J.J. Ajdler: Luhot ha-Ibbur II, BDD n° 30, pp. 7 – 48.
Additional considerations about the Molad and the mean conjunction.

We explained in the referred paper that for Hanover, the exact value of the astronomical parameters used by Maimonides could be deduced from the data that he gave for a span of time of 10000 days. On this basis, Hanover put in evidence a difference of 0.635s between the length of the average Jewish month 29 – 12 – 793 and the length of the mean astronomical lunation. Departing from this principle, Hanover considered that the difference between the molad and the astronomical mean conjunction is the result of the accumulation of this little difference of 0.635s. As soon as we know exactly the length of the span of time between these two different moments at the beginning of a Jewish month, we can calculate when the molad and the mean conjunction coincided. The molad is thus a rough estimation of a mean conjunction. The span of time between them increases linearly with the time. Hanover adopted Maimonides' epoch on Wednesday evening, 22 March 1178 at 6h 20m p.m. JMT[1] or 3 Nissan 4938 at 5 – 0 – 360. From Maimonides' radices[2] we deduced that the mean conjunction occurred (2d – 414.82hal) before the epoch, hence at 3 – 0 – 775
The molad 3 – 1 – 721 followed the mean conjunction by 1026 hal. This is the basis of his calculation of the *ytronot* (the differences between molad and mean conjunction) used in his tables. It is of course the proof of his genius and of his exceptional calculation ability.

However, the examination and the study of the astronomical books of the ancients, Ptolemy's Almagest and al-Battani's Opus Astronomicum, show that Hanover's assumption, while fully justified in Luhot ha-Ibbur I, BDD n° 29 with regard to modern astronomy, is incorrect with regard to Maimonides. There is in fact no difference between the length of the Jewish month and the length of the mean astronomical lunation. Indeed

- The mean conjunction tables of Ptolemy and al-Battani are constructed on the basis of a mean lunation of 29 – 12 – 793.
- Maimonides' epoch, radices, astronomical parameters and tables rely completely on al-Battani's astronomy. This was demonstrated and detailed in my paper the equation of Time in ancient Jewish astronomy, BDD n° 16 pp. 5 – 56.

In the mean conjunction tables of Ptolemy and al-Battani (for the latter, the tables constructed in Egyptian years of 365 days) we note that the mean conjunction shifts in 500 Egyptian years with 55' 42", which corresponds to a fraction 0.92833333 of a day. For Example in al-Battani's table we note[3]
Year 990 22d 6' 23"
Year 1490 21d 10' 41"
Now 500 Egyptian years = 182500 days ~6180.0314 lunations.
Thus more exactly: 182500 d – 0.9283333 d = 6180 lunations.
Thus Ptolemy and al-Battani lunation = 29.5305941209 d.
The Jewish lunation = 29.530594135804 d.

[1] Jerusalem (modern = Flamsteed) mean time. This is of course anachronistic. Hanover forgot that this mean time was known for only hundred years.
[2] See J. Jean Ajdler: The equation of time in ancient Jewish astronomy, BDD n° 16, pp. 5 – 56.
J. Jean Ajdler: Comment on Ariel Cohen's article, BDD n° 27, pp. 93 – 100.
J. Jean Ajdler: Luhot ha-Ibbur II, BDD n° 30 pp. 7 – 48.
[3] Ptolemy and al-Battani begin the day at noon. The day of 24 hours is divided in 60' and 3600''. Each ' is 24 m and each " is 0.40 m = 24 s = 7.2 hal.

The difference is 1.49 10^ -8 d. ~The difference accumulated during the 61069 months between *Beharad* and the molad of Nissan 4938 is only 1.31 minutes. It is negligible. Therefore the span of time between the molad and the corresponding mean conjunction is a constant; a constant in the tables of Ptolemy and another constant in those of al-Battani. This last constant should also be valid for the tables of Maimonides.

