Methodologies Used by Poskim to Determine the Orientation of the Synagogue.

Addenda et Corrigenda.

In the last issue of Hakirah (Spring 2011) Arye Shore wrote about the orientation of the synagogue and the prayer direction. I would like to address a few issues that he touches upon in his paper.

1. page 176 line 5.
The Jews of France and Germany had no problem in facing east…

This consideration results from Tossafot on Berakhot 30a ***letalpiot***. Apparently Tossafot had a rough geographical notion of the relative position of Israel. There are even good reasons to assume that they considered a flat earth: See J. Ajdler: The limits of Sabbath, B.D.D. n°24 Spring 2011.

2. page 177 line3.
The correction of the Levush was an outstanding contribution.

There is no reason to consider that Levush differed in any point with his teacher Rema, based on the ruling of R. Isaiah ben Elijah di Trani (R. Isaiah the younger). Bayit Hadash wrote that the words of Levush are already included in the words of R. Isaiah and in the glosses of Rema. In Perishah on Tur O.H. 94, the author recalls the ruling of Rema, he writes that he was more explicit in Darkei Moshe and that in the book of R. Mordehai Jaffe things are more detailed. All these authors, R. Lipman Heller included, did not remark any contradiction between Levush and his predecessors, who, certainly, could not know the rhumb line and necessarily considered only the great circle.

3. page 177 line 10.
Rabbi Eliahu ben Shmuel from Lublin.

Both the petitioner (sho'el) and the anthor (meshiv) refer to Delmedigo's Hukat ha-Shamayim page 137 and therefore the use of spherical trigonometry for the determination of the prayer direction makes sense. However it is not explicitly mentioned in the text. If this exegesis is correct, then we have another earlier reference to the problem of the great circle, dated from Adar 1712,[1] the date of publication of the response Yad Eliahu.

4. page 177 line 15.
Why the Levush adapted such a complicated system?

The procedure of Levush is indeed surprising for the modern reader. The text is relatively clear but the drawings are confusing and even the reconstructed drawings by the author of the article don't clarify the subject because of the absence of the representation of the windows. Actually Levush considers a rectangular synagogue with a window in the middle of the eastern wall. If the synagogue were perfectly oriented the luminous image of the window on

[1] And not 1710 as in the paper. This rabbi knew Sefer Elim but it is doubtful whether he mastered spherical trigonometry.

the western wall would be, on the day of the equinox at sunrise, is in the middle of the western wall.
The rectangle should rotate by about 15 to 30° so that the luminous image of the window will be exactly in the middle of the western wall on the day of the equinox at 6h 30m or 7h (local mean time). Without the representation of the window and its luminous image on the western wall, the drawings are without interest.
It seems farfetched to compare this procedure to the use of the camera obscura all the more the first edition of the astronomical works of R. Levi ben Gerson was printed only in 1560 in Riva di Trento, only 30 years, and not 100, before the issue of the Levush.
The procedure of the Levush recalls a similar procedure described by Ravia on the eve of Pessah in order to calculate the schedule of the eve of Pessah: See Sefer **Ravia**, edition Avigdor Aptowitser, Jerusalem 1964, Hilkhot Pesahim § 432 p. 64.

5. page 178 **The Great Circle**.

