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J. Jean Ajdler

The Gregorian Revolution of the Jewish Calendar

The Jewish calendar is a luni-solar calendar. It works with lunar months, having lengths of 29 or 30 days, and with years of 12 or 13 months, in order to approach, as best as possible, the length of the tropical year.

The mean Jewish year is nevertheless longer than the tropical year by 6.658 minutes. This small difference creates a slow shift in the Jewish calendar and its festivals with regard to the solar year and its seasons. Based on a historical examination of the Jewish calendar, we establish that this shift has already reached about 5.4 days. This represents half of the shift reached by the Julian calendar at the time of the Gregorian revolution. The present shift of the Jewish calendar could thus become worrying. The aim of this paper is to present three acceptable solutions for the improvement of the Jewish calendar, to discuss them thoroughly, and to compare them.

In a mathematical supplement, after the examination and demonstration of the most advanced formulas of the Jewish calendar, we generalize them in order to develop a mathematical apparatus that will enable us to establish the correspondence of these improved Jewish calendars with the Gregorian calendar. This article should be regarded as a theoretical and mathematical analysis of what the Sanhedrin might consider doing when it is re-established.

A. THE GREGORIAN REVOLUTION OF THE JEWISH CALENDAR[1]

The Julian calendar, named in honor of Julius Caesar, was introduced in 45 BCE following the research of Sosigenes, an Alexandrian astronomer of the first century BCE. Its year had 365.25 days – three years of 365 days and one year of 366 days.

1 This article should be regarded as a theoretical analysis of what the Sanhedrin might consider doing when it is re-established. It is beyond the scope of this paper to examine the conditions necessary for the implementation of a solution improving the Jewish calendar. We will not discuss in this paper whether the rules of the calendar constitute a rule that was submitted to a vote (דבר שבמניין), whether it requires only an authoritative and respected chief rabbinate, recognized by all Israel, or if it requires the Sanhedrin of 71 ordinate rabbis (*semiha*), and whether such a Sanhedrin precedes the coming of the Messiah* or follows it.** In any

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Sosigenes considered 25 March to be the day of the equinox. This year of 365.25 days is too long (by about three days in four centuries). 25 March, considered the day of the equinox, therefore arrived later and later after this equinox.

As early as 325 CE at the Council of Nicaea, which fixed the date of Easter in order to detach the church calendar from the Jewish calendar and make it independent and autonomous, the spring equinox was considered to fall on 21 March,[2] four days before the date chosen by Sosigenes at the time of the Julian reform. In fact, Sosigenes was mistaken only by one day, but the fathers of the church attributed the four days' difference to the imprecision of Sosigenes' observations; they were unable to imagine that it was the consequence of their calendar's imprecision. On the contrary, they thought that they could definitively fix the spring equinox to be on 21 March. They believed that the equinox would occur on this date each year. The main features of the rules adopted at the Council of Nicaea were as follows:

1. The Council of Nicaea's rule of intercalation states: Easter is the Sunday following the fourteenth day of the moon, which reaches this stage on 21 March

case, in order to make the most cautious reader comfortable, I wish to quote the commentary of R. Abraham Karelitz (Hazon Ish) on Hilkhot ha-Hodesh V: 2:

ודבר זה הלכה למשה מסיני הוא שבזמן שיש סנהדרין קובעין על פי הראיה ובזמן שאין שם סנהדרין קובעין על פי החשבון הזה...

who writes:

אין הכונה שנמסרו פרטותיו של חשבון שלנו בהלכה, אלא נמסר שרשות לחכמים לעשות חשבון קבוע שעל פיו יסודרו השנים ויתאימו שנות החמה ושנות הלבנה וע״פ זה קבע הלל וב״ד את חשבוננו אבל לא נמנע לקבוע חשבון אחר שגם על פיו יסודרו שנות החמה והלבנה וכדאמר שמואל, ר״ה כ׳ ב׳ יכילנא לתקוני גולה. ואם חשבון הלל מקובל מסיני מה אנו צריכים לשמואל בזה

The subject of this paper is exactly within the scope of the research considered and even encouraged by Hazon Ish, aiming at a better determination of the length of the Jewish year. This paper is thus perfectly within the boundaries of normative Judaism and cannot rise any objection.

* Maimonides in his commentary on Mishna Sanhedrin 1: 3 and Hilkhot Sanhedrin IV: 11.

