Ashkar in his responsum 96, by R' Abraham Cohen Pimentel in his book Minhat Cohen and by Bah in his commentary on the Tor. However, their understanding raises a major difficulty; it is indeed strange that a span of 0.75 miles before sunset could be considered as Bein ha-Shemashot when it is certain that it is day as long as the sun can be seen. We will examine whether it is not possible to understand Ram otherwise.
When we carefully examine the text of R' Eliezer of Metz on B. Sabbath, we observe that he uses a very peculiar terminology. He uses the day of Ezra[105] referring to the natural workday described in B. Berakhot 2b; it begins at daybreak and ends with the appearance of three nocturnal stars. The day of Ulla begins at daybreak and ends with the appearance of all the stars it corresponds to the day considered in B. Pesahim.
Therefore

צאת הכוכבים דעזרא = תחילת צאתם = ג' כוכבים
צאת הכוכבים דעולא = גמר צאתם= צאת-כל-הכוכבים החוצה =הראות הכוכבים החוצה
סוף יום דעזרא = שקיעת החמה דעזרא = ג' כוכבים
שקיעת החמה דעולא = ה' מילין קודם הראות הכוכבים החוצה = ג' כוכבים
משתשקע = קודם שקיעת החמה דעולא = קודם ג' כוכבים
ה' מילין קודם הראות הכוכבים לחוץ = תחילת צאתם = לילה מדאורייתא = ג' כוכבים בינוניים

In the whole description of R' Eliezer of Metz (Ram), there is no place for astronomical or apparent sunset. In his terminology, sunset always means the time of the appearance of three stars.[106] R' Eliezer of Metz is the only author who carefully describes his model and explains how it differs from that of R' Tam. In his system, the physical disappearance of the sun does not seem to have much significance. He refers only to the appearance of all the stars and to the beginning of their appearance, five miles

before. Therefore the sun begins crossing the firmament a little after our sunset, between the time of the appearance of one and three stars; he calls it in one place, the sunset of Ulla.
R' Eliezer of Metz writes clearly that he considers משתשקע as meaning before שקיעת החמה;
It corresponds to the appearance of three stars, with apparently no relationship to apparent sunset; it is five miles before the appearance of all the stars. Thus, שקיעת החמה represents for Ram[107] the appearance of three middle stars and משתשקע represents the beginning of BHS, 0.75 mile before the appearance of three middle stars.[108]
The end of Bein ha-Shemashot is thus the time of the appearance of the three first night stars i.e. middle stars, five miles before the appearance of all the stars. Ram writes that he did not interpret like R' Tam who considers משתשקע as meaning after שקיעת החמה, after the appearance of the three first stars, and even much later, when the sun has crossed the firmament during 4.25[109] miles nearly before the appearance of all the stars. R' Eliezer of Metz rejects this explanation, first because the time of the beginning of BHS of R' Tam is distant from the time when the hens are going to perch near to sunset. Second, he invokes a philological argument showing that משתקע means before and not after. If our understanding is correct, then the practical conclusions of Ram's theory would be very similar to the conclusions of R' Isaac the Elder of Dampierre and German Rishonim.[110] His BHS would begin 0.75 miles before the appearance of three middle stars, slightly after sunset. This would avoid being forced to accept that R' Eliezer of Metz exaggerated by placing BHS before apparent sunset.

The explanation given by R' Eliezer of Metz, as proposed in this paper,[111] helps us in our understanding of R' Tam. The BHS of R' Tam is actually at the end of the crossing of the firmament, not far from the appearance of all the stars, and although it hurts our logic and the physical reality, R' Tam really considered his three Sabbath stars to be the last stars appearing at the western horizon. The explanations of R' Eliezer exclude the speculations of those who propose that the three stars of R' Tam are the same as those of R' Eliezer of Metz and ours but that the crossing of the firmament by the sun according to R' Tam happens before the crossing of R' Eliezer. The crossing of R' Tam would end with the apparition of three stars just when the crossing of R' Eliezer is beginning. The decisive point in the discussion relies on the understanding of the following passage in Sefer Yereim:

דלשיעור שפירשתי רואים זמן התרנגולים קרוב אבל אם נפרש ג' חלקי מיל קודם להראות הכוכבים החוצה הופלג זמן וסימן התרנגולים מזו השיעור הרבה וכן נראה לי עיקר דמשתשקע החמה הוא קודם שקיעת החמה
דעולא ולא כדברי רבינו יעקב שפירש דשקיעת החמה קודם למשתשקע לפי פירוש רבותיו שפירשו ג' חלקי מיל קודם הראות הכוכבים החוצה ולא משתשקע קודם לשקיעת החמה .....

In this quotation, we must understand according to his conventional vocabulary:

הראות הכוכבים החוצה = צאת כל הכוכבים
שקיעת החמה דעולא= ג' כוכבים בינוניים

It appears clearly that R' Eliezer of Metz writes that *BHS is before the appearance of three stars (which precedes the appearance of all the stars by five miles), and then the moment when the hens are going to perch is indeed not far removed from the beginning of BHS. But if we explain that BHS is 0.75 miles before the appearance of all the stars, like R' Jacob, then the moment when the hens are going to perch would be far removed from the beginning of BHS.* Therefore it appears to R' Eliezer of Metz that משתשקע is before the "sunset of Ulla", which in the terminology of Ram represents the appearance of three stars. R' Eliezer of Metz ascertains that this is clearly in contradiction with Rabbenu Tam. According to Ram, משתשקע of R' Tam, which is the beginning of BHS, is just before the appearance of all the stars, קודם להראות הכוכבים החוצה, much after the appearance of the three stars, which marks the end of the BHS of Ram. Ram fights the opinion of R' Tam in both paragraphs, even if he mentions him

personally only in the second paragraph.[112] It is important to note that the discussion between R' Eliezer of Metz and R' Tam turns around the position of the Bein ha-Shemashot of Rabbi Judah at the beginning of Sabbath but there is apparently no discussion about the position of the Bein ha-Shemashot of Rabbi Jose just before the appearance of all the stars, five miles after the beginning of the appearance of the stars.
Finally, it appears that the position of R' Eliezer of Metz is very similar to that of Ravan. His practical conduct is also very similar: he begins Sabbath 0.75 miles before the appearance of three middle stars, slightly after apparent sunset and he could end Sabbath 5 miles after the appearance of the first three middle stars like Ravan, but in fact he does not examine this problem. The beginning of BHS is certainly slightly after sunset but Ram does not care about sunset. In order to remove any doubt on the subject let us quote a passage of Sefer Yereim relative to Yom Kippur. After explaining the subject according to R' Tam, namely that sunset in B. Pesahim is the "beginning of sunset," five miles before the appearance of all the stars, while משתשקע, in B. Sabbath represents the "end of sunset," 0.75 miles before the appearance of all the stars, he writes:

ואני פירשתי ענין אחר מפני ראיות וקושיות והרוצה לדעת פירושי יעיין למעלה בחייבי סקילה במחלל שבת בהלכות המוציא מרשות לרשות והארכתי שם בפירוש זה, **קדמתי** כניסת שבת ויום הכיפורים ויום טוב, ויזהר אדם בשיעורן כי ראיתי בני אדם משערים שלא כדין, וירא שמים יניח עצמו בארצות החיים.

Rabbi Eliezer of Metz writes clearly that he has brought forward the time of the entrance of Sabbath, the Day of Atonement and holidays, with regard to R' Tam. In fact, Ram did not change anything; it was R' Tam who delayed, at least in theory, the time of the entrance of the Sabbath. Ram complains that there are already people who begin the Sabbath too late.
The discussion between R' Eliezer of Metz and his teacher R' Tam is not a merely academic discussion about the meaning of two Talmudic texts, but it corresponds to two practically divergent positions with regard to the time of beginning the Sabbath. It seems that he began Sabbath, theoretically, 0.75 miles before the first three night stars, slightly after sunset.[113] His position and his arguments are convincing and he apparently already foresaw the abuses to which the plain application of the theory of R' Tam could lead.

6. The Understanding of R' Tam.

We examine the understanding of R' Tam by early authorities in order to prove that they understood him according to the classical understanding.

1. The understanding of R' Tam by R' Simeon[114] in Sefer Ha-Ittur,[115] R' Isaac ben Abba Mari of Marseilles (c1120-c1190).

We find elements connected to the limits of Sabbath in Hilkhot Mila, in Sefer Ha-Ittur. The text of the Ittur seems corrupt. Especially regarding the subject of the interconnection of Sabbath and Mila there are, in the present editions, two non-consecutive, and apparently contradictory, paragraphs. The first one has been recopied textually by R' Joseph Caro in Shulhan Arukh Yoreh Deah 266; 9. In the second one, the theory of R' Tam is presented:

וכ' הר"ר שמעון בשם הר"ר יעקב משתשקע החמה לאו ביאת שמשו הוא שאינו נראה ברקיע דהא משתשקע החמה עד צאת הכוכבים ד' מילין ובין השמשות לא הוה אלא ג' חלקי מיל, אלא משתשקע החמה דהיינו ביאת אורו סמוך ללילה, שאע"פ ששקעה החמה יש שהות מעט עד הלילה

There are two different ways of understanding the expressions ביאת שמשו and ביאת אורו.

According to the Geonim, followed by the Spanish rabbinical authorities, ביאת שמשו refers to the disappearing of the sun (sunset) and ביאת אורו refers to the subsequent disappearing of the light at the end of civil twilight and the appearance of the first night stars.[116] According to Tosafot in B. Berakhot,[117] ביאת אורו is the diminution of light corresponding to the "beginning of sunset" when the sun sets and begins the crossing of the firmament, five miles before the appearance of the stars. It corresponds to sunset and to the end of the direct light after the disappearance of the sun.[118] ביאת שמשו is the "end of sunset" when the sun, after crossing the firmament, crosses the thickness of the vault leading to the complete night. In the present text the passage can only be understood in the following way: the meaning of משתשקע, the beginning of BHS, is not the moment of the disappearance of the sun, when it is no longer visible in the sky, because there are four miles between this moment and the appearance of (all) the stars, and BHS lasts only 0.75 miles. Therefore משתשקע החמה represents a much later moment, near to the night, when the light has already diminished. Although the sun has already set in the firmament, there is still a little time until night, corresponding to the thickness of the opaque vault of 0.75 mile. In other words, ביאת שמשו and ביאת אורו used by the author of Ittur, must be understood according to the Spanish terminology, and this passage confirms the former understanding of R' Tam. Thus *משתשקע ביאת שמשו = sunset but it is much later, when it becomes completely dark, at ביאת אורו after the sun has crossed the firmament during 3.25 miles*;[119] it is only at this moment that begins BHS. It is further impossible to establish if the first paragraph, which is quoted in Shulhan Arukh, Yoreh Deah 266,9 contradicts the position of R' Tam and champions, according to its plain meaning, an early BHS beginning at sunset, or if it is in accordance with R' Tam and champions a late BHS according to R' Tam.[120]

2. The Understanding of R' Tam by R' Abraham ben Nathan ha-Yarhi (c1155-c1215).

He writes in Sefer ha-Manhig:[121]

נר חנוכה משתשקע החמה עד שתיכלה רגל מן השוק מלקתי עצים דקין השוק, שהן מתאחרין מכל אדם.....
משתשקע החמה כשקיעת חמה דתענית, דאמר שמואל כל תענית שלא שקעה עליו חמה לאו שמיה תענית, דהוא צאת הכוכבים דמעלות השחר ועד צאת הכוכבים יממא הוא כדכתיב קרא בעזרא ומייתינן להו בריש ברכות, דמשקיעת החמה הראשון עד צאת הכוכבים ה' מילין כדאמרינן בפסח שני וחזי כשרגא בטיהרא וליכא פרסומי ניסא אלא משקיעת החמה האחרון ואילך.

R' Nathan from Lunel[122] follows the theory of R' Tam: *one must wait for the appearance of the stars for lighting the Hanuka candles. The span of time of 5 miles after sunset belongs to the day and lighting the candles wouldn't be more remarkable than lighting a candle by day.*[123]

3. The Understanding of R' Tam by Tosafot Rid[124](c1200-c1260).

He writes in his commentary on B; Sabbath 34b:

ונראה לי לתרץ שאע"פ ששקעה החמה ונעלמה מן העין עדיין אורה שולט בעולם, שעדיין לא נעלמה מכל העולם כולו. שאלו היה עולה אדם על ההרים, עדיין היה יכול לראותה ומפני זה שולט עדיין אורה בעולם.
וכל זמן שאורה שולט אין פני המערב מאדימין. אבל כששוקעת לגמרי והולכת תחת הארץ מסתלק אורה מן העולם ואז מתחילין פני המערב להאדים מכח זהרותיה. שכל זמן שאורה בעולם, האור מכסה האדמומית ולא תמצא כמו שלא תמצא ביום ובשכלה אורה מן העולם אז נמצא אותו האודם.ותופס שעה מועטת עד שתתכסה תחת הארץ לגמרי. ומתחילת שקיעתה, שתתעלם מן העין ועד שתגמר שקיעתה להתכסות תחת הארץ, הוא שיעור חמשה מילין. ומאי דאמר התם משקיעת החמה עד צה"כ ה' מילין מתפרש מתחילת שקיעתה, שהתחילה להעלם מן העין. ומאי דאמר' הכא משתשקע החמה מתפרש משתגמור שקיעתה שמתחלה לכנס תחת הארץ.