The author of the paper considers as granted, to such a point that it seems not necessary to elaborate and give any reference and justification, that R. Mordehai Jaffe did not consider a propagation of the prayer along a great circle or orthonome but along a rhumb line or loxodrome. The prayer would thus propagate along a loxodrome, a curve presenting in each point a curvature and torsion, as if we were shipping our prayers following the navigation ocean line of the seventeenth century.
The rhumb lines became known only after the publication in 1569 by Mercator of the maps designed according to the projection system of Mercator i.e. a system of cylindrical projection perpendicular to the rotation axis of the earth on a cylinder. It became the standard map projection for nautical purpose because of its ability to represent limes of constant course, known as rhumb lines, as straight segments. It is unlikely that R. Mordehai Jaffe knew about the rhumb lines when he wrote Levush.
The properties of the rhumb lines were studied in the 16th century by Pedro Nunes (Nonius) and in the beginning of the 17th century after the discovery of the logarithms by John Napier (1614).
An erroneous understanding of the maps led people astray to considering the rhumb lines as the shortest distance between two points because of their straight line representation on the map.
There is no reason to consider that Levush differed in any point from his teacher Rema, based on the ruling of R. Isaiah ben Elijah di Trani (R. Isaiah the younger). Bayit Hadash, Perishah and R. Lipman Heller included, did not remark any contradiction between Levush and his predecessors, who, certainly, could not know the rhumb line and necessarily considered only the great circle. Even R. Isaiah ben Elijah di Trani must refer to the great circle and cannot be suspected to consider a flat earth. Indeed his grandfather in Tossefot Rid on Sabbath described sunset according to the Ptolemy cosmography by contrast with R. Tam and R.Eliezer of Metz.
The only reference known to me, imputing the theory of the rhumb line for the orientation of the Synagogue, to Levush is a paper by Judah Herscovits in Yeshurun 3, Elul 5757: בענין לאיזה צד צריך להתפלל.
It seems more likely to accept a lack of precision and even an error in the demonstration of Levush than creating a new specious theory, which makes no sense, with the only purpose to fit with R. Jaffe's explanation.

6. Page 178 line13.
The earliest reference to the problem of the great circle.

Actually the first author to take exception to R. Joffe's reasoning was R. Solomon Aviad Sar Shalom Basilea (1680 – 1749)[2] in Sefer Emunat Hakhamim[3] Chapter 24, page 46b in the second edition of the book in Yohannisburg 1859.[4] He did not object the ruling of Rema but the erroneous reasoning of Levush. R. Jaffe had considered the difference of Latitude between the considered town and Jerusalem but he had neglected the effect of the difference of longitude. R. Basilea proved that in a town like Lisbon, the prayer direction is eastwards with a slight deviation to the north and not to the south.
R. Basilea presented in an appendix, written in Italian, a complete calculation of the prayer direction for Lisbon in order to give the necessary tool to anyone to perform this calculation. The calculation was performed on a modern way, using the analogies of Napier and the logarithms (1614). The only remark is the imprecision of the longitudes adopted by R. Basilea: 9° 10' for Lisbon, 63° 30' for Jerusalem and hence a difference of Longitude of 54° 20'. These values are compatible and only slightly better than the values of Sefer Elim (1629) and the Geographia of Ptolemy (2nd century). R. Basilea found a Prayer direction eastwards slightly deviated to the north, making an angle of 82° 20' (80° 29' after recomputation because of an imprecision of his tables) instead of 87° according to modern coordinates.

7. Page 178 line 15.
In Netzah Yisrael
This author, independently from R. Basilea, raised similar objections. It is interesting to note the following points: R. Israel Zamosc's calculation rests completely on Sefer Elim and appears primitive with regard of the calculation of R. Basilea. In Sefer Elim the author still ignores the analogies of Napier. This makes the resolution of the spherical triangles much harder and longer. The resolution must be made in two steps; one must solve two rectangular spherical triangles. In fact R. Zamosc considered three examples, Tunis Toledo and Bayonne (south of France) in order to prove that the prayer direction of these town is eastwards with a deviation northwards. However the calculations are performed with ancient coordinates values taken in Sefer Elim. With the correct coordinates, in the three cases, the prayer direction is eastwards with a deviation southwards when he wanted to demonstrate the contrary.

8. Page 178 line 23. **Tables in Sefer Elim** (1629).
Despite the statement of R. Zamosc, there is in Sefer Elim only a rough little table of sine and tangent given from degree to degree. Otherwise Delmedigo refers to a set of tables available at that time called "Canon Mathematicus"[5], "where all the sine, cosine and tangents are given"

9. Page 178 three lines from bottom.

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[2] R. Basilea was one of the important Rabbis of his time together with R. Isaac Lampronti and R. Samson Morpurgo. In 1733, R. Basilea was at the center of a forgotten incident that Jews should never forget. As he was making his regular visit to prison of Mantua on a Friday afternoon, he bent over to put some money in the alms box as he was used, a Christian hooligan painted a cross on his rear. As he left the prison he was mocked by the host. He retorted: "You should not laugh if you notice where the cross has been placed". His response so infuriated the Church authorities that he was thrown into prison and held for almost a year despite his failing health. Even after his release he remained under house arrest until 1739 and the Chief Rabbi of Mantua was restricted to the ghetto until his death (Simonsohn, p. 158, History of the Jews in the Duchy of Mantua, Jerusalem 1977 and Ruderman p. 227, Jewish Thought and Scientific Discovery in Early Modern Europe, Detroit 1995).