** R. Hananel on B. Rosh ha-Shanah 20b wrote that the Sanhedrin will follow the coming of the liberator.

Nahmanides on *Sefer ha-Mitzvot* writes that Hillel had sanctified all the neomenia and the years until the coming of Elijah; afterwards we will return to the observation calendar. This seems to imply that the Sanhedrin will be re-established after the coming of Elijah.

Rashi on B. Yoma 80a writes that the re-establishment of the Sanhedrin will follow the reconstruction of the Temple. The opinion of Rashi can be deduced fromm B. Temura 8a, dealing with the possibility of condemning a town to be "עיר הנדחת" where it states that if there in no Temple there is no Sanhedrin of 71 able to deal with this problem.

2 In fact, the spring equinox of 325 CE was on 20 March, at about 10 a.m. U.T; or 7h 40m a.m. JMT, Jerusalem Mean Time.

or slightly later. When we consider that the church had the true equinox in mind, which precedes the mean equinox by two days, we ascertain that this rule is identical[3] with the rule of intercalation of Shitsar, taught by Rabbi Huna bar Avin (B. Rosh ha-Shanah 21a) if we accept that Nissan 16 may occur on the day of the *tekufa*, according to R. Hananel[4] and R. Abraham bar Hiya.[5]

2. If we compare our modern order of intercalation with theirs, we get the following scheme.

Jewish	17*	18	19*	1	2	3*	4
Christian	1	2	3*	4	5	6*	7
Jewish	5	6*	7	8*	9	10	11*
Christian	8*	9	10	11*	12	13	14*
Jewish	12	13	14*	15	16		
Christian	15	16	17*	18	19*		

The numbers of the Jewish lines represent the order of the years in the 19-year cycle. In the Christian lines, one finds the order number of the Christian years; their order number is called the gold number and is defined by: $G = 1 + [N]_{19}$.

Thus year 0 CE has 1 as gold number, and it corresponds to the Jewish year 3760 which is the 17th of the cycle: hence the correspondence between the Jewish and Christian lines.

As ibn Ezra already noted,[6] they intercalate in our years 5 and 16. Their cycle of intercalation is, thus, expressed in Jewish years: 3, 5, 8, 11, 14, 16, 19.[7] We will see below that this order of intercalation corresponds to the opinion of Rabbi Eliezer[8] in the *baraita* of intercalation,[9] and was the order of intercalation fitting the period of the third, fourth and fifth centuries. It seems thus that the Church adopted the

3 As far as we have an agreement about the dates of the mean and true equinox.
4 See note 29.
5 See p. 25 “Which Equinox was Considered in the Rules of Intercalation?” See also notes 28, 29 and 44.
6 *Sefer ha-Ibbur*, p. 6b, bottom.
7 We could also write that the Christians intercalated the years 3, 6, 8, 11, 14, 17, 19, according to our order of intercalation, if we consider that the first year of their cycle is the year 0 CE, which corresponds to the 17th year of our cycle. This is exactly what R. Abraham bar Hiya wrote in his *Sefer ha-Ibbur*, Book III, chap. 10.
8 See note 44.
9 See note 43.

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empirical order of intercalation of the Jewish calendar,[10] and imposed it as a definitive rule to satisfy their new rule of intercalation. By the eighth century, they must have ascertained that this was not true and that the equinox shifted with regard to the sun toward the winter.

The Julian year was indeed too long, by 11m 8s, which represents one day in 128 years. However, Easter was connected to the date of 21 March; it must take place on the first Sunday after the first full moon of spring, i.e. on the Sunday following the 14th day of the lunation that reaches this stage after the vernal equinox, fixed rigidly on 21 March. As the equinoxes always came earlier, because the Julian year was too long by one day every 128 years, 21 March was shifting toward the summer. Therefore, Easter was also shifting toward the summer.

In 1147, R. Abraham ibn Ezra determined the true equinox in Verona: Friday, 14 March, at 7h.[11] He notes that 130 years later the equinox would already be on 13 March.[12]

In about 1300 CE, the vernal equinox fell on 13 March[13] instead of 21 March. Moreover, the 235 lunar cycles of a 19-year cycle, assumed to be 19 x 365.25 = 6939.75 days, in reality were 6939.69 days. Thus, in about 1300 CE, 51 cycles after the Council of Nicaea, the full moon came three days earlier than was computed.[14]