R' Isaiah ben Mali explains the theory of R' Tam using the Ptolemaic elements of astronomy. Except for the strange idea that the appearing and disappearing of the reddish gleams is similar to the vision of the stars and depends on the intensity of the surrounding light; his understanding is much closer to ours. He understands that the theory of R' Tam *is connected to the late vision, a relatively important time after sunset, of reddish gleams in the western horizon. The "beginning of sunset" is our apparent sunset, when the sun disappears from our view. The reddish gleams that are seen in the western sky until BHS, correspond to the moment called משתשקע, representing the "end of sunset" and occur after the sun has disappeared under the earth.*[125] R' Isaiah ben Mali writes clearly that the "beginning of sunset" corresponds to apparent sunset.[126] In Sefer ha-Yashar and in Sefer Yereim, the authors do not explicitly care about apparent sunset.

4. The Understanding of R' Tam by Nahmanides (1194-1270).

The problem of the limits of Sabbath is examined in Torat ha-Adam.[127]

ומסקנה משתשקע החמה הוי בין השמשות עד צאת הכוכבים ואם כן התוספת הזה קודם שקיעת החמה הוא וזה דבר של תימה שתהא חמה זורחת כנגדו והוא חייב בעינוי ואסור במלאכה ותנו נמי וכולן בית הלל מתירין עם השמש.....הא למאן דקים ליה אע"פ שנסתלקה מראש הדקלים, כל זמן שלא שקעה מותרין בכל מלאכה וסוגיין בכולי תלמודא כל שהחמה זורחת מותרין בכל מלאכה..... .

.....נמצא שהוא עושה יום מעלות השחר ועד צאת הכוכבים, לפיכך עביה של רקיע משעת שקיעת החמה ועד זמן שהוא לילה מהלך ארבעת מילין.

ולפי דבריו ופירושו זה אני אומר דמשעה שהתחיל השמש להשתקע עד זמן בין השמשות הוא הזמן הנזכר בפרק במה מדליקין, הוא בכלל תוספת מחול על הקודש והוא מהלך שלשת מילין ורביע מיל, רצה להוסיף.....

Nahmanides accepted and acclaimed enthusiastically the theory of R' Tam. Besides being in agreement with the apparent support of the Talmud for the position of Rava, who was looking towards the west to see the reddish gleams, the theory of R' Tam creates for him an empty period of time that does not belong to the former day of Friday and that does not yet bear the interdicts of the Sabbath. This time, which is quite long -3.25 miles-, fits this additional time at the entrance of the Sabbath, תוספת שבת. Indeed, Nahmanides seems to attach great importance to this problem of Tosefet which cannot, he considers, start before apparent sunset. This preoccupation[128] with finding an adapted moment for this addition is probably the first reason for adopting R' Tam's theory.
As we can see in the above quotations, Nahmanides repeatedly and explicitly mentions the apparent sunset and he identifies it as being the "beginning of sunset." It precedes the "end of sunset" by 3.25 miles and the apparition of the stars by four miles.

Similarly Nahmanides writes in his commentary on Shemot XII; 6:
והזמן הזה הוא כל עת זרוח השמש ברקיע אבל בשקיעת החמה והוא כמו שעה ורביע על דעת רבותינו אינו זמן השחיטה ואינו נקרא ערבים, אבל הוא ערב יום.....

Ramban seems to make no difference in this passage, between sunset, which precedes the apparition of (all) the stars by 1h; 1/5[129] and Pelag ha-Minha which precedes it by 1h; ¼. [130]

5. The Understanding of R' Tam by R' Solomon ben Menahem Meiri (1249-1316).

.....שבין השמשות לרבי יהודה הוא משתשקע החמה, סוף שקיעתה ר"ל שנכנסה כולה וכל שפני מזרח מאדימים ופרשו למטה צד המערב.....ומה שאמרו בפסחים משקיעת החמה עד צאת הכוכבים ארבעה מילין פירושה מתחילת שקיעתה כשמתחלת ליכנס והיא נראית עדין מעט מעט עד שתכנס כולה. מ"מ גדולי הדורות משתמשין בתחילת שקיעה זו לומר בה שהיא בכלל תוספת חול ר"ל שקודם שתתחיל לשקוע כלל אינה בכלל תוספת ומשתחיל לשקוע.....

The terminology used by Meiri is confusing.[131] We do not know from the text if the "beginning of sunset" is something preceding apparent sunset, when the altitude of the still visible sun decreases, or if it is apparent sunset. Similarly we do not know if the "end of sunset" is the moment of apparent sunset, when the sun disappears at the horizon, or if it occurs about an hour later, much later than apparent sunset, "when it has set." Meiri tells us nevertheless that at the "beginning of sunset" there are reddish gleams in the western horizon. But the most important element is the reference to the fact that the "beginning of sunset" has been used by Nahmanides[132] as the beginning of the time of the Tosefet, the additional time that one adds to the Sabbath. This moment is undoubtedly apparent sunset.[133]
Therefore the above quotation must be understood as follows: BHS of Rabbi Judah begins at the moment: משתשקע החמה, "the end of sunset" which occurs much after apparent sunset. But as long as the western horizon is reddish, it is still day. When B. Pesahim speaks of a twilight of four miles, it is counted from the "beginning of sunset" corresponding to apparent sunset, which is the end of the setting process, when the sun disappears little by little.
Furthermore we have noted the two following parallel passages, related to the complex problem of the time of the prayer of Ne'ila; they offer more decisive evidence:
In his commentary on B. Taanit 26a he writes:

וכבר ידעת שבתלמוד המערב נחלקו בזמנה אם בנעילת שערי שמים והוא קרוב לצאת הכוכבים אם בנעילת שערי היכל והוא בתחילת שקיעת החמה והלכה בנעילת שערי היכל. מדקאמר הכא, כיון דסמוך לשקיעת החמה היא כנעילה דמיא, אלמא שנעילה עצמה כך דינה.

In his commentary on B. berakhot 26a he writes:

אע"פ שאין זה מקומה תפילת נעילה זמנה שיתחיל אותה על צד שיהא משלים בה תיכף לשקיעת החמה ויש מפרשים שיתחיל בה סמוך לשקיעת החמה ושאפשר לו לשהות בה עד סמוך לצאת הכוכבים.

We see here clearly that he calls the common sunset: "the beginning of sunset". It is far from the appearance of the stars by 1.2 hours so that those who begin Ne'ila at sunset dispose of enough time (four miles) to end it just before the appearance of the stars. By contrast those ending before sunset[134] must begin at least when the sun is at the top of the trees, about half an hour before sunset. They dispose of only half an hour. This is the reason why the Shulhan Arukh insists so much on the necessity to hurry and shorten the prayer of Ne'ila in order to end it in time before sunset..

6. The Understanding of R' Tam by R' Solomon Aderet (c1235-c1310).

    1. In his Novellae on B. Berakhot 2a he writes:

ואינו נראה כלל דהאי ודאי לר' יהודה אע"פ שאינו מתפלל תפילת המנחה מפלג המנחה ואילך, לא מפני שהוא לילה קאמר, אלא דתפילות כנגד תמידים תקנום ותמיד של בין הערבים היה קרב והולך עד פלג המנחה ומשום הכי תפילת המנחה שהיא כנגדה אינה אלה עד אותו זמן ומשם ואילך ראוי לתפילת הערב מפני איברים ופדרים שקרבין והולכין אבל אינו לילה ותדע לך שהרי עדיין השמש על הארץ כדי מהלך שתות המיל והאיך הוא לילה וזמן שכיבה. ועוד שהרי לרבי יהודה בין השמשות אינו אלא שני חלקי מיל סמוך לצאת הכוכבים וקודם הזמן הזה כהנים טובלים ויום הוא ופלג המנחה קודם ליציאת הכוכבים מהלך ד' מילין ושתות. כיון שכן, האיך אפשר שקורא בו לר' יהודה.....

This quotation is not connected to the problem of Sabbath; it belongs to his commentary on B. Berakhot. It allows us clearly understanding his position. R' Solomon follows the theory of R' Tam and uses long temporary hours calculated on the basis of an extended day beginning at daybreak and ending at the appearance of the stars (all the stars according to the theory of R' Tam). Therefore, at Pelag ha-Minha, the sun is still visible for 1/6 mile, the same time already given by Nahmanides.

Rabbi Solomon ben Aderet explains that *Minha is said until Pelag ha-Minha because this was the time limit for bringing the Minha offering. After this time, one is already allowed to say the prayer of Ma'ariv although it is not yet night or bedtime. Indeed, at this moment of Pelag ha-Minha, the sun is still visible on the earth for 1/6 mile and the coming of the stars occurs only 4 1/6 mile later. The BHS of Rabbi Judah is only 2/3 mile*[135] *near to the appearance of the stars and before this BHS, the priests are allowed to immerse themselves because it is still day.*
Pelag ha-Minha is thus slightly before apparent sunset, which coincides with the "beginning of sunset," and the priests are allowed to immerse themselves until the beginning of BHS, 4 1/6 miles – 2/3 miles = 3.5 miles after Pelag ha-Minha or 3 1/3 miles after apparent sunset.[136]

2. In B. Berakhot 2b he writes at the entry: ד"ה רבי חנינה

דשקיעת החמה ידועה טפי ומתפרסם טפי משיעור דעני.....

In B. Sabbat 34b, he writes at the entry: ואזדו לטעמייהו

ולפי שאין הכל בקיעין בתכילת השקיעה האחרונה כדי שיעמוד ממנה על סוף השקיעה ושיהא מותר לו לאכול בתרומתו קא יהיב השתא שיעורא דכולי עלמא בקיעין בו דמכי הוי שמשא בראש הכרמל ירד משם ויטבול בים ויעלה וכשיעלה שם ידע שכבר נשלם זמן בן השמשות ומותר לו לאכול בתרומתו.....

Thus the *"beginning of sunset" corresponds to sunset; it is a tangible and precise phenomenon that all people know. By contrast, "the end of sunset" or beginning of BHS is something intangible; it must be determined indirectly.*

7. The Understanding of R' Tam by R' Yom Tov ben Abraham Ishbili, Ritva (c1250-1330).

He follows the theory of R' Tam of his teacher R' Solomon ben Abraham Aderet.
He writes in his commentary on B. Sabbath 34b:

ונמצא שמתחילת השקיעה עד זמן בין השמשות שלשת מילין ורביע וכתב הרמב"ן ז"ל שזהו הזמן הנזכר בכל התלמוד לתוספת שמוסיפין מחול על הקודש שאי אפשר לומר שיהיה בעוד שהשמש זורחת על הארץ.....אלא ודאי זמנו של תוספת היינו מתחילת השקיעה שאין השמש נראה על הארץ עד זמן בין השמשות.....

In his commentary on B. Taanit chap. IV about the time of the prayer of Ne'ila, he quotes a text of Y. Yoma according to a variant reading, different from the common reading and writes:

שמע מינה דנעילת שערי שמים הוא ביום והוא קודם בין השמשות שהיא סוף שקיעה..... ובש"ס דילן ..... אלמא תפילת נעילה סמוך לשקיעת החמה, כלומר סוף השקיעה שאף הוא ראוי לתפילת המנחה ואין להקדים אותה קודם תחילת השקיעה בעוד שהשמש בעולם דאיכתי ליכא נעילת שערי שמים.

Thus according to Ritva *the prayer of Ne'ila must be said before the closing of the heaven's gates which occurs before the "end of sunset" corresponding to the beginning of Bein ha-Shemashot. Therefore the prayer of Ne'ila must be said before the "end of sunset" and it must not begin before the "beginning of sunset", when the sun is still (visible) in the world because at that moment it is not yet the time of the closure of the heaven's gates.*
Because of his special reading, Ritva has a very personal theory of the time of the prayer of Ne'ila. He is the sole author to remain coherent with the theory of R' Tam in the problematic of Ne'ila. According to him the prayer of Ne'ila and the blessing of the priests must be said by day, therefore it may be said until the "end of sunset", 3.25 miles after sunset.

Without any doubt, he calls sunset, the "beginning of sunset" and it is still day after sunset during a span of time of 3.25 miles.

8. The Understanding of R' Tam by R' Nissim ben Reuben Gerondi (c1310-c1375)

R' Nissim follows the theory of R' Tam according to the understanding of Ramban.
He writes indeed in his commentary on the Rif on B. Sabbath 34b:

כבר פירשתי למעלה שאף לאחר שנסתלק השמש מעל הארץ הוא יום גמור כשיעור שלשה מילין ורביע ואותו זמן הוא ראוי לתוספת.....

However, in B. Taanit 26a, he writes in his commentary on the Rif, the following surprising passage:

וזמן תפילת נעילה אפלוג בה בירושלמי אי נעילת שערי שמים והיינו עם חשכה, אי נעילת שערי היכל והיינו עם שקיעת החמה כשחמה היא בראשי האילנות הגבוהים ומוכח התם שאין בין זה לזה אלא שיעור תפילה למי שמאריך בתפילתו וסוגיין דהכא מוכח כמן דאמר נעילת שערי היכל דאמרינן שכיון דמצלו לה סמוך לשקיעת החמה כתפילת נעילה דמיא.

There is thus a divergence whether Ne'ila must be said before the closure of the heaven's gates, when it becomes dark, whether it must be said when the sun is on top of the tall trees, before the closure of the sanctury's gates which occurs at sunset. The span of time between these two moments, the closure of the heaven's gates and the sanctury's gates is only a short span of time corresponding to the additional time necessary to someone who lengthens his prayer.

In the first quotation it is still day after sunset during 3.25 miles. In the second quotation the span of time between sunset and the night is not more than a short span of time, allowing the lengthening of the prayer of Rav.[137] It is then likely that R' Nissim does not any more follow the theory of R' Tam, but he considers that about 20 minutes after sunset it is night. This fits another passage in his commentary on Rif on B. Pesahim[138] where he explains in the name of Rabad the expression אור לארבעה עשר, because there is still light in the beginning of the night when the research of the leaven must be done.