[3] Mantua 1730.

[4] The Public Library of New York restricts the access to the old editions when there are more recent editions available. Idem for Sefer Elim.

[5] See page 149 in the Odessa edition1864.

The actual problem of prayer direction to Yerushalayim is not discussed in Sefer Elim.

This is not true. Delmedigo (1591-1655) writes, incidentally, in Mayan Hatum, a part of Elim:

ומכאן תבין שמצב ההיכל שבבתי כנסיות אינו מכוון נגד ירושלים יפה ואינם מתפללים דרך ארצם שלדרים בצפון והם מערביים לא יפה עושים אותו נגד מזרח. ובעלי מקרא מדקדקים בזה מאד כנראה בספר האדרת להחכם אליהו בשיצי.[6] והוא ותלמידו החכם כלב אפינדרפולו[7] היו תוכנים תלמידי מהר"ר מרדכי כומטיאנו[8] איש כלול בכל חכמה כנראה בכמה חבורים שכתב למודיים וטבעים, גם כל ספרי הראב"ע פירש.[9]

Delmedigo adopted clearly the solution of the great circle. Remark that R. Heller wrote in Tossefot Yom Tov on Mishnah Berakhot I, 1 about Delmedigo in the most over polite terms: מצאתי לרופא מומחה וחכם כולל מהר"ר יוסף שלמה דלמדיגא מן קנדיאה בספרו, בחלק שממנו שקראו בשם גבורת …..Therefore the earliest reference to the problem of the great circle is in Delmdigo's Elim.

10. Page 178 last line. **Sefer Elim**.
Sefer Elim is an encyclopedic book about mathematics, physics and astronomy. Yessod Olam and Almagest are old stuff; Sefer Elim is resting on the latest scientific contributions, known by the author. In this book printed in 1629 Delmedigo mentions (but does not use) the logarithms known only since 1614: "Nowadays even a child can solve the problems of triangles…….by wonderful methods…….the sine and logarithmic tables".[10]

11. Page 179 line3.
R. Jacob Emden (1697 – 1776) raised again the problem in Mor u Ketsiah Altona 1761-68, glosses on Shulhan Arukh. In O.H n° 150 he recopied the objection of Sefer Emunat[11] Hakhamim and mentioned also the similar objection of the astronomer R. Israel Zamosc.

12. Page 179 line 5.
Basing himself on the Netzah Yisrael.

This is a pure conjecture.
It could also be Emunat Hakhamim but there is however a similitude between the calculation of R. Zamosc and R. Shneour Zalman: both don't calculate the direction of the prayer; they only calculate whether the prayer direction is north or south to the east. In fact his method is exactly the same as that of R' Zamosc.[12]

12. Page 179 note 13.

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6 Elijah Bashyazi , Andrinople-Constantinople, 1420-1490.

7 Caleb Afendopolo, Andrinople-Constantinople, 2nd half of the 15th century.

8 R. Mordehai Comitiano (1420- ~ 1487). His most important pupil was R. Elijah Mizrahi (~ 1450-1526).

9 P. 435 in the Odessa edition1864. After ending this letter I could consult the little book שימת עין מאת חיים יחיאל בארנשטיין, ורשה תרע"ג, thanks to R' Samuel Pinson of Brussels. Borenstein saw the book of Bashiazi (ענין תפלה פרק ג') and he noted that his calculations were primitive and assimilated spherical triangles to planar triangles.

10 P. 151 in the Odessa edition1864.

11 The agreement of R. Jacob Emden is noteworthy because in Mitpahat Sefarim R. Emden wrote a refutation of Sefer Emunat Hakhamim.