10 We must be very cautious regarding this simplistic statement since we don't know how the leap years were fixed in the calendar of Hillel when it was instituted. There are pieces of evidence proving that the Christians reproached the Jews for not keeping the rules of the equinox (see Jaffe, pp. 49-50 and Stern, pp. 69 and 78) during the fourth and fifth centuries. The Christians placed the true equinox on 21 March. If the mean vernal equinox of the Jews was on 22 March (the astronomical truth at that time), they could have Pesah on March 21 and the eve of Pesah (the Pascha) on 20 March – if they considered the rule of Shitsar according to the understanding of R. Abraham bar Hiya and R. Hananel (see note 60). No wonder that the Christians considered that the Jews began Passover too early. In any event, it is likely that the rule of intercalation was the rule of Shitsar and that it lead to the empiric order 3, 5, 8, 11, 14, 16 and 19, which was also adopted by the Church.

11 I am assuming that we are discussing Jewish hours, thus 1h a.m. (contradicting what I wrote incorrectly in *B.D.D.* 16, p. 26 n. 74). His equinox would then be about 12h in advance. This explains why ibn Ezra spoke of a shift of the *tekufa* of Adda of two days. His equinox was 0.5 day in advance and therefore the true shift was only 1.5 days.

12 *Sefer ha-Ibbur*, p. 9b.

13 In 1300 CE the spring equinox was on 12 March, 16h U.T. or 13h 40m JMT, thus between 12 and 13. During the period 325–1300, the shift is 975/128 = 7.62 days. Thus, approximately, we have March 20.32 – 7.62 = March 12.70.

14 51 x (0.75–0.69) = 3.06 days. Today, in the ecclesiastical calendar of the Orthodox church, this difference has reached: 88 x (0.75 – 0.69) = 4.86 days.

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The computation for Easter was thus completely wrong. Easter was celebrated on the wrong days, and flesh was eaten during Lent; the Jews and the Mohammedans of the time regarded this with derision. The internal quarrels of the Church in the succeeding centuries prevented any corrective measure from being taken; it took nearly three more centuries until a solution was adopted. For a long period, the problem was a familiar one. It was raised at the Council of Constance in 1414; it was raised again at the Council of Trent (1545–53). In 1475, the Pope summoned the great German astronomer Regiomontanus to Rome to seek his advice on the urgently needed reform of the calendar. Regiomontanus informed the Pope that new observations would be needed to provide a reliable basis for improved rules. However, he died in Rome the following year and matters remained as they were. The calendar was in disorder; the ancient rules of intercalation had not been sufficiently accurate, and the discrepancies had become unduly large. In the 16th century, the vernal equinox fell on 11 March instead of 21 March, and the full moons came three days too early.

It was Pope Gregory XIII who finally imposed a new calendar in 1582. Chosen from many propositions, the adopted solution was one put forward by a lecturer in medical science at the University of Perugia, Luigi Lilio, mostly known by his Latinized name, Aloysius Lilius. The solution was criticized by such important scholars as the French mathematician François Viète,[15] the astronomer Michael Maestlin, Johannes Kepler's professor at Tübingen, and the renowned scholar Joseph Justus Scaliger.[16] They criticized the new ecclesiastical calendar of the moon, which would remain far from the true movement of the moon. Clavius[17] was a tireless defender of the reform, and wrote many books to this effect, to such a point that his name is better known than that of the inventor. The choice of Lilio's solution was motivated by its simplicity. The solution consisted in the removal of ten days, by jumping suddenly from 4 October to 15 October, and the omission of three leap years every 400 years. The solution also consisted in keeping the 19-year cycle and having the time of full and new moons pushed back one day eight times in 2,500 years. This should be done in the years 1800, 2100, every 300 years until 3900, and then in 4300 and again every 300 years. Now, if the calendar reform had come before the Reformation, nothing would have stood in the way of its general acceptance. However, because of the great religious discord, the calendar reform

15 François Viète (1540–1603), the father of modern algebra.

16 Joseph Justus Scaliger (1540–1609), philologist and chronologist, author of *De Emendatione Temporis*.

17 Christoph Clavius (1537–1612), Jesuit and mathematician.

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was accepted immediately only by Spain, France, and Poland. The Protestant countries accepted it much later. In the course of the 18th century, the Gregorian calendar was introduced everywhere in Protestant Europe and in England (in 1752). Russia followed only in the 20th century. We see thus how long the process was from the recognition of the problem to the will to solve it, the research of a solution, the definition and the implementation of the solution, and finally to its general acceptance.