9. The Understanding of R' Tam by R' Jeroham (c1290-1350).

Rabbi Jeroham[139] first brings the opinions of Rif, who follows Rabbah and forbids the performance of any work after sunset, and then he mentions the opinion of R' Tam, who considers a first sunset (discussed in B. Pesahim) when the sun enters the thickness of the firmament and its light disappears from the earth and a second sunset (discussed in B. Sabbath), 3.25 miles later, when it begins to come out of the thickness of the firmament. He then writes:

ואנו יש לנו להחמיר ואסור לעשות מלאכה מתחילת שקיעת החמה כדאמר רבה ...

This proves that R' Jeroham considers that the BHS of R' Tam begins much later than that of Rif but *"we must adopt the most stringent position and refrain from any work as early as the first sunset."* In other words, R' Tam begins BHS at the beginning of the second sunset, but we must accept the interdicts of Sabbath as soon as the first sunset begins, which corresponds to the sunset of the Rif, or apparent sunset. In Hilkhot Ta'anit[140] he rules like R' Tam that "one must fast until sunset" means after the end of sunset, five miles after sunset, at the appearance of the stars.

10. The French and German Rishonim with regard to the theory of R' Tam.

It appears that all the noted authorities near the time of R' Tam, from the twelfth and thirteenth centuries undoubtedly understood that the "beginning of sunset" or in later terminology "the first sunset" must be identified with apparent sunset. Nevertheless, R' Eliezer of Metz seems to understand this expression "beginning of sunset" as the first appearance of three night stars. He calls this moment "sunset of Ulla". The only exception is given by R' Hayim Or Zarua[141] in his responsum 186.
He writes:
משתשקע החמה הוי בין השמשות. ופר"ת זצ"ל מסוף שקיעה. ונ"ל דכשאין החמה נראית יותר בראשי ההרים הוי סוף שקיעה, מדאמר פ' במה מדליקין רבי נחמיה אומר כדי שיהלך משתשקע החמה חצי מיל, פ' זה הוי שיעור אורך בין השמשות. אמר רבי חנינא הרוצה לידע שיעורו של רבי נחמיה יניח חמה בראש הכרמל וירד ויטבול בים ויעלה וזהו שיעורו של ר' נחמיה. ופי' תוס' לא שירד הטובל מראש הכרמל אלא הוא עומד על שפת הים ורואה חמה בראש הכרמל ומשירד ויטבול ויעלה הוי לילה. אלמא דכשאין החמה נראית בראש הכרמל הוי סוף שקיעה.

All the preceding German authorities have ignored the position of R' Tam in their practical decision.[142]This is also the case with R' Hayim's father, R' Isaac of Vienna in Or Zarua. Therefore I am absolutely not sure that R' Hayim has changed his position and now follows R' Tam after reinterpreting him according to the German standards. I would rather say that he takes exception to the theory of R' Tam and ascertains that according to the rule of Rabbi Hanina, the moment of משתשקע is not later than the moment when the sun disappears at the head of the Carmel. R' Hayim considers that this moment, when the sun disappears at the head of the Carmel, is, in any case, the latest moment that can be taken into account by R' Tam for the end of sunset. This does not mean that R' Hayim reinterprets the theory of R' Tam and considers a "beginning of sunset" nearly an hour before sunset and an end of sunset slightly after sunset.
The influence of R' Tam on the French Rabbis was tremendous; nearly all of them followed him without any contradiction. The only exceptions are his two pupils R' Eliezer of Metz and R' Isaac the Elder of Dampierre.[143] His influence spread to Provence and Spain through Provencal and Spanish pupils learning in Northern France.[144] The role of Nahmanides was probably preponderant. In Germany things were different and the theory of R' Tam could never assert itself. The German Rabbis did not apparently accept the theory of R' Tam. This could be connected to the old German practice of accepting Sabbath very early, so early that the Rabbis could hardly justify this practice. According to R' Tam Sabbath begins theoretically 3.25 miles, or even 4.25 miles after sunset. On the contrary the German rabbis accepted to begin Sabbath at Pelag ha-Minha, 1.25 temporary hour before sunset or even two temporary hours before sunset.[145] The necessity of finding a justification for the early acceptance of Sabbath,[146] which was a general practice, obliged them to consider Pelag ha-Minha before sunset and to use therefore short seasonal hours, calculated between sunrise and sunset; this was the only way to have Pelag ha-Minha substantially before sunset. Practically the theory of R' Tam was never accepted in Germany until the publication of Shulhan Arukh in the sixteenth century.[147] And even then, the theory of R' Tam was only accepted in little towns and in the country where it was common to accept Sabbath a quarter of an hour before the night, but it was never accepted in Frankfort am Main.

1. Rabbi Eliezer ben Samuel of Metz, (Ram, c1115-c1198).

He was probably the first to contradict his teacher. According to the proposed understanding,[148] the BHS of Rabbi Judah begins 0.75 mile before the appearance of the three first night stars,[149] probably slightly after sunset. He does not speak about the BHS of Rabbi Jose and the end of Sabbath. It is likely that, similarly to Ravan, he places the BHS of Rabbi Jose 5 miles after the end of the BHS of Rabbi Judah, at the appearance of all the stars.

2. Rabbi Isaac ben Samuel the Elder of Dampierre, Ri ha-Zaken, (c1115- c1185).

He seems to share the opinion expressed in Tosafot[150] that the minor public fasts can be ended at sunset, [151] probably because this moment is the beginning of BHS.
He rules further that BHS ends about one temporal hour before the appearance of (all) the stars.[152]
From one of his responsa[153] we learn that he believed that there is still light in the sky at the moment of the first vision of the three stars marking the end of Sabbath.
We can conclude that R' Isaac the Elder considered that the first appearance in the eastern sky of three night stars (middle stars) while the western sky is still lightened, relatively shortly after sunset and about one (temporary) hour before the appearance of all the stars, marks the beginning of the night.
Therefore, the beginning of Sabbath is at sunset but the end of Sabbath is 4 miles later like Ravan.

3. Rabbi Eliezer ben Nathan of Mainz, Ravan, (c1090-c1170).

If we accept the reading of R' Posen in Or Meir, Ravan considers a BHS of Rabbi Judah of 0.75 mile beginning slightly after sunset. But, he adds, that *we do not rely on the end of BHS of Rabbi Judah for estimating Sabbath's end, indeed we consider that it is day until the appearance of all the stars, five miles after sunset. Therefore we must consider that BHS of Rabbi Jose is five miles after sunset, at the appearance of all the stars.* Therefore he begins Sabbath at sunset[154] or slightly after together with the opponents of R' Tam[155] but he ends Sabbath together with R' Tam, as the rule is to accept Sabbath according to the ruling of Rabbi Judah and end it according to the ruling of Rabbi Jose.

4. Rabbi Meir ben Barukh of Rothenburg, Maharam, (c1215-1293).

Maharam said the evening prayer early on Friday evening, before sunset, according to the German custom, but after Pelag ha-Minha.[156]
He would not eat Friday evening's meal immediately upon returning from the synagogue, when it was still early, but at around sunset.[157] Even when he would fast on Friday, he would eat at sunset (except for a dream fast) and he considered that this is the meaning of "fasting and completing." Even on weekdays, he would not require fasting on minor fasting days until the appearance of the first night stars, but only until sunset. It is also said in his name that the quotation "he fasts and completes" means that he completes the fast only if he wants.[158] **He rules that Ne'ila on Yom Kippur must end before sunset, during the day.**[159]

5. Rabbi Eliezer ben Joel ha-Levi of Bonn, Ravia, (1140-1225), grandson of Ravan.

Ravia accepts Sabbath slightly before sunset[160] but certainly ends it in the same way that he ends the fasts, at the appearance of the stars, when the sun has *completely* set, four miles after sunset.[161] Ravia follows completely his grand-father, the Ravan: he begins Sabbath at the beginning of BHS of Rabbi Judah and he ends it at the end of BHS of Rabbi Jose.
Ravia nevertheless uses short temporary hours, counted from sunrise to sunset, corresponding to the hours of a sundial.[162]

6. Rabbi Isaac of Vienna, Or Zarua, (c1180-c1250).

Or Zarua begins Sabbath before sunset[163] because of Tosefet Shabbat and he ends it at the appearance of three middle stars although the sky is not completely dark.[164] He uses short temporary hours.[165]

7. Rabbi Jacob Moellin, Maharil, (c1360-1427).

It seems that he counted the hours of his religious Jewish day in equinoctial hours and it is only because he wanted to justify an early acceptance of Sabbath that he writes *that we could consider, as Tosafot did, temporary hours, counted from sunrise to sunset, as the astronomers demonstrate it on their spherical astrolabe.* He writes finally that *according to those who begin Sabbath at Pelag ha-Minha, one must consider 1.25 seasonal hours before sunset.*[166]
However elsewhere[167] he seems to count *1.25 hours of Pelag ha-Minha and even 2 hours before sunset of Maharah (Rabbi Hayim Or Zarua)*[168] *for early acceptance of Sabbath, in temporary hours allowing justifying the custom of the communities*.
He writes that the custom of Maharam to eat at sunset was influenced by the French authorities. But he justifies the German custom of saying Shema and the evening prayer much before that, after Pelag, and having the Friday evening meal before sunset in the courtyard without the candles at the dinner table. The candles should nevertheless be lit at home until they are somehow useful.[169]

8. Rabbi Joseph Yuspa ben Phinehas Seligman Hahn Nordlingen of Frankfurt, (1570-1637)

This rabbi comes much later than the former authorities, but maintains a straight continuation of their thoughts and is apparently not at all influenced, in this matter, by the Shulhan Arukh.[170] He defends the early acceptance of Sabbath before sunset, when the sun is on top of the trees.[171] He writes that *one should hurry home after evening prayer in the synagogue to have the Friday evening meal.* He takes exception to Maharam[172] *who waited for sunset before sitting at the table* and says that *only the end of the meal should be near to the night*[173] *or at least until sunset*.[174]
He ends Sabbath slightly after the appearance of three stars,[175] much before 4 miles after sunset[176] and he counts 1.25 equinoctial hours before sunset for Pelag ha-Minha. Indeed he writes that *even if the hours were temporary hours, Pelag ha-Minha would not precede sunset by two equinoctial hours*.[177]
He probably counts his temporary hours from sunrise to sunset[178] and writes that *even if Pessah is very late (in the eighth year of a cycle) the limit of four hours allowed for eating hamets, will never be before 9 a.m.*[179]

All these opinions are very similar and represent, with some slight shades, our current behavior. Nevertheless, Ravan ended Sabbath much later, at the end of the BHS of Rabbi Jose, theoretically at the appearance of all the stars, together with R' Tam. Of course, we do not know how R' Tam behaved practically. It is likely that R' Tam used his theory only on the stringent side at the end of Sabbath but he surely began Sabbath together with his elder brother, his father and his maternal grandfather near to sunset.

9. Conclusion.

The theory of R' Tam was never accepted among ancient German Rabbis. R' Tam was at the origin of many original theories and sometimes also at the origin of the revival of old theories and practices. But most of them remained theoretical and they did not foist themselves upon the Halakha. The success of this problematic theory must be connected to its success in southern France and its propagation in Spain. It is certainly connected to its enthusiastic acceptance by Nahmanides. This general acceptance of the theory of R' Tam by the French Tosafists and by the Spanish rabbinical authorities led R' Joseph Caro to decide in its favor. This has favored the spread of this theory in Eastern Europe in a way it had

never been used before i.e. the practical late acceptance of Sabbath until a quarter before night, as was so much deplored by R' Joel Sirkes.[180]