12 See Simat Ayin, Borenstein 1913. This book can be consulted at http://www.daat.ac.il/daat/vl/tohen.asp?id=521. I thank R. Shmuel Pinson, Rabbi of the Jewish Community Ma'ale in Brussels for this information.

Indeed the prayer direction in New York is north-eastwards, about 54.2° from the north and 35.8° from the east.
This prayer direction following the great circle joining New York to Jerusalem seems to be the only valid solution. It is very likely that the alternative solution, i.e. 95° from the north or 5° south of the east direction, was adopted in some cases because of an erroneous interpretation of the maps representation and the belief that the straight line of the map represents the shortest distance. This solution is less shocking for the common sense. However, as we showed, most if not all authorities followed the first solution. *Lehavdil*, Muslims faced the same problem. The first mosques established in North-America were oriented according to the quibla following the rhumb line, probably because of the incorrect understanding of the maps. Today all the mosques are oriented with a quiblah following the great circle theory.
A very nice graphical representation can be visualized on http://kosherjava/zmanim project/bearing-to-yerushalayim-and-zmanim-map/

The synagogues in Manhattan and Brooklyn are generally oriented according to the disposition of the streets and avenues. Therefore many synagogues have a prayer direction toward the south the north or even the west. Even a circular synagogue like the Lincoln Square synagogue in Upper west Side, which had the opportunity to establish a correct orientation, has a prayer direction toward the south instead of a north east direction: 54.2° from the north as it was adopted in the new synagogue of Lakewood.

13. Page 180 7 lines from bottom.
And the distance from prime meridian is 17 degrees west (French meridian).

17 degrees is a misprint that a careful reading allows correcting. Indeed a little further we read explicitly 12° and the detail calculation 66° - 12° = 54° confirms it.
West: this is a crude mistake resulting from the erroneous conviction that the prime meridian was the meridian of Paris. If this were the case the distance in longitude between Jerusalem and Bayonne would be 66 + 12 = 78° and not 66° – 12° = 54° as adopted by R. Zamosc.
French Meridian: This is also a crude mistake. The longitude of Jerusalem is 35.2° and the difference of longitude between Jerusalem and Paris is 32.9°. R. Zamosc considered a longitude of Jerusalem of 66°. We won't suspect him of such a mistake! The truth is that R. Zamosc followed the data of Sefer Elim who was still much influenced by the values given by Ptolemy in his Geography.[13] The longitude of Jerusalem was 66° with regard of a prime meridian passing through the Fortunate Islands.[14] It represented the limit of the inhabited world before the discovery of America.
Apparently R. Delmedigo, Basilea and Zamosc still referred to the same prime meridian, that of Ptolemy.
Therefore on page 186 the § about the Paris Meridian is without any utility.

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[13] Thus from the time of Ptolemy onwards until the beginning of the 18th century, the origin of the longitudes or first meridian was the meridian passing through the Fortunate Islands. Important geographical concepts used in rabbinical writings of the 12th century were related to this prime meridian: קצה המערב or western end, corresponded to this prime meridian, טיבור הארץ, the center of the inhabited world, corresponded to 24° east of Jerusalem and קצה המזרח corresponded to 114° east of Jerusalem. The ancients overestimated the distance of Jerusalem to the western extremity and underestimated its distance to the eastern end.

[14] Today the Canary Islands (Las Palmas), about 18° West. Note that 35.2° + 18° = 53.2° to compare with 66° given by Ptolemy, Elim and R. Zamosc and 63° 30' adopted by R. Basilea. In the book Luhot ha-Ibbur part I by R' Raphael ha-Levi from Hanover, Leiden 1756, the longitudes are already given with a very high precision but this was not the case in Tekhunot ha-Shamyim of the same author, Amsterdam 1756, which was the transcription of a manuscript written in 1734.

14. Page 185 fig5.

The remark: **And the distance from prime meridian is 17 degrees west (French meridian)**: see above.