B. WHEN DID THE JEWISH CALENDAR DEFINITIVELY SETTLE DOWN?[18]

1. The Date of the Institution of the Jewish Calendar

According to a responsum of R. Hai Gaon, written in 992 CE and mentioned by R. Abraham bar Hiya,[19] the fixed calendar was instituted in 670 S.E. (Seleucid Era=*minyan shtarot*), i.e. 358/59 CE by Hillel II, the Patriarch. We have already shown[20] that as early as about 325 CE, a calculated and predictable calendar was communicated to Babylonia, probably year by year. What then does the date of 358/59 represent? I have suggested that 358/59 CE could represent the date of the official and irreversible institution of the fixed calendar. It seems very likely that the calendar calculated in around 325 CE was still a semi-empirical calendar, calculated year by year. It was probably a flexible calendar and it is very likely that the neomenia were still intended to coincide with the first observation of the new moon. Indeed, the transition to a fixed calendar requires the choice of a *molad* (conjunction), the length of a synodical month, and a rule of intercalation (to respect the lunisolar character of the Jewish calendar). It also requires a shift of about two days of the neomenia, to shift the neomenia from the day of first visibility of the moon to the day of the mean conjunction. It is likely that it took about 34 years to define all these elements, during which time the calendar evolved from the former semi-empirical calendar to a fixed calendar.

18 Sections 2–5 contain historical data based on the research and assumptions of Borenstein and Jaffe, accepted by Rahamim Sar Shalom in his *Shearim le-Luah ha-Ivri*, and on personal assumptions that have already been explained in my paper "The Equation of Time in Ancient Jewish Astronomy," *B.D.D.* 16. Some of these conclusions were contested by R. Casher in *Torah Shelemah*, 13. However, the latter must accept the existence of Talmudic evidence of later enactments in the Jewish calendar. These data are known only by specialists, and cannot be considered as being universally accepted.

19 *Sefer ha-Ibbur*, Book 3, chap 7.

20 J. Jean Ajdler, *Hilkhot Kiddush ha-Hodesh* (Jerusalem, 1996), p. 695.

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2. Further Evolution of the Jewish Calendar

It is likely that these elements mentioned above, a synodical month, a *molad* and rules of intercalation, were not adopted at once definitively but evolved and were subject to research, debate, and evolution.

a. Rosh ha-Shanah on a Sunday

We know from a passage in B. Sukkah 43b that, in about 325 CE (the time of Rabbin), Rosh ha-Shanah could fall on a Sunday. Similarly, we know from Y. Megilah I, 2 that later, in about 350 CE at the time of R. Yose (Youssa), Purim could fall on a Wednesday. This implies that now, when the calendar has become non-variable between Purim and Rosh ha-Shanah, Rosh ha-Shanah can fall on a Sunday. We know from a passage in B. Niddah 67b, that at the time of R. Yeimar (head of the academy of Sura after R. Ashi, 427–32 CE), Rosh ha-Shanah could still fall on a Sunday. We know further from a passage of the epistle of R. Sherira Gaon[21] that in 817 S.E., i.e. 4266 AMI, Purim could still fall on a Wednesday[22] and Rosh ha-Shanah on a Sunday. This situation could have continued until the middle of the seventh century.[23]

b. The Length of the Jewish Lunation

The length of the Jewish lunation adopted in our calendar is 29d 12h 793ch. The date of the introduction of this value of the Jewish lunation is the subject of endless discussions that are outside the scope of this article. Stern[24] considers that the first allusion to a Jewish month of this length appears in a liturgical poem of R. Pinkhas,[25] which refers to the division of the hour into 1,080 parts.

c. The Molad

According to the beginning of the fifth chapter of the *baraita* of Samuel, as it appears in our printed text,[26] the *molad* of Tishri 4537 AMI was on Tuesday, 17 September 776 CE at 18h, i.e. (4) – 0 – 0 instead of the modern value of (4) – 3 – 363; thus a difference of about 3h 20m.

21 Edition Aharon Heyman, p. 85, part 3, chap 4.
22 Adar 4 was a Sunday this year.
23 This at least was the opinion of Borenstein, but we have no direct evidence.
24 Sacha Stern, *Calendar and Community* (Oxford University Press, 2001), p. 204.
25 Late eighth or early ninth century.
26 Based on the edition of R. Nathan Amram, Salonika, 1861.

d. The Letter of the Resh Galuta of 4596 AMI[27]

From this letter we know that the fixing of the years 4596 and 4597 AMI were different than in our calendar.