[1] These two systems of division are similar to the two systems of division considered in the Mishnah Berakhot I, 1.
[2] References:
Carcopino, J. La vie quotidienne à Rome à l'époque de l'Empire, Paris, 1939, 1967 and 1972.
Carcopino, J. Daily Life in Ancient Rome. New Haven, 1940, 1963 and 2003.
Google: History of Timekeeping. Roman article.
Dupont, Florence, Daily Life in Ancient Rome, 1993.
Ginzel, F.K. See following note.
[3] Ginzel, F.K. Handbuch der mathematischen und technischen Chronologies des Zeitrechnungswesen der Volker. Vol. 2, 1911,. Zeitrechnung der Römer, p. 162.
Bilfinger, G. Die mittelalterlichen horen und die modern stunden. Ein Beitrag zur kulturgeschichte. 1892.
[4] Same references as those mentioned in note 4**.** These two systems of division are similar to the two systems of division considered in the Mishnah Berakhot I, 1.
[4] References:
Carcopino, J. La vie quotidienne à Rome à l'époque de l'Empire, Paris, 1939, 1967 and 1972.
Carcopino, J. Daily Life in Ancient Rome. New Haven, 1940, 1963 and 2003.
Google: History of Timekeeping. Roman article.
Dupont, Florence, Daily Life in Ancient Rome, 1993.
Ginzel, F.K. See following note.
[4] Ginzel, F.K. Handbuch der mathematischen und technischen Chronologies des Zeitrechnungswesen der Volker. Vol. 2, 1911,. Zeitrechnung der Römer, p. 162.
Bilfinger, G. Die mittelalterlichen horen und die modern stunden. Ein Beitrag zur kulturgeschichte. 1892.
[5] There was surely a gnomon in Hezekiah's palace, around 700 B.C.E. as mentioned in Isaiah XXXVIII: 8 and II Kings XX: 11. The sundial is ascribed to King Ahaz and is called "Maalot Ahaz." See Encyclopedia Judaica, entry: sundial. See also Benish, P. Ha-zemanim ba-halakhah, 1996, pp. 77-88. Sundials are mentioned in Mishnah Eduyot 3, 8 and Kelim 12, 5. The use of the gnomon is described in the Braita of Samuel, chapter II, 2.
[6] Marcus Vitruvius Polio, architect of the first century described the extant sundials in the book IX of his treatise.
[7] Toomer, G.J. Ptolemy's Almagest. Chater II, 5 and 6: Calculation of the lemgth of the shadow of a gnomon in function of the season and the latitude (climate).
[8] However Maharil (~1360-1427) and R' Israel Isserlein (1390-1460) used mainly equinoctial hours and must have lived in areas were the transition occurred earlier. They do not seem to know that temporary hours were earlier in use. For them the temporary hours have a theoretical character; they both know them only through the text of Tosafot.
[9] Dohrn-Van Rossum, G. History of the Hour: Clocks and Modern Temporal Orders. Chicago 1996.
Google: Hour. Wikipedia.
[10] In some areas 24h corresponded to sunset.
[11] R' Azariah de Rossi (~1511-~1578). In the first chapter of his book *Me'or Enayim*, entitled *Kol Elokim*, he described the earthquake of 1571 in Ferrara, using the Italian hours. See pp. 4b and 5a of the Vienna edition, 1829.
[12] R' Joel Sirkes (1561-1640): see his commentary Bayit Hadash on Tor Orah Hayim 261.
[13] R' Samuel Aboab (1610-1694) mentions the "Italian" and the "German" hours in his responsa *Devar Shemuel*, n° 213. He writes that the difference between the two systems is not significant; it is however half an hour.
[14] R' Samson Morpurgo (1681-1740), the latter's pupil, gives numerical indications about the time schedule of Friday afternoon in Italian hour (see *Shemesh Tsedaka*, responsa 7 and 8). Some congregations were accepting Sabbath at 10h and in any case, it should not be accepted later than 11h, sunset being at 11h: 30m. He writes also about the tekiot of the eve of Sabbath, that the first call was at *Minha Ketana*, at 9h (or 21h), the second call at 9h: 30m (or 21h 30m) and the third call at 10h (or 22h), all these times are expressed in Italian hour. Therefore one can accept Sabbath from 22h on. Placing the first call at 9h Italian hour, corresponding at the equinox to 9h: 30m temporary hour is in accordance with Maimonides, Hilkhot Shabbat V: 20 but the third call at 10h, 1.5 hours before sunset is in contradiction with the same reference of Maimonides according which the third call is near to sunset. In connection with the early acceptation of Sabbath, I came recently across an important quotation of Flavius Josephus proving that the early acceptation of Sabbath was already an old tradition when Augustus (– 62 - +14) confirmed the privileges of the Jews to send money to the Temple of Jerusalem and to keep the

Sabbath. He wrote "that they be not obliged to go before any judge on the Sabbath day, nor on the day of the preparation to it , after the ninth hour." Flavius Josephus: Jewish Antiquities, book 16, chapter 5, 2; Kregel Publications p. 536. Paul Maier wrote in his commentary: This is an authentic evidence that the Jews, in the days of Augustus, began to prepare the celebration of the Sabbath at the ninth hour of Friday, as the tradition of the elder did, it seems, then require of them.

[15] Rabbis like R' Joel Sirkes (Bah), R' Samuel Aboab and R' Samson Morpurgo made use of the Italian clock.

[16] Bigourdan, G. L'Astronomie, 1916. p. 79.

[17] See Google: Hour Wikipedia.

[18] Except Maharil and R' Israel Isserlein, who considered equinoctial hours. Temporary hours had for them a theoretical character; they knew them only through the Tosafot. The temporary hours must already have been abandoned for a long time in their countries.

[19] Maimonides writes that explicitly in his commentary on Mishna Berakhot I: 2 and in responsum 44 in Pe'er ha-Dor, Amsterdam 1765.

[20] Toomer, Ptolemy's Almagest, 1984, p. 23 and pp. 99- 101. The civil time was given in temporary hours and the equinoctial hours were only used in the astronomical tables.

[21] Talmudic Metrology II: The Mile as a Measure of Time. J. Ajdler. B.D.D. n° 20, 2008.

[22] If we exclude Maimonides, who was certainly aware of this, we must wait until the seventeenth century for R' Joseph Solomon Delmedigo for a correct notion of the evolution of the length of twilight during the year. It is only in the middle of the eighteenth century, in 1766, that R' Raphael ha-Levi from Hanover publishes a table giving daybreak, sunrise, sunset and the time of the appearance of three stars during the year. This table will contribute, little by little, to the calculation of more correct Jewish timetables. As for Maimonides, see our paper: The Equation of Time in Ancient Jewish Astronomy, BDD 16. Note that Maimonides considered three types of hours:

1. Equal hours or equinoctial hours: סאעאת אלאסתוי : see Mishna Pesahim III: 2.
2. Temporary hours or seasonal hours: סאעאת זמאניה: see Mishna Berakhot I: 3.
3. Corrected hours: סאעאת אלאעתדאל :see Mishna Berakhot I : 1. It deals about the length of the astronomical twilight at the equator. These hours are not equal but they are not proportional to the length of the day. The length of dawn is 1h 12m at the equinox and it varies during the year according to the time necessary for the sun's depression to increase from 1°, corresponding, according to Maimonides, to the time of apparent sunset, to 19° corresponding according to Maimonides to the end of the astronomical twilight (today we consider the values of 0.85° and 18°). This explains the divergences in the translation of the litigious passage in the Commentary of Maimonides on Berakhot I: 1. See reference in note 21.

However R' Kafih translated in Maimonides' commentary on Mishna Berakhot I;1: 1h;12m equinoctial hours.
See Prof. Leo Levi, Zemanim ka-Halakha p. 42, Jerusalem 5752.

[23] One must, nevertheless, remain cautious. In Sefer Yereim, ed. Schiff, Vilna 1899, § 274 (in old edition § 102), in the section on the beginning of Sabbath, R' Eliezer of Metz writes that R' Tam explained it according to his teachers. In other editions, it writes "according to R' Guershom," see Or Meir p. 40. The theory of R' Tam may not be original and it could be older. Nevertheless, if it was Rabbenu Guershom's theory then it would be strange that it did not have a greater echo and more defenders among German Jewry.
It is also interesting to note that Rashi already notes, in B. Berakhot 26a: תפילת מנחה עד הערב, **עד חשכה** and more explicitly in B. Aboda Zara 25a: היה אותו יום מעלות השחר עד יציאת הכוכבים that an ordinary day lasts from daybreak until Tseit ha-Kokhavim like the Jewish religious day as defined by Nehemiah and quoted in B. Berakhot; Rashi could then also consider long temporary hours calculated on the basis of an extended day.

[24] See the paper of R' Zalman Koren.
קורן, זלמן. על שיטת בעל תרומת הדשן. יד יצחק, כרם ביבנה, תשס"ג, עמ. 9- 76

[25] Sefer ha-Ibbur, ma'amar I, chapter 10 and Sefer Mahalekhot ha-Kokhavim chapter V, p. 29. See also R' Isaac Israeli in Sefer Yessod Olam II, 13, p. 30 and R' Ovadia ben David on Hilkhot Kiddush ha-Hodesh chapter 6, 2.

[26] See Commentary on Mishna Berakhot I, 2 and IV, 1. See also response Pe'er ha-Dor 44 and Hilkhot Tefila III, 4.

[27] Rashi apparently begins BHS at sunset or slightly later, when the sun disappears on top of the Carmel. He writes it in his commentary on B. Sabbath 35a about the quotation of Rabbi Hanina about the one who wants to know the length of the BHS of Rabbi Nehemiah. In his responsum 96, R' Moses Al-Ashkar quotes this passage and proves that Rashi's BHS begins at sunset and his night rapidly follows sunset. On this basis he proves that Rashi shares the position of the Geonim and contradicts R' Tam. See however the former note n°23.

[28] See Maimonides' commentary on the Mishna Megila II:4 :
ומה שנאמר שנחשוב יום מעת עלות השחר לאמרו בעזרא מעלות השחר עד צאת הכוכבים וקרא את הזמן יום והוא אמרם והיה לנו הלילה למשמר והיום למלאכה, ומה שאמרו וכולם שעשו מעלות השחר כשר למי שעבר ועשה בעת הצורך הגדול.

[29] We find this length of dawn and dusk in Tosafot, in the following references.
B. Pesahim 94a: בד"ה רבי יהודה.

B. Sabbath 35a: בד"ה תרי תלתי
B. Zevahim 56a: בד"ה מנין לדם שנפסל בשקיעת החמה
B. Menahot 20b : בד"ה נפסל בשקיעת החמה כמותה
[30] This is explicitly mentioned by Rashi ad locum. These 40 miles are the walk of an average walker during an extended day while 32 miles are the walk covered between sunrise and sunset. It is also mentioned implicitly by R' Hananel ad locum. Thus 40 miles is the distance covered by the average walker during an extended day while 32 miles is the distance covered between sunrise and sunset. The consecutive value of 22.5 m for one mile is accepted by nearly all later authorities like Ramban, Rashba, Ran and Ritva. It may also be assumed that Tosafot accept this value of 22.5 m for one mile. It is also the value advocated by the Gra and R' Jacob Reicher. The two important exceptions are Maimonides who equates a mile to 24 m and R' Israel Isserlein who adopts the value of 18 m for one mile. But most of the later authorities considered that these 18m are long temporary minutes and represent in fact 22.5m or 24m (equinoctial minutes). This is the objection of the Gra against the Mehaber in Orah Hayim 459. However R' Isserlein will be followed by R' Joseph Caro in Shulhan Arukh, by Rema in his glosses and by R' Mordekhai Jaffe in Levush. The latter writes clearly that we deal with equinoctial minutes but it is not clear whether the two first authors were aware that R' Israel Isserein counts the hours of the day between sunrise and sunset and therefore a mile is really 18 m equinoctial (see Talmudic Metrology II: The Mile as Measure of Time, BDD 20, 2008). On the contrary it is highly improbable that they were aware of that and it would be likely that these 18 m are enlarged minutes corresponding to 18*40/32 = 22.5 m.
[31] This would correspond at the equinox, in Jerusalem, to a depression of 19.8°.
[32] See following references in Tosafot.
B. Berakhot 2b: בד"ה דילמא.
B. Pesahim 11b: בד"ה אחד.
B. Sanhedrin 41b: בד"ה אחד
B. Aboda Zara 34a: בד"ה מתענין.
Sefer ha-Yashar on Sabbath 34a.
Sefer Yereim on Sabbath.
Tosafot Rid on Sabbath.
Sefer Ravan, responsum 2.
We also find five miles in Rashi on B. Berakhot 2b: בד"ה מקדמי.
[33] R' Tam reads Rabbah instead of Rava in B. Pesahim.
[34] See following references: (non limitative)
Mordekhay on Sabbath: § 293 and in the remark.
Rosh on Ta'anit, first chapter, § 12.
[35] *Bayit Hadash* on Tor Orah Hayim 562. Bah attributes even five miles "*to the other Tana, following Rabbi Judah*!"
[36] See references note 21.
[37] R' Johanan could then perhaps be this other Tana following (not blindly) R' Judah.
[38] Respecting the condition dawn/day = 1/10
[39] Each mile is then 18 minutes. Therefore in B. Pesahim 11b, Tossafot בד"ה אחד 5miles = 5 * 18 = 90m = 1.5 hours and in B. Pesahim 2a בד"ה והא: 3 1/3 miles represent 3 1/3 * 18 = 60m = 1 hour. However the quotation in B. Menahot 20b Tossafot בד"ה נפסל: that there are 15 miles from noon till sunset and then 5 miles until the appearance of the stars remains a conundrum.
[40] This explanation was already given by Maharal of Prague on B. Shabbat 34b and by the Gaon of Vilna in his glosses on Shulhan Arukh O. H. 261.2.
See: R' Judah Loeb ben Bezalel: Novellae on tractates Shabbat, Eruvin and Pesahim, Lvov 1863.
[41] What the Talmud calls צאת הכוכבים.
[42] This can be inferred from the dictum of Ulla in B. Pesahim 93b where he says that a walker covers 40 miles in an (extended day): 5 miles during dawn and 5 miles during twilight and 30 miles during the day, 15 miles from the morning (sunrise) until the middle of the day and 15 miles from the middle of the day until the evening (sunset).