The dependence of Maimonides on al-Battani is evident when we consider the way on which Maimonides calculated his epoch, radices and the variation of his astronomical parameters. Here is an additional demonstration based on the calculation of the mean conjunction of 20 March 1178. This year corresponds in al-Battani's tables, to the year 1489[4] of the era of Dhu'l qarnayn.[5]
According to al-Battani's tables of the mean conjunction in Roman years:

1479	10d	30'	24"
10	9	47	37

1489 March 20 18' 1"
This conjunction occurred on Monday March 20, at 7h 223.2 hal p.m. or (3) – 1 – 223 in ar-Raqqah. The molad of Nissan 4938 was (3) – 1 – 721.
molad–conjunction of al-Battani (ar-Raqqah): [(3) – 1 – 721] – [(3) – 1 – 223] = 498 hal.

Acccording to al-Battani the longitude of ar-Raqqah is 73°; 15'
Jerusalem is 66°; 30'
The difference is 6°; 45' corresponding to 27m = 486 hal. The mean conjunction in Jerusalem was thus [(3) – 0 – 223] – 486 = (3) – 0 – 817.

The mean conjunction occurred (2d – 414.82hal) before the epoch. The epoch is thus [(3) – 0 – 817] + (2d – 414.82hal) = (5) – 0 – 402 on Wednesday evening 22 March 1178, at 6h 22m 20s p.m. Formerly, in BDD 19 we had found that Maimonides' epoch followed mean noon (according to the principle of determination of mean time of al-Battani) in ar-Raqqah by a span of time between 6h 49m and 6h 50m. We adopted the interval of 6h 49m and adopted the epoch at 6h 49m p.m. ABaRMT or at 6h 22m p.m. ABJMT[6] or 6h 38.5m JMT[7]. The coincidence is perfect.

According to modern astronomy the mean conjunction of Monday 20 March, 1178 CE was on JD 2151401.2017 TD or Monday 20 March 1178 at 16h 50m 24s Dynamic Time[8] or 16h 33m 28s UT. Indeed in 1178 CE, ΔT was about 17m.

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[4] Year 1DQ = –310 CE and therefore 1489 DQ = 1178 CE.
[5] Its epoch is 0 March –310, corresponding to Nissan 3450, 6 months after the epoch of *Minian Shtarot.* We can also write: 3450 AM1= 1SE and therefore 4938 = 1489SE.
[6] Al-Battani Jerusalem mean time.
[7] Jerusalem modern mean time.
[8] The Universal Time UT, or Greenwich Civil Time, is based on the rotation of the earth. The UT is necessary for civil life and for the astronomical calculations where local hour angles are involved.
However the earth's rotation is generally slowing down and moreover, this occurs with unpredictable irregularities. For this reason UT, is not a uniform time. This fact was unknown to the Ancients.
The astronomers need a uniform time for their calculations. From 1960 to 1983, they used the *Ephemeris Time* ET and defined it by the laws of the dynamics. In 1984 the ET was replaced by the *Dynamical Time*. Today it is defined by atomic clocks. One can consider that the dynamical Time TD is, in fact, a prolongation of the Ephemeris Time. The relation is: UT = TD – ΔT. The exact value of ΔT can be deduced only from the records of ancient observations. From 2000 onwards the TD, Dynamical Time, is replaced by TT, the terrestrial Time.

The table of al-Battani was written in 880 CE when ΔT[9] was about 36m but in 1178 CE ΔT was about 17m. Therefore the data of al-Battani's table for the year 1178 CE must be corrected by 36 – 17 = 19m. We must furthermore add 16.44m[10] in order to get modern mean time. Therefore the mean conjunction of March 20, 1178 CE, according to al-Battani, expressed in Greenwich modern mean time is:
19h 12.4m[11] + 16.44m – 19m – 2h 36.2m = 16h 33.64m UT.
The correspondence is perfect.