The *molad* of Nissan 4596 was thus less than (3) – 13 – 642. Otherwise, the *molad* of Tishri 4597 would be *zaken* and Rosh ha-Shanah would be delayed until Saturday, 16 September. The *molad* was probably still in accordance with the *molad* of the *baraita* of Samuel, near (3) – 12 – 720.

The *molad* (3) – 16 mentioned in the letter of the Resh Galuta was probably a Babylonian approximation deduced from the value of the Almagest, (3) – 14 – 1041, by a translation from Alexandria to Bagdad. In conclusion, the *molad* used by the Palestinians in 4596 was still different from the modern *molad*.

Table 1: The Situation According to Our Modern Calendar

4596 AMI	835 CE	Tishri 1	Nissan 1
385 days		Saturday, 28 August *Molad* (6) – 22 – 660	
	836 CE		Thursday, 23 March *Molad* (3) – 15 – 811 *Molad Zaken* if *Molad* > = (3) – 13 – 642
4597 AMI		Saturday, 16 Sept. *Molad* (5) – 20 – 169 *Molad Zaken*	

Table 2: The Data According to the Letter of the Resh Galuta

4596 AMI	835 CE	Tishri 1	Nissan 1
383 days			
	836 CE		Tuesday, 21 March
4597 AMI		Thursday, 14 Sept.	

Therefore, the proposition of Borenstein and Joffe, according to which the definitive rules of the Jewish calendar were fixed in 4599, seems likely. However, there still remained a difference between the Palestinians, who fixed the first *molad* in Nissan

27 See Rahamim Sar Shalom, *Shearim le-Luah ha-Ivri*.

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of year 1 AMI on (4) – 9 – 0, and the Babylonians, now associated with the process, who fixed the first *molad* in Tishri of year 2 AMI on (6) – 14 – 0. This last difference of 642ch, which apparently subsisted between the *molad* of the Palestinian Council of Intercalation and the *molad* of the Babylonian scholars, would create the dispute of 922–24 between Ben Meir and R. Saadia Gaon. The victory of R. Saadia Gaon will definitively fix the *molad* to its modern value, and inevitably undermine the dominant position of the Palestinian council regarding the calendar.

3. Which Equinox was Considered in the Rules of Intercalation?

It is generally accepted that the rule of intercalation that was adopted to determine the regular years (12 months) and the leap years (13 months) is that set out by Rav Huna bar Avin to Rava in B. Rosh ha-Shanah 21a: "When you see the winter season prolonging itself until the 16th of Nissan, intercalate that year and do not worry [about contradictory opinions (Rashi), or about the two other signs of maturity (Tosafot)]."

The existence of such a rule of intercalation (together with concurrent rules mentioned in B. Sanhedrin) implies that the 19-year Metonic cycle was not yet instituted in Hillel's calendar. The exact significance of this passage has often been discussed. The first problem was the meaning of "until the 16th of Nissan." According to Rashi and Maimonides, we intercalate only if the equinox occurs on the 16th of Nissan. According to others, such as Tosafot, Savasorda (R. Abraham bar Hiya)[28] and R. Hananel,[29] we intercalate only if the equinox occurs on the 17th of Nissan.

A second problem, which interests us particularly, is whether the *tekufa* (the mean equinox) was the *tekufa* of Samuel, the *tekufa* of Rabbi Adda or another *tekufa*. In any case, it must have been a mean equinox and not, as some claim, a true equinox. A true equinox is the passage at the vernal or autumnal point of the true sun, while a mean equinox is the passage at these points of the mean sun. True vernal equinox occurs nowadays two days before the mean vernal equinox, and true autumnal equinox occurs two days after the autumnal mean equinox.

Some have claimed that the *tekufa* of Adda coincided well with the true vernal equinox in the fourth century, and that this *tekufa* was already in use when Hillel's calendar was instituted.[30] I consider these assumptions false, and believe that this

28 *Sefer ha-Ibbur*, Sha'ar V, p. 92.
29 B. Rosh ha-Shanah 21a. His commentary on B. Sanhedrin 13a-b raises difficulties.
30 Y. Loewinger, *Al ha-Sheminit*, pp. 160-62.