פשו לה תלתין, חמיסר מצפרא לפלגא דיומא וחמיסר מפלגא דיומא לאורתא

There is other Talmudic evidence:

שבחמש חמה במזרח, חצי שש וחצי שבע חמה עומדת בראש כל אדם ובשבע חמה במערב. פסחים צ"ד ע"א

Thus between 5h 30 and 6h 30 the sun is in everyone's zenith.
In B. Pesahim 12b: שית יומא בקרנתא קאי. At 6 hours (thus at the beginning of the 7th hour) the sun is in the meridian, at equal distance from the corners (from the east and the west). The explanation of this quotation is the subject of the responsum n° 134 of Maimonides, Blau Vol. 1, p. 251.
In B. Pesahim 58a the Mishna details the horary of the sacrifice of Minha. When the eve of Pessah was on Friday, the sacrifice was slaughtered at 6h 30 i.e. 0h;30m p.m., because, so said Rava:

אלא אמר רבא מצותו דתמיד משינטו צללי ערב, מאי טעמא, דאמר קרא בין הערבים מעידנא דמתחיל שמשא למערב

In B. Yoma 28b: if the eve of Pesah was on Friday the sacrifice of Minha was slaughtered at 6h 30, i.e. 0h; 30m p.m. when the walls become darker (because the sun is no more at the zenith).
Rashi writes, ad locum: ובתחילת שבע שעות שהחמה באמצע הרקיע בראש כל אדם.
R' Hananel writes, ad locum, משעה שביעית משיכנס השמש במערב.
Maimonides writes (Hilkhot Temidim u Musafim I:3:
תמיד של בין הערבים שוחטין אותו משיעריך הצל ויוכר לכל שהאריך והוא בשש שעות ומחצה ומעלה....
In Bereshit Rabbah 48:8 it writes:כחום היום, בשישה שעות.
[43] Let us consider Tosafot **אחד** in B. Pesahim 11b and Tosafot **אחד** in B. Sanhedrin 41 b, which present the evidence for the consideration by Tosafot of long temporal hours, the beginning of the second hour being before sunrise: הנץ החמה.
Generally we understand that this Tosafot is constructed on the basis of a day of 30 miles and an extended day of 40 miles. One mile is then 24 m equinoctial and 18 m temporal and therefore five miles represent 1.5 temporal hours and two equinoctial hours at the equinox. All the hours mentioned in this Tosafot are long temporal hours. If we consider now that this Tosafot follows the column "A" of Talmudic Metrology II, BDD 20, that we ascribed to Rabbi Johanan, then one mile is 18 m equinoctial or 14.4 m temporal and five miles represent 90 m equinoctial or 72 m temporal. This Tosafot can be explained under this assumption, but we must then explain that the mentioned 1.5 hours are equinoctial.
We can then summarize: there are many divergent opinions in France and Germany.

1) Rashi considers twilight of four miles and explicitly says that there are 32 miles between sunrise and sunset. The mile is then 22.5 m.
2) Tosafot, whenever twilight has a length of four miles, have the same opinion.
3) R' Tam in Sefer ha-Yashar objects from Rabbah to Rabbah and considers a mile of 24 m. This could be an arguing position but it would not represent his final position.
4) Other Rishonim who consider twilight of five miles can follow the position of Rabbi Johanan and column A. The mile is 18 m equinoctial or 14.4 m long temporary time. Twilight lasts 72 m long temporary time corresponding at the equinox to 90 m equinoctial.

Rij (Rabbi Isaac the Elder of Dampierre) in Tosafot on B. Pesahim 2a **והא קיימא לן** :says that the apparition of the stars occurs an hour, שעה גדולה, after the end of Bein ha-Shemashot, which in this passage has a length of 2/3 mile, according to Rav Joseph. According to Tosafot and case 2, this span of time is 3 1/3 mile.
We ascertain: {3 1/3}/40 * 12 = 1 long temporal hour
{ 31/3}/32 * 12 = 1.25 equinoctial hour
שעה גדולה, is then exactly a long temporal hour.
Ravan in his responsum 2 writes that the length of twilight is more than an hour. If we consider that Ravan follows the fourth method according to column A, then
5/40 * 12 = 1.5 equinoctial hour.
But 5/50 * 12 = 1.2 temporal hour. Ravan then writes correctly: יותר משעה אחד.
Note also that Ravan begins Sabbath according to Rabbi Judah (the rule of Rabbi Johanan) at sunset or slightly later according to another reading in his responsum, but he ends Sabbath according to Rabbi Jose five miles later than sunset i.e. 1.2 temporal hours later. As he considers the BHS of Rabbi Jose for all purposes other than the Sabbath, he can certainly consider long temporary hours. His Pelag ha-Minha would be 1.25 temporal hours before the end of twilight, i.e. 0.05 long temporal hours corresponding to 3 temporary minutes before sunset or, at the equinox, 3.75 equinoctial minutes before sunset.
[44] Indeed in the morning the eye, accustomed to the night, will consider as day and light what it considers in the evening as night and darkness. It is a purely physiological phenomenon. See also Y. Berakhot I, 1 the objection of Rabbi Hanina the friend of the Sages and the statements of Rabbi Abba, Rabbi Ba and Rabbi Jose berabbi Bon.
[45] See Y. Berakhot I:1, באחד בתקופת ניסן ובאחד בתקופת תשרי היום והלילה שווים
Leviticus Rabbah XXVI: 4:.....תניא באחד בתקופת ניסן ובאחד בתקופת תשרי היום והלילה שוין .
[46] See B. Pesahim 94a: חצי שש וחצי שבע חמה עומדת בראש כל אדם.
B. Pesahim 12b: שית, יומא בקרנתא קאי. In Pe'er ha-Dor n° 44, Maimonides comments on this passage.
Midrash Rabbah: Bereshit 48; 8: בששה שעות טולא ושמשא שריבין כחדא.....אמר רבי תנחומא, בשעה שאין לבריות צל תחתיו.
See Niddah 63b:ורבי יוסי מתיר מן המנחה ולמעלה .
רש"י. היינו משש שעות ולמעלה מכי ינטו צללי ערב
תוספות. פירש רש"י סוף שש שמאז נוטים צללי ערב ופ' רשב"ם דאף על גב דאמר בתמיד נשחט דזמן המנחה לכל המוקדם הוי משש שעות ומחצה ואילך היינו משום דלא קים לן שפיר בשיעור שעות היום אבל מן הדין הוא מתחילת ז '
See also B. Pesahim 58a, Rashi:
בין הערבים, משש שעות ולמעלה שהצל נוטה קרינן בין הערבים .
See B. Pesahim 93b, Rashi
ושעת שחיטה מחצות היום ואילך דבין הערבים כתיב ביה בתמיד ותמיד אע"פ שנשחט בשש ומחצה זמנו מן התורה מתחילת שעה שבע ואילך שמתחלת החמה להתעקם כלפי מערב .....

See B. Sabbat 9b, Rashi:
מנחה גדולה התחלת זמן תמיד של בין הערבים מכי מתחיל להעריב דהיינו מחצי שבע ולמעלה דהא חצי שש וחצי שבע חמה עומדת בראש כל אדם וקודרת כנגדה למטה ואינה נוטה לשום צד ומחצי שבע ולמעלה נוטה ברקיע למערב סמוך למנחה מהתחלת שבע ...
See also Magen Avraham on Orah Hayim 233 § 4 who writes that the end of the sixth hour must coincide with true noon.
[47] Today this method of counting the temporary hours is called "the method of the Magen Avraham" This denomination is quite strange because Magen Avraham describes both methods in O.H. 233 and does not decide in favor of one of them.
[48] With nevertheless the notable exceptions of R' Israel Isserlein (see Talmudic Metrology II in B.D.D. 20, *Ma'agalei Tsedek* by M. Kossover p 19, ha- Zemanim ba-Halakha p. 113), R' Mordekhai Jaffe (Levush) and Gra.
[49] The attribution of these different schemes will be examined in the following of the present paper. A striking example of this evolution is given by R' Moses Sofer in his responsum O.H. 80. Hatam Sofer ended Sabbath (at least for the problem of circumcision) earlier that we do today according to the Geonim. However he wrote that they followed R' Tam.
[50] The word שקיעה refers certainly to the sunset at the horizon as we find in Vayikra Rabba 31, 9:
ר' אושעיא בריה דר' שמלאי דקסרין בשם ר' יצחק זעירא, לעולם אין גלגל חמה שוקע עד שהוא נעשה כמין חרדל של דם.
It is the prefix מ which raises the issue.
[51] We have explained the passage of B. Sabbath according to the explanation of Nahmanides in Torat ha-Adam. This explanation is coherent and it is in accordance with a similar passage in Y. Berakhot I: 1, quoted in the next paragraph. It would imply that the BHS of Rabbi Nehemiah begins slightly after sunset.
Rashi, ad locum, has explained this passage differently. The man is on the shore and the time necessary to immerse himself and get out would represent the length of the BHS of Rabbi Nehemiah i.e. 0.5 miles. This explanation is difficult: first, it is in contradiction with the parallel passage in the Y. Berakhot. Second, the time necessary for this immersion seems shorter than 0.5 mile. Third, the span of time of 0.5 mile is clear, so why would it be necessary to give another definition of this time? Rashba, in his Tosafot on B. Berakhot, has proposed a similar explanation as Nahmanides, but he understands that the man, after immersion, not only gets out of the sea, but he climbs the Carmel again in order to reach his starting point. This explanation allows him to delay the beginning of the BHS of Rabbi Nehemiah, the משתשקע, in order to fit the theory of R' Tam.
[52] Like Teruma, evening prayer (Sages) or fasting limits. Teruma is the offering to the priest levied before the tithe to the Levite.
[53] The Y. Berakhot makes a clear distinction between הכסיפו which means to gray, or to fade, and השחירו which means to darken.
[54] The offering to the priests.
[55] This opinion is parallel to similar proposition in B. Sabbath 35a. But here it is clear that it aims at defining משתשקע , the beginning of BHS. This moment is slightly later than sunset.
[56] This is thus in contradiction with the theory of R' Tam who places the night and the three stars at the end of the crossing of the firmament. This is also in contradiction with R' Eliezer of Metz who places the appearance of the three stars at the beginning of the crossing. This is further in contradiction with the modern interpretation of R' Tam, according to which the three stars of R' Tam are the same as ours, but the crossing of R' Tam ends when the crossing of R' Eliezer of Metz begins.
[57] For reasons of symmetry, the beginning of the morning crossing must happen with the same solar depression as the end of the evening crossing.
But we want also that the span of time between the beginning of the morning crossing and the beginning of the afternoon crossing should be equal to the span of time between the beginning of the evening crossing and the beginning of the morning crossing. These two constraints imply that the morning crossing begins at the equinox at 5h 24m and ends at 6h 36m. Similarly the evening crossing must begin at 17h 24m and end at 18h 36m.
The morning crossing belongs to the day, the evening crossing belongs to the night.
R' Abraham Cohen Pimentel considers that the morning crossing must begin, at the equinox, at daybreak at 4h 48m and end at 6h (or more precisely, if we take into consideration the difference between the notions of true and apparent sunrise, from 4h 52m until 5h 56m) and the evening crossing must begin at sunset at 18h and end at 19h 12m (or more precisely if we take into consideration the difference between the notions of true and apparent sunset, from 18h 4m until 19h 16m) and therefore he must correct the text and write "if you don't attach the morning and the evening crossing to the night, then the day and the night are not equal at the equinox. But this new reading doesn't fit the discussion in progress.
[58] This text appears in three different places: Y. Berakhot I,1,2c; Y. Yoma III,2, 40b (in a different version) and in Bereshit Rabbah 50, 10. In the two last quotations the text mentions Rabbi Hanina. These quotations must be related to the dictum of Rabbi Hanina in B. Pesahim 93b where Rabbi Hanina refers also to the distance of Sodome to Tsoar, which is greater than 4 miles; but it was covered as if it was 4 miles because of the angels. The origin of the reading R' Hazna could be the result of the agglutination of the two letters נ י giving a צ . I thank the reviewer of the paper for this remark.
[59] See in Zohar on Shemot, p. 10a: אילת השחר. דכד נהירו אנפוי דמזרח ואתפרשא חשוכא דלילא, חד ממנא אית דסטר ומשיך חד חוטא דנהירו דסטר דרום. עד דאתי ונפיק שמשא ובקע באינון כוי רקיעא ואנהיר עלמא. וההוא חוטא אפריש חשוכא דלילא, כדין אילתא דשחרא אתי...