The Molad was derived from the first conjunction of Ptolemy.

Slonimski, Hayim Selig (1810 – 1904) made an exceptional discovery about the meaning of the molad.[12] He noted the dependence of the molad on the mean conjunction of Ptolemy.
1. In the table of conjunction of Ptolemy, the first conjunction occurred on Toth 24; 44' 17"of the first year of the era of Nabonassar.
Toth 1 of the first year of the era of Nabonassar began at noon of Friday 26 February – 746 of the proleptic Julian calendar and therefore Toth 24 began on Friday 21 March at noon and the first conjunction was on Saturday 22 March – 746 at 5h 42m 48s a.m. in Alexandria or Nissan 3014 at (7) – 11 – 770.4 according to the Jewish notation.

2. The first year of Nabonassar corresponds to the Jewish year 3014 AM1.
The number of months between Beharad and the beginning of that year (in the proleptic Jewish calendar) is given by
Ft= Int [(235 * 3013 + 1) / 19] = 37266. The molad Nissan 3014 was then
Mol = [31524 + (37266+6) * 39673] $_{181440}$= 169020 = 6 * 25920 + 12 *1080 + 540 = (7) – 12 – 540 or Saturday morning at 6h 30m.

According to Ptolemy the longitude of Alexandria (where he lived) was 60°; 30' and the longitude of Jerusalem was 66°, hence a difference of 5°; 30' = 22m = 396 hal.
The conjunction of Ptolemy was thus (7) – 11 – 770 + 396 = (7) – 12 – 86.

3. The number of months between the molad Vayad and the molad Nissan 3014 was 37272 – 12 = 37260. During this span of time the molad shifted by 24300 hal = 22.5 hours.
Thus if the molad Nissan 3014 is Mol, then the molad of year 2 AM1 is Mol – 22.5h or [(7) – 12 – 540] – [0 – 22 – 540] = (6) – 14 – 0.

It is thus clear that the first Meabrim wanted to have a principal molad in hours without *halakim* and therefore they rounded of the molad of Nissan 3014 and in the same time they got a main molad in hours. They added 850 hal to the first conjunction of Ptolemy, 396 hal correspond to the difference of longitude and 454 hal were added in order to round off and get a principal represented by a molad in hours without halakim.[13] It is clear that the molad was derived from the first conjunction of the table of Ptolemy.

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[9] See Meeus: Astronomical Algorithms, Willman-Bell 1991, chapter 9: Dynamical Time and Universal Time. In the second edition of 1998, chapter 10 p. 77.
[10] See BDD n° 16 p. 14.
[11] (3) – 1 – 223 as seen above.
[12] Yessodei ha-Ibbur, Slonimski, Zitomir 1865 (2nd edition) p. 49.
[13] It is likely that it is at this point that the dispute between Ben Meir and Sa'adia Gaon found its origin. It seems that the Palestinians, instead of adding 454 hal, subtracted 188 hal and adopted the molad (7) – 11 – 978 for Nissan 3014. The molad of Tishri 3014 was then (5) – 7 – 540 and the molad of Nissan preceding Vayad was (4)

Comparison of the mean conjunctions of al-Battani with those of Ptolemy.

The conjunction of 16 April 603 CE.

The first entry of al-Battani's table of mean conjunctions calculated in Egyptian years is the year 915 of Dhu'l qarnayn and the first conjunction occurred on Toth 22d 14' 44". This year is 915 + 436 = 1351 of Nabonassar. 1 Toth of that year corresponds to 1448638 + 1350 * 365 = 1941388 JD which corresponds to 26 March 603 C.E. or 26 Adhar 914 DQ[14] (al-Battani's Roman years). The first day of the era of Nabonassar is indeed JD 1448638. The mean conjunction occurred on Toth 22, 1351 Nabonassar or April 16, 603 CE, which al-Battani notes 16 Nissan 914 SE (al-Battani's Roman years).

al- Battani in ar-Raqqah, longitude 39.05°.
Using the tables of mean conjunction in Roman years, we find:

903	18d	33'	15"	p. 84
11 years	28	41	39	p. 86
914 March	47d	14'	44"	
or April	16d	14'	44"	

The conjunction occurred on 16 April 603 at 5h 53m 36s p.m.