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rule – the rule of R. Huna bar Avin – without any doubt involved a mean equinox. In B. Sanhedrin (13b), the Talmud seems concerned with the timing of Sukkot, that the occurrence of the 21st of Tishri should be during autumn, and with the timing of Passover, that the occurrence of the 16th of Nissan should be during the spring. This co-occurrence can be reached only by applying the intercalation rule to the mean equinox. Indeed if we apply an intercalation rule to the spring equinox, then the rule concerning the position of Sukkot with respect to the true autumnal equinox cannot be respected.[31]

It can be demonstrated further that the view that this was the mean equinox was the view of all rabbinical authorities and all Jewish calendar specialists throughout Jewish history. It is the mean equinox that is involved in all the Talmudic rules of intercalation. For example, Tosafot in B. Sanhedrin (13a; 13b) consider that the *tekufa* used was the *tekufa* of Samuel. Maimonides thinks that the rule of Shitsar was applied with the *tekufa* of Adda,[32] and he considers that this *tekufa* is a mean equinox.[33]

Rabbi Abraham bar Hiya ha-Nasi gives the definition of true and mean equinox, and he writes explicitly that the Jewish *tekufot* are based on the mean movement of the sun.[34]

R. Judah ha-Levi[35] considers that the *tekufa* of Rabbi Adda coincides with the observation of Al-Battani.[36] This implies that Judah ha-Levi compares the *tekufa* of Adda of Sunday, 16 September 882, at 21h 0min 23 sec Jerusalem mean time with the mean equinox of Al-Battani. The true equinox of Al-Battani occurred on 19 September 882 at 1h 15m ar-Raqua mean time, and the mean equinox was then on 17 September 882 0h 42min Jerusalem mean time.

In the *baraita* of Samuel, chapter V, an observed autumnal equinox is mentioned on Tuesday, 17 September 776 at 16h JMT.[37] This time was obviously a mean equinox as the true equinox was on Thursday, 19 September 776 at 8h 51m JMT with the mean equinox on 17 September 776 at 8h 51m. The precision was quite

31 Indeed, the distance between 16 Nissan and 21 Tishri is 182 days. The distance between 22 March and 21 September (mean equinoxes) is 183 days. The distance between 20 March and 23 September (true equinoxes) is 187 days.
32 Hilkhot Kiddush ha-Hodesh XI, 6.
33 Hilkhot Kiddush ha-Hodesh XI, 7.
34 *Sefer ha-Ibbur*, Book III, chap. 2, last lines of the chapter.
35 *Sefer ha-Kuzari*, Book 4, chap. 29.
36 Al-Battani, *Opus Astronomicum* (Milan, 1903), pp. 42 and 210.
37 Z. H. Jaffe, *Korot Heshbon ha-Ibbur* (Jerusalem, 1931), p. 64.

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good (a difference of about seven hours).

In his *Sefer ha-Ibbur*, R. Abraham ibn Ezra notes:

1. That the *tekufa* of Adda is, in contradiction with common opinion, a mean equinox. These four points are on the circle of the sun and not on the ecliptic.[38]
2. The rule of intercalation is based on the *tekufa* of Adda.[39]

In his famous book *Yessod Olam*,[40] the 14th-century Jewish astronomer Isaac Israeli of Toledo writes that it is the mean vernal equinox that is considered for the fixing of Passover.

4. The Adoption of a Stable Intercalation Cycle

According to Borenstein[41] and Jaffe,[42] the adoption of a stable intercalation cycle happened during the eighth century, before the adoption of the definitive *molad*. Until that period, the intercalations were probably fixed empirically based on the rule of Shitsar, according to which Nissan 16 should occur, at the earliest, on the day of the mean equinox (Gregorian 22 March) and, at the latest, 29 days later. The decision to adopt a fixed intercalation rule implies that the mean Jewish year is equal or nearly equal to the tropical year.

According to a *baraita*,[43] probably dating from that period, three orders of intercalations were proposed. The first is the order attributed to Rabbi Eliezer.[44] It seems that this was the order of intercalation that worked and appeared empirically during the third, fourth, fifth, and beginning of the sixth centuries.[45] The second is attributed to the Sages,[46] and must correspond to the empirical order used during the sixth, seventh and eighth centuries. The third order of intercalation is attributed to Rabban Gamliel, and corresponds to the years of intercalation of our modern

38 P. 6b, bottom.

39 P. 8b-9a.

40 Book 4, chap. 2, p. 3 column 2.

41 *Divrei Yemei ha-Ibbur*, p. 268.

42 *Korot Heshbon ha-Ibbur*, p. 84.