About the figure 5: This figure is incomprehensible for the reader and one does not see its interest.
If we present the figure slightly differently it becomes familiar and we can easily explain the method of Zamosc.
R. Zamosc did not calculate the prayer direction, which is actually the important data that we research; but so did also R. Shneour Zalman proceed but his procedure remains involved and even problematic. R. Zamosc calculated only whether the direction is north of the east or south of the east and he made this calculation on a very astute manner.
The point B is the pole of the meridian ACN passing through Bayonne, C. The length of AC is 42°, the angle B is also 42°. ACB is a spherical triangle rectangular in A and C.
Now in the spherical triangle A'C'B rectangle in A', tang b' = tang B * sin c' where B = 42° and c' = c – 54° = 90° - 54° = 36° Thus tang b'= tang 42° * sin 36°.
Hence b'= 27.89° = 27° 53' < 32° and b' < latitude of Jerusalem. Thus the point representative of Jerusalem on the meridian A'C'N of Jerusalem is between C' and N. The great circle passing through Bayonne and perpendicular to the meridian passes under Jerusalem.

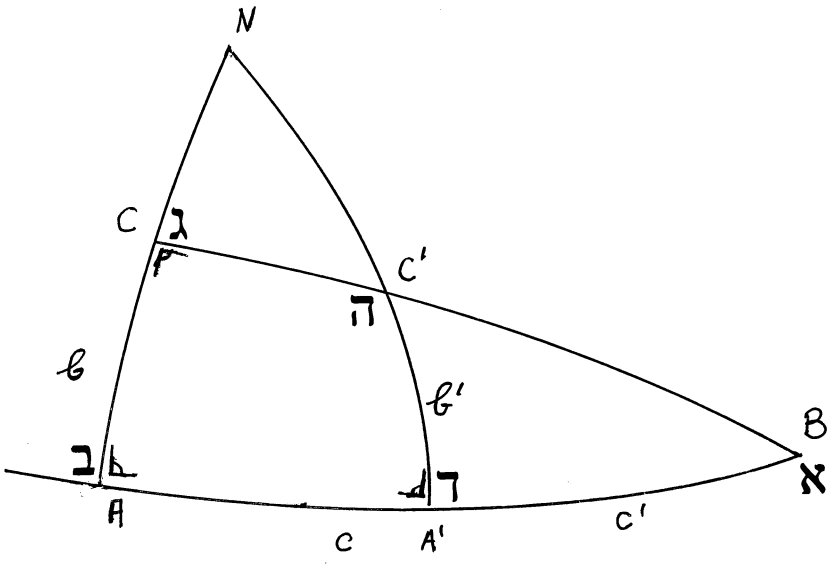

Figure 1: Method of R. Israel Zamosc. The Hebrew letters are the same as in the printed drawing and the Latin letters were chosen in order to have the angles A and A' right in order to use the classical formulas. N is the North pole, C is Bayonne, NCA is the meridian of Bayonne, NC'A' is the meridian of Jerusalem, AA'B is an arc of 90° of the equator, B is the pole of the meridian of Bayonne and CC'B is a great circle perpendicular to the meridian of Bayonne. The angles A, C, A' are right angles; b = 42°, N = 42°, c = 90°. AA' is the difference of longitude i.e. 54° and c' = 36°.

Conclusion. The great circle passing through Jerusalem and Bayonne is above the great circle CC'B and the prayer direction in Bayonne is north of the east direction. If Jerusalem was exactly on C', the prayer direction would be the east direction and if Jerusalem was between A' and C' the prayer direction would be south of the east.
Jerusalem would be in C' if sin c' = tang 32° / tang 42 or if c' = 43.95° and $\Delta\lambda = 46.05°$.
Thus, at the latitude of Bayonne: if $\Delta\lambda < 46.05°$ the prayer direction is south of the east.
if $\Delta\lambda = 46.05°$ the prayer direction is exactly the east
if $\Delta\lambda > 46.05°$ the prayer direction is north of the east
In fact the difference of longitude between Bayonne and Jerusalem is only about 36.6°, b' is given by tang b' = tang 42° * sin 53.40° hence b' = 35.86° > 32°. The conclusion is reversed, Jerusalem is between A' and C' and the prayer direction is south of the east.