Ptolemy in Alexandria, longitude 29.9°.

The last year considered in the table of Ptolemy is 1101 Nabonassar. But in 250 vague years, the moment of the mean conjunction of Ptolemy shifts with – 0d 27' 51", we can establish the following table:

Ptolemy in Alexandria	conjunction on 22 Toth 1101	22^d 41' 45" Toomer p. 278.
	Shift in 250 years	- 0^d 27' 51"
	Conjunction in 1351 of Nabonassar:	22^d 13' 54" Toth.

According to al-Battani, the difference of longitude between ar-Raqqah and Alexandria is 73°15° – 60° 30' = 12° 45' corresponding to 51m.[15]

We have thus in Alexandria: al-Battani: 17h 53m 36s – 51m = 17h 02m 36s.
Ptolemy: = 17h 33m 36s.

The conjunction occurred according to al-Battani 31m before Ptolemy.[16]

We observe a good correspondence of the moments of mean conjunctions between the tables of Ptolemy and al-Battani and we can consider al-Battani's table as the prolongation of that of Ptolemy, with, however, a discontinuity at the junction. We note a jump of – 20 minutes between the mean conjunction Ptolemy in Alexandria and the mean conjunction of al-Battani in ar-Raqqah or a jump of 31m between the mean conjunctions considered in the same place; the mean conjunction of Ptolemy following that of al-Battani by 299 hal.

– 9 instead of (4) – 9 – 642. The molad of Palestine was 642 hal less than our molad. See Jaffe, *Korot Heshbon ha-Ibbur*, 1931 pp. 109 – 110.

[14] 603 – (– 310) +1 = 914

[15] In fact the difference of longitude between ar-Raqqah and Alexandria is 39.05° – 29.9° = 9.15° corresponding to 36m 36s.

[16] The molad of Ben Meir was 642 hal before the molad of Babylonia (our molad). If Ben Meir had known the book of al-Battani, but it is not proved, this result would have convinced him of the truth of his tradition.

The corresponding molad was the molad of Iyar 4363: (4) – 0 – 378.
molad–conjunction of Ptolemy (Alexandria): [(4) – 0 – 378] – [(3) – 23 – 604.8] = 853.2 hal
molad–conjunction of al-Battani (ar-Raqqah): [(4) – 0 – 378] – [(3) – 23 – 964.8] = 493.2hal

Comparison
If we want to compare Ptolemy and al-Battani with modern data we must consider that both Ptolemy and al-Battani were not aware of the irregularities of their mean time due to the irregularities of the earth's rotation.
The table of Ptolemy was written in about 150 CE when ΔT was about 135m but in 603 CE, ΔT was about 64m. Therefore the data of Ptolemy's table for the year 603 CE must be corrected by 135 – 64 = 71m. We must furthermore add 17.57m in order to get a modern mean time.[17] Therefore the mean conjunction of April 16, 603 CE, according to Ptolemy, expressed in modern mean time is:
17h 33.6m + 17.57m - 71m – 1h 59.6m = 14h 40.57m UT.

The table of al-Battani was written in 880 CE when ΔT was about 36m but in 603 CE ΔT was about 64m. Therefore the data of al-Battani's table for the year 603 CE must be corrected by 36 – 64 = – 28m. We must furthermore add 16.4m in order to get modern mean time. Therefore the mean conjunction of April 16, 603 CE, according to al-Battani, expressed in modern mean time is:
17h 53.6m + 16.44m + 28m – 2h 36.2m = 16h 01.84m UT.