43 The text of the *baraita* can be found in *Yessod Olam*, Book 4, chap. 2. See also Pirkei de Rabbi Eliezer, end of chap. VIII. See also Borenstein mentioning the text of this *baraita* in *Sefer ha-Ibbur of Rashbam* (still in manuscript): *Divrei Yemei ha-Ibbur ha-Aharonim*, p. 18 and Z. H. Jaffe, *Korot Heshbon ha-Ibbur*, p. 83.

44 Cycle: 3, 5, 8, 11, 14, 16 and 19. i = 18 – Rabbi Eliezer is known for his transmission of the old traditions. See B. Sukkah 28b.

45 Exactly the order of intercalation that the Church adopted as a rule in its calendar.

46 Cycle: 3, 6, 8, 11, 14, 16 and 19. i = 19.

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calendar.[47] If we consider that the first opinion, attributed to Rabbi Eliezer, corresponded to an old order of intercalation and that the doubt was in determining the most suitable order between the last two systems, then the assumption that this *baraita* dates from the eighth century, when a cycle of intercalation was adopted, seems likely. There was indeed a possibility of doubt with regard to the best system of intercalation between the different systems. It seems even that the system of intercalation definitively adopted in the eighth century was, in fact, implemented too early when it did not yet fit perfectly; during the eighth century the order of intercalation of the Sages would probably have worked better. It was only a century later that the new order of intercalation worked perfectly. It would prove very efficient and precise when the modern calendar was definitively fixed. As we show in the next chapter, the Jewish calendar, with the new order of intercalation, was perfectly centered during the ninth century with regard to the solar year.

5. The Jewish Calendar was Definitively Fixed in about 4599, not During the Fourth Century: Additional Evidence

We have established two tables, the first for the 243rd cycle, probably the first cycle working under the definitive rules of the Jewish calendar, with the corrected *molad* probably calculated according to the Almagest. It is striking to note that the 16th of Nissan of the critical year 16 of the cycle occurs on the fictitious Gregorian 22 March, the date of the mean spring equinox and that Tishri 21 of the same year occurs on the fictitious Gregorian 20 September, corresponding to the mean autumn equinox. It appears that the intention of the founders of the Jewish calendar was probably twofold: they wanted to satisfy two conditions, one in spring and the second in autumn.

Apparently, Nissan 16 and Tishri 21 were allowed to coincide with the day of the mean equinoxes once in each cycle of 19 years, and must always occur during the month following these mean equinoxes.

This coincidence reinforces the conviction that the last changes in the Jewish calendar were performed at this time, and that the rules of intercalation must be understood as explained before. Now, when we examine the second table, calculated for the 247th cycle, we observe that the conditions, both in spring and in autumn, are still satisfied.

Nevertheless, we observe also that the results of the Julian shift of one day are already noticeable. Within a century, the shift toward the summer, because the

47 Cycle: 3, 6, 8, 11, 14, 17 and 19. i = 1. This cycle did not fit before the ninth century.

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Jewish year is too long, will bring the critical day of Nissan 16 of year 8 of the cycle out of the spring. Of course, this seems not to have been noticed. On the contrary, the world was struck by the precision of the Jewish calendar with regard to the Julian calendar.

Table 3: The Dates of Nissan 16 and the Following Tishri 21 During the 243rd Cycle

N	Year	Jewish Year	Nissan 16 Julian	Nissan 16 Gregorian	Tishri 21 Julian	Tishri 21 Gregorian
1	839	4599–4600	4 April	8 April	3 October	7 October
2	840	4600–4601	24 March	28 March	22 Sept.	26 Sept.
3	841	4601–4602	11 April	15 April	10 Oct.	14 Oct.
4	842	4602–4603	31 March	4 April	29 Sept.	3 Oct.
5	843	4603–4604	21 March	25 March	19 Sept.	23 Sept.
6	844	4604–4605	7 April	11 April	6 Oct.	10 Oct.
7	845	4605–4606	27 March	31 March	25 Sept.	29 Sept.
8	846	4606–4607	16 April	20 April	15 Oct.	19 Oct.
9	847	4607–4608	6 April	10 April	5 Oct.	9 Oct.
10	848	4608–4609	25 March	29 March	23 Sept.	27 Sept.
11	849	4609–4610	12 April	16 April	10 Oct.	14 Oct.
12	850	4610–4611	2 April	6 April	1 Oct.	5 Oct.
13	851	4611–4612	22 March	26 March	20 Sept.	24 Sept.
14	852	4612–4613	10 April	14 April	9 Oct.	13 Oct.
15	853	4613–4614	29 March	2 April	27 Sept.	1 Oct.
16	854	4614–4615	18 March	22 March	16 Sept.	20 Sept.
17	855	4615–4616	7 April	11 April	6 Oct.	10 Oct.
18	856	4616–4617	27 March	31 March	25 Sept.	29 Sept.
19	857	4617–4618	14 April	18 April	13 Oct.	17 Oct.