Thus Ayelet ha-Shahar corresponds to the definition of daybreak, by the apparition of the first ray of light and it precedes closely the lighting of the orient. The reference to this passage was given by R' David Hofman in Melamed le-Hoïl 30.
[60] In the dictum of Rabbi Hanina it speaks of: עד שיאיר המזרח. In B. Sabbath 34b it speaks of: פני מזרח. In Mishna Yoma III :1, in Mishna Tamid III :2, in B. Yoma 28b and, in B. Menahot 100a it speaks about פני כל המזרח. But the reading is not certain, there are other readings: פני המזרח, In Rambam's commentary on Mishna Yoma he speaks about פני המזרח instead of פני כל המזרח. This reading is more satisfactory but in Hilkhot Temidim u mussafim the reading is again פני כל המזרח. It seems however that the opinion of Rabbi Matathias (Mathias) ben Samuel aims at making sure that it is already really day. The additional precision « עד שבחברון » shows that at this moment only a little spherical segment of the eastern vault is already lightened, corresponding to a little lightened arc of the eastern horizon. But the entire eastern horizon is not yet lightened because it would imply that the whole eastern vault is lightened until the zenith; this happens much later when the civil dusk is reached, when the solar depression has diminished until about 6-6.5°. The moment when the lighting of the vault reaches Hebron would then be close to daybreak.
In Y. Erubin II,1,22c it writes that eht :stniop gniwollof eht neewteb desirpmoc noziroh latneiro eht fo cra eht si פני מזרח point of sunrise of the sun at summer solstice and the point of sunrise at winter solstice.
אמר רבי יוסא אם אין יודע לכוין את הרוחות צא ולמד מן התקופה. ממקום שהחמה זורחת באחד בתקופת תמוז עד מקום שהיא זורחת באחד בתקופת טבת **אלו פני מזרח .....**
The middle of this arc is the east. In Jerusalem this arc is comprised between the points of Azimuth – 118.0470° and – 61.9530. Its length is 56.0940°; it is less than the third of the eastern horizon. Thus פני מזרח is a little arc of the eastern horizon centered on the east point. At daybreak only one point of the eastern horizon is lightened but a few minutes later a part of this little arc is already lightened; this is the meaning of האיר פני מזרח or האיר כל פני מזרח. It is also not certain that in B. Yoma it represents the whole arc of 56°. It could also represent a littler arc centered on the east point. Its illumination is required to make sure that the process of sunrise really began. Therefore the difference between the first Tana and Mathias ben Samuel in Mishna Yoma III:1 and in Mishna Tamid III:2 would be of only a few minutes. Indeed when the whole eastern horizon is lightened then the zenith is also lightened and this corresponds to the civil twilight when the solar depression is 6°; this is certainly later that the moment defined by Mathias ben Samuel where it says that the vault is lightened only until the level of Hebron. Therefore the reading elbanoitseuq si פני כל המזרח, the readings ro המזרח פני במזרח כל פני would better fit because, at that moment, only a little arc of the eastern horizon is lightened and not the entire eastern horizon.
It is interesting to note the position of Maimonides about the beginning of the day.
In Hilkhot Kriyat shema I: 12," à posteriori, if he read Shema after daybreak he fulfilled his duty".
In Hilkhot Tefila III: 7, "If, in case of emergency, he prayed after daybreak he fulfilled his duty".
In Hilkhot Temidin u Mussafin I:2, " The slaughtering of the morning daily sacrifice is made before sunset when (all) the orient is lightened".
In Avodat Yom ha-Kipourim I:8, " The slaughtering could not begin before we know that it is daybreak, with certitude".
This last formulation proves that the time given by Mathias ben Samuel was not far away from daybreak and its purpose was to make sure that daybreak had effectively been reached. Therefore it seems that the correct reading in Hilkhot Temidin u Mussafin is ton tub משיאיר פני כל המזרחמשיאיר כל פני המזרח. It is even likely that it does not concern the whole arc of 56° but a littler arc and therefore the best reading would beמשיאיר פני המזרח. Maimonides ruled according to Rabbi Mathias ben Samuel only to make sure that daybreak occurred with certitude. Apparently, it is only for the Temple service that security was needed. For Tefila and Shema such delay of security is not needed. Therefore the assumption of R' David Hofman in *Melamed le-Hoïl* that the delay between the enlightening of פני מזרח and daybreak is only a short period of about 6 minutes seems very likely. R' David Hofman supposed that the assertion of R' Hanina was corrupt and he proposed the following reconstruction of the original dictum:
מאילת השחר עד הנץ החמה הם ד' מילין וכן מהאיר פני המזרח עד עלות השמש-עליית כל גוף השמש- הם ד' מילין.
This reconstruction would imply that the time between daybreak and the lighting of Pnei Mizrah is the same as the time between the beginning of apparent sunrise and the time of the complete true sunrise. This span of time would be around 6 minutes.
This would agree with Maimonides' statement in Hilkhot Kriyat Shema I; 10: ושיעור זה כמו עישור שעה קודם שתעלה השמש. The span of time between the beginning of apparent sunset and the complete and true sunrise is 6 minutes. This statement was never understood and is still a conundrum. I propose the following explanation.
At the equinox, Yom Kippur is not far from it, and neglecting the special topography of Jerusalem which delays slightly the times of sunrise, we can write:

daybreak:	4h; 44m	solar depression: 16.1°
האיר פני מזרח :	4h; 50m	solar depression: 14.8°
Beginning of the time of Shema :	5h; 04m	solar depression: 12°. (B. Cohn)
Beginning of the time of Shema :	5h; 22m	solar depression: 8°; 05'. (Raphael Hanover)
האיר פני כל המזרח :	5h; 32m	solar depression: 6°
Apparent sunrise:	5h; 56m	solar depression: 0°; 35' + 0°; 16' = 0°; 51'

End of apparent sunrise: 5h; 58m 31s solar depression: 0°; 35’ – 0° 16’ = 0° 19’
End of true sunrise: 6h;01m 15s solar depression: - 0°16’(without refraction)

I have already expressed the assumption that, according to Maimonides, the solar depression at the beginning of apparent sunrise, is 1° (see: J. Jean Ajdler. The equation of Time in Ancient Jewish Astronomy, pp. 23-24 , B.D.D. 16 and J. Jean Ajdler. Talmudical Metrology II, The Mile as a Measure of time, note 76, B.D.D. 20). This would imply that the horizontal refraction is 0°; 44’ instead of 0°; 35’. Under this assumption The beginning of apparent sunrise is at 5h; 55m 18s and the end of true sunrise is at 6h; 01m 15s. The span of time between the beginning of **apparent** sunrise and the end of **true** sunrise (without taking the refraction into account) would then be 5m 57s or 6m. This would be the meaning of the difficult passage of Hilkhot Kriyat Shema I: 10 and probably also of the dictum of Rabbi Hanina, according to the brilliant reconstruction proposed by R’ David Hofman.

[61] See an important table showing the evolution of the eastern and western sky around sunset in Benish, ha-Zemanim ba-Halakha p 408.

[62] Benish, P. Ha-zemanim ba-halakhah 1996, p. 407.

[63] The Geonim begin BHS at sunset. The basis of their opinion is a responsum of R’ Sherira and Hai Gaon. It was already quoted by R’ Moses Al-Ashkar in his responsum 96. The complete text of the responsum is to be found in ha-Zemanim ba-halakha, Benish 1996, p. 645. The great difficulty of this text is that it advocates a BHS beginning at sunset but it follows Rava and explains in many instances that the visible phenomena are to be found towards the west.

[64] R’ Tam in Sefer ha-Yashar. See infra.

[65] The rule is like Abaye in six cases known as יע"ל קג"ם. See B. Sanhedrin 27a, see also Rashi ad locum for references for the different cases.

[66] See R’ Hananel on B. Sabbat 23b: תנא ליה ההוא סבא, ובלבד שלא יקדים ובלבד שלא יאחר אלא בעת שקיעת החמה קודם כמעט תהיה ההדלקה.

[67] Rambam, Hilkhot Sabbath V : 4.

[68] It is generally accepted that Maimonides does not consider a Sabbath addition, תוספת . He nevertheless considers such an addition for Yom Kippur, see Hilkhot Shevitat ha- Assor I:6 and commentaries ad locum.

[69] Rambam, Hilkhot Terumot VII; 2 and Hikhot Kiddush ha-Hodesh XIV: 6. We see thus that the night begins about 20m after sunset, i.e.20m after sunset at the time of the equinox and about 20m after sunset during the year. These 20m must correspond to 0.75 miles = 18m corresponding to the BHS of Rabbi Judah and an additional 2m probably necessary to reach the BHS of Rabbi Jose.

[70] See J. Ajdler: The Equation of Time in Ancient Jewish Astronomy, B.D.D 16, August 2005, note 77.

[71] Rabbi Abraham ben David of Posquière (a suburb of Lunel in South of France) notes in B. Pesahim,see Ran, at the bottom of page 1a , in the beginning of the Rif, about the expression אור לארבע עשר that in the beginning of the night there is still light in the sky. Furthermore, he does not contradict Maimonides in Hilkhot Sabbath V:4; this implies agreement. Rabbi Abraham ben David and Maimonides had a good opinion of each other. Maimonides writes in his letter to R’ Samuel Ibn Tibbon: וכן כשבא אצלנו ר' מאיר החכם היקר שהיה למד אצל רבי אברהם בן רבי דוד, הרב הגדול מפושקיירש ואצל ר' יעקב הרב זצ"ל ואצל ר' אברהם בן עזרא נ"ע. Rabbi Abraham ben David, from his side, writes in his critical remark on Hilkhot Kilaiym VI: 2, חיי ראשי, לולא כי מלאכא גדולה עשה, באסיפתו דברי הגמרא והירושלמי והתוספתא... . As a side point, R’ Meir could most probably be Rabbi Meir ben Isaac of Trinquetaille (suburb of Arles in South France), father of R’ Nathan, Nahmanides’ teacher and grandfather of Estori ha-Parhi and R’ Jacob could probably be R’ Tam. The latter’s name is followed by the benediction of the deceases. Indeed R’ Tam lived only until 1171, much before R’ Abraham ben David and Maimonides who lived until 1198 and 1204. At the first glance, it seems strange that R’ Jacob Tam and Rabad would have a common pupil. Indeed it is well known that both were strong and exclusive personalities. Rabad could hardly tolerate the invasive influence of R’ Tam: see in Temim Deim 50, the last paragraph which, according to Urbach, in Baalei ha-Tosafot p.62, refers to R’ Tam. Nevertheless we know that a younger pupil of Rabad, R’ Abraham ben Nathan ha-Yarhi, was a pupil of both Rabad and R’ Isaac the Elder, from Dampierre. Therefore it is not impossible to imagine that R’ Meir, the greatest pupil of Rabad, stayed for a while under R’ Jacob Tam. We learn here that he paid a visit to Rambam in Cairo. Anyhow, Rambam seems to know about R’ Tam.

[72] 18h; 22m – 13.5m = 18h; 8.5m.

[73] The sun has at this moment a depression of 6°.

[74] 18h; 28m – 13.5m = 18h; 14.5m.

[75] Rabad teaches that in the case of a fast day falling on Friday one should end the fast at sunset, because sunset is the beginning time of the תוספת, the additional time one should add before the beginning of the Sabbath. This opinion is reported in the commentary Magid Mishneh, by R’ Vidal of Tolosa on Maimonides, Hilkhot Ta’aniot V: 5. This opinion of Rabad is also mentioned in Beit Joseph on Tor Orah Hayim 249. It is also mentioned in Beit ha-Behira of Meiri on B. Ta’anit at the end of chapter II. This would then imply that the BHS of Rabad begins slightly later than sunset. If he considers that three stars appear 20 minutes after sunset, his BHS of Rabbi Judah begins 6.5 minutes after sunset. The time allowed to the tosefet is then limited to 6.5 minutes.

[76] Ravan, Rabbi Eliezer ben Nathan of Mainz (c.1090-c.1170), author of the book Even ha-Ezer. His opinion is written in § 2. (see Or Meir Meir Posen, London 1973 p.26 and ha-Zemanim ba-Halakha, Benish 1996, p.374). Or Meir reads: שמדליקין בה את הנר היינו משקיעת החמה **ואילך** and concludes that BHS of Rabbi Judah, according to Ravan, begins slightly after sunset. Benish doesn't read ואילך and he ascertains that, according to Ravan, the BHS of Rabbi Judah begins at sunset.

[77] Rabbi Eliezer ben Samuel of Metz (c. 1115-c.1198) in Sefer Yereim. See infra.

[78] Abaye can follow Rabbah or Rav Joseph. It seems likely that he follows Rav Joseph and that BHS begins as soon as the western horizon fades and the reddish color disappears in the eastern horizon, marking the moment defined by the expression משתשקע. If this moment were sunset, it would be meaningless to watch the disappearing of the reddish color in the eastern horizon because such a determination is not very precise and the determination, towards the western horizon, of the moment of sunset would be more convenient. This supports that משתשקע is probably slightly later than sunset. Abaye seems thus to try to narrow the moment known as משתשקע ,marking the beginning of BHS of Rabbah, or to determine the beginning of the BHS of R' Joseph, when the eastern horizon begins to gray and fade. Apparently, Maimonides decides in Hilkhot Sabbath that BHS begins at sunset as a measure of security, because of the sanctity of Sabbath, but he accepts in Hilkhot Kiddush ha-Hodesh that after sunset, at the appearance of the first night star, as long as the second star is still invisible, it is still day. These rulings are in accordance with the ruling of R' Johanan in B. Sabbath that the rule is according to Rabbi Judah for the beginning of Sabbath and according to Rabbi Jose for other matters.

[79] According to the reading of R' Kafih and of Ernest Weill (Berlin 1894). The traditional text in the printed editions omits the two last words .וצאת הכוכבים I thank R' Y. G. Weiss for this information.

[80] Rabbi Levi ben Haviv concluded that the first part of this rule is related to the beginning of the evening following the 29th day, before the appearance of two stars and thus still belonging to the 29th day. He reached this conclusion by a comparison with the third quotation. His conclusion is thus a necessary conclusion, but Maharal ibn Haviv doesn't understand it clearly in the text. On the contrary, R' Simon Veltch in his commentary on Hilkhot Kiddush ha-Hodesh: Na'ava Kodesh, Berlin 1786 also gives the correct explanation, but he understands it by the syntax of the text itself. R' David Ibn Avi Zimra also raised the issue in responsa 1353, 1379 and 1442. His explanations are "pilpulic" and hardly acceptable.

[81] I haven't seen any commentator giving this plain explanation. Maharal ibn Haviv explained that for Kiddush ha-Hodesh the court could apply the strict ruling, while for laymen, on the eve of the Sabbath, one must be more cautious. Similarly R' Isaac Eizik Haver in Seder Zemanin, Warsaw 1844 and Jerusalem 5754, pp. 30-33 proposes an even more refined distinction between the two cases. Nevertheless the proposed explanation is not very different in its essence than that of Maharal ibn Haviv, but it is simple and genuine and it does not require two big folio pages.

[82] B. Sabbath 35a.

[83] The suggestion made by R' Kafih that ביאת השמש in the text of Maimonides is always the moment of the appearance of the second star, 15 minutes after sunset and that the third star appears 20m later, 35m after sunset is completely untenable. See his Mishneh Torah vol. IV, p 690.