The difference between Ptolemy and al-Battani is then more than 81m and not –31m, as written above.

According to modern astronomy the conjunction of Tuesday April 16, 603 CE was on JD 1941409.2017 TD or at 16h 50m 30s Dynamic Time and 15h 46.5m UT. The assumed error of al-Battani is then about 15m while that of Ptolemy is of about – 66m.

The conjunction of August 153 at the time of Ptolemy.

According to the tables of Ptolemy the conjunction of Toth 901 of Nabonassar was on
23d 4' 2", thus on 23 Toth 901 at 4' 2" or 1h 36m 48s p.m. (Alexandria) or 11h 37m 12s a.m. UT. Now 1 Toth was 16 July 153, therefore the mean conjunction was on Monday 7 August (2) – 19 – 662.4 (Alexandria).
The molad of the proleptic Elul 3913 was (2) – 20 – 432.
molad–conjunction of Ptolemy (Alexandria): [(2) – 20 – 432] – [(2) – 19 – 662] = 850 hal.
According to modern astronomy the conjunction of Monday 7 August, 153 CE was on JD 1777160.0866 TD or at 14h 4m 44s Dynamic Time and 11h 50m UT. The assumed error of Ptolemy was of about 13m; his conjunction preceding the modern estimation of the mean conjunction.

The conjunction of March 880 C.E. at the Epoch of al-Battani

According to al-Battani, book 2, p 84, the conjunction of March 880 occurred on 14^d 31' 49" March or Tuesday 15 March 0h 43m 36s aRABMT i.e. Monday 14 March 22h 24m UT. Indeed the longitude of ar –Raqqah is 39°; 03' and there is a difference of 16.44m between

[17] Ptolemy mean time + 17.57m = modern mean time. Modern mean time means the mean time as defined by Flamsteed in 1672. The difference between the ancients and the moderns concerns the calibration of the mean time with regard to the true time. See BDD n° 16: The equation of time in ancient Jewish astronomy.

modern mean time and al-Battani mean time. Therefore the moment of the mean conjunction expressed in UT is 0h 43m 36s – 2h 36m 12s + 16m 44s = 22h 24m 08s.
According to Meeus' Astronomical Algorithms[18] the mean conjunction was on Tuesday JD 2042551.4590 TD on March 14th at 23h 01m Dynamic Time. ΔT was about 36m in 880 C.E; this gives 22h 25m UT. The difference between al-Battani and the modern value is about 1m; it would be 17m if we neglect to add 16.44m[19] to al-Battani's mean time, as Nallino and Schiaparelli did.
The molad of Nissan 4640 was (3) – 7 – 203.
molad–conjunction of al-Battani (ar-Raqqah): [(3) – 7 – 203] – [(3) – 6 – 784.8] = 498.2 hal.

We can conclude that Ptolemy and al-Battani determined the mean conjunction, at the time of their observation, with a very good precision. However their model and tables did not behold this precision as soon as one significantly moves apart from their time. Indeed they were not aware of the slowing down of the rotation of the earth.

The conjunction of January 2000 C.E.

The mean conjunction of the beginning of January 2000 occurred on Thursday 6 January JD 2451550.0976 TD[20] or 14h 20m 37s TD i.e. about 14h 21.7m UT or 16h 42.5 JMT.
The molad of Shevat 5760 was (6) – 0 – 733 or 18h 40.7m.
The molad is thus delayed by 118.2m but this delay becomes 135.77m if we take into consideration that the molad was defined according to the scale of mean time of Ptolemy.[21]

Is the increasing span of time between the mean conjunction and the Molad worrying?