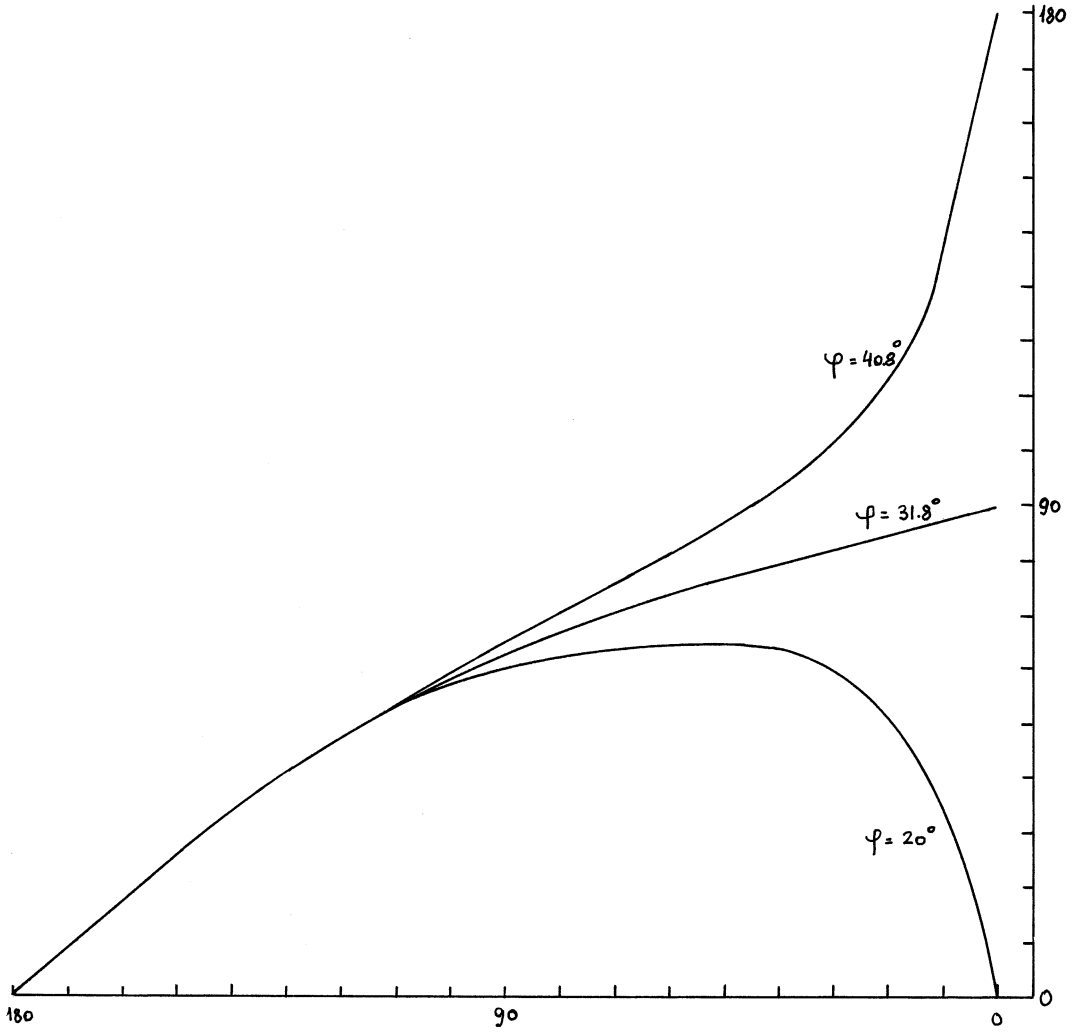

Figure 2. Prayer direction: angle between the north direction and the direction of Jerusalem for towns situated to the west of Jerusalem, in function of the difference of longitude for three different latitudes, φ = 40.8° (as New York), φ = 31.8° (as Jerusalem) and φ = 20° (similar to Mexico). Levush had considered Central Europe i.e. a difference of longitude between about 10° and 30° till maximum 35°. This explains why he did not mention or apprehend that the prayer direction gets a northern component for a greater difference of longitude and why for the latitude of 31.8° he considered that the prayer direction is to the East.