[84] There are different allusions or mentions in the Talmudic and Midrashic literature of the cosmographical model of the Sages of Israel:

Y. Berakhot I: 1:בד"ה.מאן פליג רבי חנינא חברהון דרבנן (3b Vilna ed.)
Y. Rosh Hashanah II: 4 (12b Vilna ed.)
Pirkei de Rabbi Eliezer ha-Gadol VI
Exodus Rabbah Bo, chap 15, § 22.
Yalkut Shimoni Melakhim I, 186
Yalkut Shimoni Tehilim 673
Midrash Soher Tov 19

[85] This model is nevertheless at the origin of the formulation of the theory of R' Tam. R' Moses al-Askar has rejected the theory of R' Tam because it rests on a false model.

[86] This dictum is mentioned in the Guide of the Perplexed II: 8, a chapter dealing with the music of the spheres.

[87] This citation has been used in the sixteenth century by R' David Gans to illustrate the debate between the old astronomy of Ptolemy and the modern astronomy of Copernicus. This is indeed the terminology used by David Gans in chapter 25 (relating the meeting of David Gans with the great astronomer Tycho Brahe in his observatory) of the first part of his book נחמד ונאים, in a completely different debate: the confrontation between Ptolemaic astronomy and heliocentric astronomy. The use of this Talmudic dictum is only a pretext in order to raise the problem of the confrontation of ancient astronomy against "modern" astronomy and to make Jewish tradition appear triumphant by proclaiming right from the beginning that planets revolve around the sun. This adaptation and explanation of this dictum in B. Pesahim 94b is nevertheless completely out of its context; this was already noted by Solomon Munk in his notes on Moreh Nevuhim, The Guide of the Perplexed, II: 8, pp 77-78)

[88] Translucent is better than opaque: the vault is translucent: the sun, during the night, is sufficiently above the vault and cannot be seen but the stars are seen as soon as the darkness is sufficient.

[89] See note 84.

[90] pupil of R' Tam.
[91] Sefer Yereim, Vilna 1899.
[92] Rabbi Moses Cases, rabbi of Mantua from 1586 to 1617. He wrote treatises on the Mishna and the Talmud. See *Hidushei Rabbenu Moses Cases on Sabbath, Rosh ha-Shanah and Sukkah* by R' Eliahu Dov Pines, Jerusalem 1988.
[93] In order to account for the disappearance of the sun at sunset.
[94] During the time of the crossing of the firmament, an average person can walk 4 or 5 miles.
[95] But R' Hananel writes four miles.
[96] See B. Sabbath 34b.
[97] 4.25 miles corresponds to a thickness of the firmament of five miles. In the Talmud B. Pesahim, the thickness of the firmament is fixed at four miles, so the span of time between the beginning of sunset and the end of sunset is then 3.25 miles.
[98] Bah on O.H. 261 calls it, following Ram: גמר יציאתם.
[99] The contradiction, between B. Sabbath 34b and B. Pesahim 94b, is presented differently in Tosafot than in Sefer ha-Yashar. In Sefer ha-Yashar it is presented as a contradiction between two statements of Rabbah: the statement of Rabbah, in B. Pesahim 94a (and not Rava) according to the reading of R' Tam, according which the time necessary for the sun to cross the firmament is 1/6 of daylight and the statement of Rabbah, in the name of Rabbi Judah, according to which BHS is 0.75 mile.
On the contrary, in Tosafot (B. Sabbath 35a and in B. Pesahim 94b) the contradiction is presented between two statements of Rabbi Judah: Rabbi Judah in B. Pesahim says that twilight is four miles and in B. Sabbath he says that twilight is 0.75 mile.
In Sefer Ha-Yashar astronomical twilight is five miles, in Tosafot it is four miles.
[100] The cosmographical model of R' Tam was also accepted by his opponents; it is in fact the model of the sages of Israel. But the opponents of R' Tam place BHS at the beginning of the crossing of the firmament. The length of BHS of 0.75 miles has then nothing to do with the thickness of the celestial vault.
[101]We follow the version of Tossafot according which the thickness of the firmament is 4 miles.
[102] During the crossing by the sun of the firmament, during dawn and twilight, an average walker covers 4 miles.
[103] This is the argument of those who want to consider that R' Tam and Geonim refer to the same three night stars, i.e. the first night stars. Thus for both, R' Tam and Geonim BHS is 0.75 miles preceding the appearance of the third star. R' Y. G. Weiss writes "Unless we understand this "window" of 0.75 miles thickness before the moment when the zenith becomes dark enough for three stars to be seen, the model of R' Tam is untenable".
[104] See note n°46.
[105] Nehemiah belongs to Sefer Ezra; it is the 23rd of the 24 books. Referring to the same passage of Nehemiah IV:15, Maimonides writes in his commentary on Mishnah Megilah II:4 that we find in **Ezra** that he calls day the span of time between daybreak and the appearance of the first stars.
[106] It is then likely that the same thing occurs in the description of the model of R' Tam: the" beginning of sunset" would then be the appearance of the first three night stars and the "end of sunset would" be 3.25 or 4.25 miles later.
It would then be logical that for him the "beginning of sunset" = the appearance of the three first night stars and the end of the natural day correspond to ביאת אורו , while the "end of sunset" = the appearance of all the stars and the end of the crossing of the firmament correspond to ביאת שמשו . See Tosafot **דילמא** on B. Berakhot 2b.
[107] Ram is the current abbreviation for R' Eliezer of Metz.
[108] Bah in his old response n° 126 follows this terminology and identifies "sunset" with the appearance of the first three middle stars, 5 miles before the "end of their appearance". However in his commentary on Tor O.H. 261 he follows the traditional understanding of Ram and considers BHS of Ram before sunset, which is corresponding to the "beginning of sunset".
[109] R' Eliezer of Metz like R' Tam in *Sefer ha-Yashar*, speaks of a thickness of the firmament of 5 miles.
[110] See below.
[111] See also another approach whose conclusions are similar to those championed in this paper:
ואלטר, שי. שיטת ספר יראים. יודעי בינה, ד, תשס"ז עמ. 89- 115
[112] Those who want to understand in the text of *Yereim* that the BHS of R' Tam ends with the appearance of the first three stars cannot explain the passage: הופלג זמן וסימן התרנגולים מזו השיעור הרבה.
[113] Generations have considered that he places BHS before sunset because of the literality of the text:
וכן נראה לי עיקר דמשתשקע החמה הוא קודם שקיעת החמה. But the meaning of שקיעת החמה is here : just before the appearance of three middle stars. Indeed, R' Eliezer wrote already that at the appearance of the first middle star it is still day and only at the appearance of the second star there is a doubt whether the sun began already the crossing of the firmament. At the appearance of the third star we are sure that the sun has began the crossing of the firmament. It is then likely that BHS begins 0.75 miles before the appearance of the third middle star.
[114] Rabbi Simeon ben Nathaniel was a pupil of R' Tam.

[115] Sefer ha-Ittur Vilna, 1874 and Warsaw 1877.
[116] See B. Berakhot, ha-Maor ha-Katan, very beginning.
[117] B. Berakhot 2b, Tosafot דילמא. The text of Tosafot presents some difficulty.
[118] As we wrote earlier, it is likely that in the original theory of R' Tam, sunset represents the first appearance of three night stars and the end of natural daylight; hence ביאת אורו.
[119] Thus the Geonim understood ביאת אורו as the moment when the light has sufficiently diminished making any external activity impossible; it corresponds to the end of our civil twilight and corresponds to the apparition of the first stars. At this moment there is no more light on the earth but there is still light in the sky in the western part of the celestial vault. The Spanish rabbis following R' Tam understood now אורו ביאת as being at the end of the astronomical twilight when the last rays of the suns disappear.
[120] Apparently R' Joseph Caro seems to have understood the first paragraph of the Sefer ha-Ittur according to its plain meaning; BHS begins at sunset. Indeed from Shulhan Arukh Yoreh Deah 262. 5 and 6 we learn that at the appearance of the three first night stars marking the end of Sabbath there is still light in the sky.
R' Shneour Zalman of Liady considered that R' Joseph Caro changed his mind with regard to his former ruling in Orah Hayim 261. Bah and Magen Avraham do not agree with this change of mind and consider that R' Joseph Caro did not change his mind.
[121] Vol 2, Hilkhot Hanukah, p. 529.
[122] He learned under Rabad in Provence and afterward under R' Isaac of Dampierre and other Tosafists.
[123] It is thus impossible to understand, as some want to do, that it is the "last sunset" which is "sunset".
[124] R' Isaiah ben Mali De Trani (c.1200-c.1260). Tosafot Rid, Lemberg 1862-1868.
[125] The exact significance of this text is clear. Nevertheless, if we read only the two last sentences we could be confused and think that the "end of sunset" is when the sun disappears under the earth, i.e. our sunset.
[126] His apparent sunset could be slightly before horizontal sunset because of the mountains of eastern Italy.
[127] *Kitvei Rabbenu Moses ben Nahman*, R' Hayim Dov Shavel ed., Mossad ha-Rav Kook, Jerusalem 1964, Vol II, pp. 252-254.
[128] The commentators didn't wonder about this position. I would propose to connect it to a ruling of Rabbi Abraham ben David according to which, if one of the four fasts falls on Friday, one must not fast on Friday after sunset because at this time it is already the time of the Tosefet of Sabbath. See details and references in note 75. Nahmanides was a privileged "disciple" of Rabad through his teacher R' Nathan of Trinquetaille. The latter's father was the most important pupil of Rabad. This important point in the teaching of Ramban would then be the consequence of a teaching of Rabad.
[129] B. Pesahim 94b.
[130] B. Berakhot 26b.
[131] In the same way as Tosafot Rid in the last sentences of the above quotation.
[132] He doesn't mention him by name, but by his usual nickname: גדולי הדורות.
[133] Rabad, R' Abraham ben David considers explicitly that the period of Tosefet begins at sunset. See Magid Mishneh on Rambam, Hikhot Ta'aniot V:5 and Meiri's Beit ha-Behira on B. Ta'anit, end of chapter II. This opinion of Rabad is certainly the origin of the opinion of Nahmanides.
[134] See Shulhan Arukh O.H. 623, 2.
[135] Rashba works with 2/3 mile which is the length of the BHS of Rav Joseph. Generally, the rabbis adopted the value of 0.75 miles of Rabbah. I don't think that much importance must be given to this. We have even more flagrant cases. Rashi writes explicitly in B. Berakhot 2b and in B. Niddah 53a, that the BHS of Rabbi Judah is 0.5 mile! (the value of R' Hanina).
[136] The BHS of Rabbi Judah is in fact 0.75 mile. The assertion of Rashba that at Pelag ha-Minha the sun is still visible 1/6 mile on the earth is similar to the assertion of Nahmanides that at Pelag ha-Minha the sun is still 333 cubits on the earth. Because both Rashba and Nahmanides consider a span of time of four miles between sunset and the stars, we can write:
1.25 temporal hour – 4 miles = 1/6 miles.
Thus1.25 temporal hour = 4 1/6 mile.
From this relationship we deduce that 12 temporal hours = (12/1.25) * 4 1/6 = 40 miles.
Therefore 1 mile = 18m temporal.
Now 40 miles – 4 miles – 4 miles = 32 miles = 12 equinoctial hours = (32/40) * 12 = 9.6 temporal hours.
Thus 9.6 temporal hours = 12 equinoctial hours or 1 temporal hour = 1.25 equinoctial hour and therefore
1 mile = 18 temporal minutes = 22.5 equinoctial minutes.
Rashi has the same opinion: he writes explicitly in B. Pesahim 94a that 40 miles are walked between daybreak and the end of twilight and 32 miles are walked between sunrise and sunset. Therefore 1 mile = 18 temporal minute = 22.5 equinoctial minute. Tosafot seem to agree. All the medieval rabbinic authorities (except Maimonides) have always considered that 1 mile = 22.5 equinoctial minutes. The first authority which advocated the value of 18 minutes was R' Israel Isserlein, who was followed by R' Joseph Caro, R' Moses Isserels and R' Mordekhai Jaffe. Nevertheless, many later authorities until the