The first purpose of the Jewish calendar, like all calendars, is to remain in concordance with the sun and uphold the feast of Pessah in the month of the spring. However, a second purpose, probably less known, is to maintain the Neomenia in general and of Rosh ha-Shannah in particular, correctly balanced between the vision of the old moon and the vision of the new crescent.
At this level, the molad plays the function of a mean conjunction and has a pivotal importance. The continuation of the shift of the molad with regard of the mean conjunction would increase the probability to see the old crescent on the eve of Rosh ha-Shanah or another Neomenia. This possibility is, so far, not completely excluded, but should it become of significant or frequent occurrence, it would become very worrying. It would require shifting back the molad in order to bring it near to the mean conjunction, if such occurrence would become frequent. It should prevent in any case a late vision of the old crescent on the morning of Rosh ha-Shannah or of another Neomenia, which would be a catastrophic situation.

Another explanation of the discrepancy Molad – mean conjunction.

Rabbi Isaac Yisraeli in Yessod Olam[22] was aware of the fact that Ptolemy and al-Battani had constructed their tables on the basis of a lunation of 29 – 12 -793. He was much concerned to find an explanation to the discrepancy between the molad and the mean conjunction. He devoted a long chapter to this question. Apparently he did not know the tables of mean

[18] Based on the publications of F.R. Stephenson and L.V. Morrison.
[19] al-Battani mean time + 16.44m = modern mean time.
[20] From 2000 onwards TD was replaced by TT.
[21] Ptolemy mean time + 17.57m = modern mean time.
[22] Ma'amar 4, chap 7.

conjunction of these two authors, because he tried to find indirectly some mean conjunctions through a few observations by Abraham Zarkali, Isaac ibn Sid and al-Battani. On the basis of his calculations, which are impossible to check because of the importance of the misprints in the numerical data, he thought to have demonstrated that the molad was calculated for the place located 23° 30' (or 24°) east of Jerusalem. This place with a longitude of 66°30' + 23°30' = 90° in the system of measure of the longitude of the ancients, and with latitude of 0° was considered as the center of the inhabited hemisphere[23], the upper hemisphere. However this statement would have some merit if the difference between the molad and the mean conjunction in Jerusalem were about 1.57h. However the molad followed the conjunction of al-Battani, considered in Jerusalem by only 984[24] hal. Furthermore, this explanation, which was acceptable by the ancients, seems today unrealistic.[25]

Recapitulative table: the molad versus the mean conjunction.

Julian date	Jewish date	Molad	Ptolemy	Ptolemy	al-Battani	al-Battani	Modern Astrono
			Alexan Dria	UT h m	ar-Raqqah	UT h m	UT h m
7 August 153	Elul 3913	(2)-20-432	4' 2"	11h 37.2			11h 50
16 April 603	Iyar 4363	(4)-0-378	13' 54"	14h 40.6	14' 44"	16h 02	15h 46.5
14 March 880	Nissan 4640	(3)-7-203			31' 49"	22h 24	22h 25
20 March 1178	Nissan 4938	(3)-1-721			18' 1"	16h 33.64	16h 43.5
6 January 2000	Shevat 5760	(6)-0-733					16h 42.5

Table 1: The indications in the column "modern astronomy" depend on the accuracy of the term ΔT. Therefore these times are only approximate and the accuracy diminishes when we move back in the past. It is comforting to note that the data according to the modern astronomy are in good concordance with the data of Ptolemy and al-Battani for the period of their observations. In the case of al-Battani, the coincidence is even exceptional. This is not surprising as the model of the evolution of the irregularities of the rotation of the earth in the past is based on the examination and the analysis of the astronomical observations of the past. The model of the ancients was imperfect and furthermore they were not aware of the irregularities of the rotation of the earth.

[23] Tibur ha-Arets.
[24] According to the difference of longitude between ar-Raqqah and Jerusalem, which was 73°15' – 66°30' = 6°45' corresponding to 27m or 486 hal instead of 39°05' – 35°12' = 3°53' corresponding to 15.53m. The difference between the molad and the mean conjunction of al-Battani in 880 C.E. and in Jerusalem was thus 498.2 + 486 = 984.2 hal.
[25] See the former paragraph.