19th century including R' Moses Sofer, considered that it was a mistake and that these 18 m were temporal minutes corresponding to 22.5 m equinoctial minutes.
[137] See note 156.
[138] P. 1 at the bottom.
[139] Toldot Adam ve-Havah, part 1, road n° 12, p. 65a; Venice, 1553, Bragadine.
[140] Toldot Adam ve-Havah, part 1, road 18, p. 163b.
[141] Rabbi Hayim Or Zarua is an Austrian Rabbi, from the thirteenth century, the son of Rabbi Isaac of Vienna. For the complete reference, see note 145.
[142] See below.
[143]See below.
[144] Rabbi Yona Gerondi and Nahmanides completed their Provencal education at the Tosafists' school. Other pupils of the school of Rabad went also to Northern France and adopted the theory of R' Tam like R' Abraham ben Nathan ha-Yarhi.
[145] Tshuvot Maharah Or Zarua, ed. Menahem Abbittan, Jerusalem 2002: Responsum 185. In the responsum it writes that one can accept Sabbath from 10 hours on. It is likely that he considered short temporary hours, from sunrise to sunset, the only way to fit the practical conduct of the German rabbis.
[146] See Terumat ha-Deshen n° 1.
[147] Venise, 1565.
[148] The generally accepted understanding of the theory of R' Eliezer of Metz derives from responsum 96 of R' Moses Al-Ashkar and from Bah on Tor Orah Hayim 461.
[149] This is, according to my understanding of Ram, the meaning of משתשקע החמה , is "before the sunset" of Ulla, which is the appearance of the first three night stars. Note that R' Joel Sirkes wrote exactly the same in his responsum 126 (ancient responsa):
על פי דברי הר"א ממי"ץ בספר יראים..... שמפרש דהתחלת בין השמשות הוא ג' רביעית מיל קודם יציאות הכוכבים.....
[150] Tosafot B. Menahot 20b: נפסל.:*fasting after sunset until 4 or 5 miles later seems to be only a custom* and Tosafot B. Aboda Zara 34a : משמע דבשקיעת חמה סגי והוא ה' מילין קודם צאת הכוכבים: מתענים
[151] See Benish p. 375, § 9 and p. 573, § 3.
[152] See Tosafot B. Pesahim 2 a: והא. The span of time between the end of BHS and the stars is 3.25 mile and we ascertain that (3.25/40) * 12 = 0.975 temporary hour. Now R' Isaac speaks of a BHS of 2/3 mile. It is then possible that he made following calculation {(3 1/3)/40} * 12 = 1 temporal hour.
This Tosafot has already been used by R' Solomon Zalman of Liady in his Siddur to prove that Rij doesn't share the opinion of R' Tam; see Benish p. 371 note 22 and p. 375 note 36.
[153] This responsum is quoted in Or Zarua, Zitomir 1862, Hilkhot Mila § 102 p. 51. It is also quoted with slight differences in Tschuvot Maharam mi-Rothenburg, Prague 1608, §219. The position of Ri could then be the following: B.H.S of R' Judah begins immediately after sunset but the B.H.S. is much later four or five miles after sunset. He would then rule, following Rabbi Johanan's ruling, according to Rabbi Jose for the Sabbath's end but according to Rabbi Juda for the Sabbath's beginning, Mila and fasting. Similarly blood would become unsuitable at sunset. The mile would represent 18m according to column A or C of table 1 p. 13 of B.D.D. 20 (Talmudic Metrology II: The mile as a measure of time). He would then contradict absolutely R. Tam.
[154] Introducing a little Tosefet before the beginning of Bein ha-Shemashot.
[155] This is the point of view of Or Meir p. 26 and Benish in Ha-Zemanim ba-Halakha pp. 374-375. On the contrary R' J. G. Weiss considers that Ravan rejects the sugia of Sabbath in favor of the sugia in Berakhot Thus, according to him, Ravan begins Sabbath slightly after sunset, as a tosefet, an addition to the Sabbath but not as a safek, a doubt about a possible part of Sabbath. Thus according to this opinion, the BHS, the safek Sabbath would begin much later, near to the end of the 5 miles after sunset, near to the bein ha-shemashot of Rabbi Jose. I do not feel able to decide between them. The origin of this difference is the ambiguity of the text of Ravan. After explaining that BHS of Rabbi Judah is near to sunset he adds that "*we admit that it is day until tseit ha-kokhavim*." According to the first opinion one must understand: "although we consider on Friday evening the BHS of Rabbi Judah as an area of doubt between day and night, on Saturday afternoon we consider that all the span of time until the appearance of the stars, five miles after sunset, must be considered as day until the last moment where we place the BHS of Rabbi Jose. According to the second opinion one must understand that the opinion of Rabbi Judah in B. Sabbath is rejected and Ravan lights the candles at or slightly after sunset as a tosefet, an addition.
In Bah on O.H. 261(p. 21b, line 3) it writes that according to Ravan § 2, *BHS of Rabbi Judah begins at sunset* and, he adds, *at the end of this BHS, it is certainly night*. Apparently Bah agrees with the first opinion: on Friday evening the BHS of Rabbi Judah is a safek Sabbath.
[156] Benish p; 370 note 14, mentions an opinion that the Pelag ha-Minha of Maharam (R' Meir ben Barukh of Rothenburg) is close to sunset. This seems highly unlikely because all the sources accept that Maharam prayed on Friday evening Sabbath's prayer (Maariv) and came back home before sunset. Similarly he fasted until sunset. His BHS must begin at sunset, representing the beginning of the night. Therefore his Pelag ha-Minha must be 1.25 temporal hour before sunset.

This is the only way to account for his conduct. Furthermore, later German authorities, especially Maharil, who refers nearly exclusively to Maharam, write explicitly that Pelag ha-Minha is 1.25 temporal hour before sunset. Note that Maharam waited for sunset before making Kiddush to make sure that the candles and the meal were in honor of Sabbath. See Yossef Omets § 600. The remark of Benish finds its origin in *Sefer Orot Hayim* by R' Hayim Druk, Jerusalem 5730, p. 277; the author wants to ascribe to *Hagahot Maïmoniot* a Pelag ha-Minha slightly before sunset according to the theory of R' Tam. In fact there is here a incorrect understanding by R' Druk of the exact meaning of the *Hagahot Maïmoniot* on Hilkhot Tefila III: 6.
The text of Maimonides is the following:ותפילת נעילה זמנה כדי שישלים אותה סמוך לשקיעת החמה

Hagahot Maïmoniot writes:

ולי נראה דאין לדקדק דאית ליה לרב דתפילת נעילה בלילה מדאמר תפילת נעילה פוטרת של ערבית, שהרי גם תפילת ערבית סבירא ליה שזמנה ביום מפלג המנחה ואילך, כדאיתא בפרק תפילת השחר, דרב צלי של שבת בערב שבת ומדקדק מינה דס"ל כר' יהודה דאמר מפלג המנחה חשיב לילה לענין תפילה ע"ש. **ולכל היותר נוכל לדקדק לרב מהכא דזמן תפילת נעילה מפלג המנחה ואילך.** והיינו כדאיתא בירושלמי שתמשך עד שקיעת החמה כדפירישית ודלא כמקצת גדולי אשכנז שכתבו שאין תפילת נעילה אלא בלילה מהא דרב.....

Thus *" Rav prays the prayer of Arvit from Pelag ha-Minha on and therefore, all the most, we can deduce from it that the prayer of Neila, according to Rav, must be at least after Pelag ha-minha.This corresponds to what was taught in the Talmud of Jerusalem that this prayer may linger until sunset as I explained, and not as some German rabbis wrote that Ne'ila must be said during the night.*"

[157] This was his understanding of the passage in B; Sabbat 23b: ובלבד שלא יקדים ולא יאחר.

[158] *Sefer Tashbets* from R' Samson ben Isaac, New York 1970, p. 3 Hagahot R' Perets.
Tshuvot Pessakim u Minhagim from Rabbi Meir ben Barukh of Rothenburg. Jerusalem, Mossad ha-Rav Kook, 1957.

[159] *Tshuvot, Psakim u Minhagim*, part 1, n° 134, p. 126-130. See also note 156.

[160] Before sunset for Tosefet Shabbat: Sefer Raviah, ed Avigdor Aptovitser, Jerusalem 1964. Hilkhot Sabbath § 199 p; 267.

[161] Idem Hilkhot Ta'anit, § 858, p. 617.

[162] Idem Masekhet Pesahim §432, p. 64

[163] *Or Zarua*, Part. 2, Erev Sabbath, § 20 p. 10.

[164] Idem, Hilkhot Ta'anit, § 404, p. 164

[165] See Hilkhot Kriyat Shema 1 where he writes, according to R' Hananel, that all Israel is accustomed to pray Minha until the night according to Rabanan. Therefore one should not pray Arvit and Shema after 12 hours but only later when it is really night at the appearance of the stars. Thus 12h is certainly sunset but one must wait for the appearance of three stars.

[166] *She'elot u Tshuvot Maharil ha-Hadashot*, Jerusalem 1977; 45, § 4, p. 55. He writes explicitly that Pelag is 1.25 h before sunset. Moreover, this is evident because he even ate on Friday evening, after Kiddush, before sunset. We can deduce from it that his 12 temporary hours are counted between sunrise and sunset. This is still confirmed by his reference, in both the response (see note167), to the astronomers and their spherical astrolabe: he counts his temporary hours as the astronomers. This is the reason why I translated "two hours before the night" by "two hours before sunset". Indeed, R' Hayim Or Zarua accepts Sabbath at 10h of the day which is thus two hours before sunset as he counts certainly his temporary hours from sunrise till sunset like the German rabbis.

[167] Sheélot u Tshuvot Maharil ha-Yeshanot 152 (163), which is similar to the former.

[168] Responsum 185 of R' Hayim Or Zarua. He writes that one can accept Sabbath from 10h. He certainly counted his temporary hours from sunrise to sunset, like the surrounding population and the German rabbis. R' Jacob Weill (first half of the 15th century) writes explicitly in responsum 116 (*She'elot u Tshuvot Rabbi Jacob Weil*, Jerusalem 2001, p. 144) about those who accept Sabbath one or two hours before **sunset**.

[169] Idem; 45 p. 56.

[170] Although he writes in § 600:
איך יעלה על דעת אדם שלא לנהוג כמו הטור ושולחן ערוך דבכל מילי בתרייהו אנו גרירין לכן .....

[171] *Yossef Omets* Frankfort am Main, 1928; § 565.

[172] R' Meir ben Baruch of Rothenburg.

[173]In order to benefit from the candle's light. *Yossef Omets*; § 600.

[174] *Yossef Omets* : § 601.

[175] *Yossef Omets* §10.

[176] In *Yosef Omets* § 677 it writes that *some people, when they have a meal between Minha and Maariv, are used to ask a gentile woman to light, is incorrect. Shulhan Arukh allows this during Bein ha-Shemashot at the entrance of Sabbath in order to have oneg Shabbat but during Bein ha-Shemashot at the exit of Sabbath there is no great necessity: indeed before it becomes dark it is still possible to see a little, when it is completely dark we are already allowed to light, therefore we should not be lenient and allow the Shevut, the rabbinical forbidding to ask the gentile to perform the work of lighting for this short wile of a quarter of an hour between these two periods.*

We can infer from this text that the end of Sabbath occurs about a quarter of an hour after a more serious darkness, this corresponds to the first apparition of three middle stars as he requires in § 10.
Now R' J. G. Weiss wants to deduce from this paragraph that Yosef Omets follows the theory of R' Tam. Indeed he considers that Bein ha-Shemashot is a quarter of an hour before Tseit ha-Kohavim . This corresponds to the position of R' Tam, especially according to the modern interpretation. Nevertheless I do not think that this is a good argument. Even those, who consider that Bein ha-Shemashot begins at sunset, will at the end of Sabbath, reduce BHS to its true length because it would be a leniency to extend it until sunset. However, one can object to Yosef Omets, that taking into account that the halakha is like Rabbi Jose, the whole concept of Bein ha-Shemashot at the end of Sabbath, is questionable.
[177] This is verbatim the argument of *Terumat ha-Deshen* responsum 1. Again the calculation is very rough.
[178] Indeed he prays Minha before Pelag and Maariv after Pelag (§490) and he nevertheless makes Kiddush before sunset!
[179] In *Yosef Omets* § 722 it writes that the limit for eating Hamets on the eve of Passover is four temporary hours corresponding to one third of the day. When this day is long, the limit for eating hamets is about 9 a.m.
When the eve of Pessah is late, for example on April 24 in 1625 (eighth year of a cycle), then the solar declination is about 13° and in Frankfurt, latitude of 50.1 ° we find:
Daybreak (assumed depression of 12°) : 3h 30m Alot ha-Shahar
Night (assumed depression of 7°;05') : 19h 53m Tseit ha-Kokhavim
Length of the day :16h 23m
End of the fourth hour: 3h 30m + 5h 27m = 8h 57m ~ 9h.
The only way to get a limit of 9h for the end of the third of the day is to consider an extended day comprised between an early daybreak and an early night. On the same day, apparent sunrise is at 4h 51m and sunset is at 7h 09m leading to a day of 14h 18m and the end of the third of the day would be 9h 37m.
Thus, at the first glance, *Yosef Omets* must consider an extended and non symmetrical day.
If one considers Alot ha-Shahar 72m before apparent sunset and Tseit ha-Kokhavim 33m after apparent sunset we get:
A lot ha-Shahar : 4h 51m – 72m = 3h 39m
Tseit ha-Kokhavim :19h 09m + 33m =19h 42m
Length of the day : 16h 03m
End of the fourth hour: 3h 39m + 5h 21m = 9h.
If one considers Alot ha-Shahar 72m before apparent sunset and Tseit ha-Kokhavim 72m after apparent sunset we get:
A lot ha-Shahar : 4h 51m – 72m = 3h 39m
Tseit ha-Kokhavim : 19h 09m + 72m = 20h 21m
Length of the day : 16h 42m
End of the fourth hour: 3h 39m + 5h 34m = 9h 13m.
One could thus think that he counts his temporary hours for the limit of eating hamets on the basis of an early Alot ha-Shahar and an early Tseit ha-Kokhavim. But this is impossible because he writes explicitly that the time of Minha begins exactly at 12h 30, half an hour after true noon: תיכף אחר חצות, שזמנה ממש הוא בשש שעות ומחצה (§ 487).
Counting the hours of the day as proposed above, on the basis of a dissymmetric day, would give him for Minha Guedola 6h 30 of the day, at about 12h, half an hour to early; this seems impossible! The clock to which he refers, worked according to the system of "the little clock": 12 h was exactly at true noon. His day was necessarily symmetric, from sunrise to sunset.
Sunrise 4h 51m.
Sunset 19h 09m.
Length of the day 14h 18m.
Minha gedola 4h 51m + 7h 44m = 12h 35m but he considers half an equinoctial hour after noon.
End of the fourth hour 4h 51m + 4h 46m = 9h 37m.
The earliest limit for ending eating hamets was 9h 37m which he rounds off, for security to 9 a.m. Note that the half hour after true noon that is considered for Minha Gedola, is also considered by him in equinoctial time.
[180] Rabbi Joel Sirkes,(1561- 1640) in *Bayit Hadash* (Bah) on Tor Orah Hayim 261 (end).