Table 4: The Dates of Nissan 16 and the Following Tishri 21 During the 247th Cycle

N	Year	Jewish Year	Nissan 16 Julian	Nissan 16 Gregorian	Tishri 21 Julian	Tishri 21 Gregorian
1	934	4694–4695	4 April	9 April	3 Oct.	8 Oct.
2	935	4695–4696	23 March	28 March	21 Sept.	26 Sept.
3	936	4696–4697	10 April	15 April	9 Oct.	14 Oct.
4	937	4697–4698	31 March	5 April	29 Sept.	4 Oct.
5	938	4698–4699	21 March	26 March	19 Sept.	24 Oct.
6	939	4699–4700	8 April	13 April	7 Oct.	12 Oct.
7	940	4700–4701	27 March	1 April	25 Sept.	30 Sept.
8	941	4701–4702	16 April	21 April	15 Oct.	20 Oct.
9	942	4702–4703	6 April	11 April	5 Oct.	10 Oct.
10	943	4703–4704	26 March	31 March	24 Sept.	29 Sept.
11	944	4704–4705	12 April	17 April	11 Oct.	16 Oct.
12	945	4705–4706	2 April	7 April	1 Oct.	6 Oct.
13	946	4706–4707	22 March	27 March	20 Sept.	25 Sept.
14	947	4707–4708	9 April	14 April	8 Oct.	13 Oct.
15	948	4708–4709	29 March	3 April	27 Sept.	2 Oct.
16	949	4709–4710	18 March	23 March	16 Sept.	21 Sept.
17	950	4710–4711	7 April	12 April	6 Oct.	11 Oct.
18	951	4711–4712	26 March	31 March	24 Sept.	29 Sept.
19	952	4712–4713	14 April	19 April	13 Oct.	18 Oct.

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6. The Precision of the Jewish Calendar and the Consecutive Problems

a. Comparison of the Jewish Calendar with the Gregorian Calendar

The length of the Gregorian year is 365.2425 days, and 19 Gregorian years have a length of 6939.6075 days. The Jewish month has a length of 29d 12h 793ch. One cycle of 19 years has the length of 235 months, representing 6939.689621914 days. The Jewish mean year is thus too long, and the difference at the end of 19 years is 0.082121914 d. This difference reaches one day after 231.36334 years. It reaches one Jewish month after 6832.2969 years, corresponding to 359.5946 cycles of 19 Jewish years.

b. Comparison of the Jewish Year with the Tropical Year

The length of the tropical year is 365.24219878 d, and 19 years have a length of 6939.60177682 d. The difference between 19 Jewish mean years and 19 tropical years is thus 0.08784509 d. This difference reaches one day in 216.2898 years. It reaches one Jewish month after 6387.1670 years, corresponding to 336.1667 cycles of 19 Jewish years. The only Jewish calendar specialists who were aware of this problem were Maimonides[48] and Abraham ibn Ezra.[49] Isaac Israeli in about 1310 does not notice any problem of precision with regard to the *tekufa* of Adda.

c. Improvement of the Jewish Calendar

The Jewish calendar shifts by one day every 216.2898 years. If we consider that the Jewish calendar was perfectly centered at the beginning of the 243rd cycle, then the shift today is (1167 / 216.2898) = 5.4 days. The shift will increase with time. In the 725th cycle, Nissan 16 will fall at the earliest on 2 May and at the latest

48 Maimonides writes in *Hilkhot Kiddush ha-Hodesh* X: 6 and 7: The *tekufa* of Adda is better than that of Samuel but it remains an approximation. Nevertheless, he must have considered it to be sufficiently precise, since he believed that the Sanhedrin considered this *tekufa* in order to appreciate if the year must be intercalated. However, from another point of view Maimonides should have noted, on the basis of his own data, that the mean spring equinox was on 15 March 1178 at about 14h 27m JMT, while the *tekufa* of Adda was on 17 March 1178 at 7h 28m JMT. See J. Ajdler, *Hilkhot Kiddush ha-Hodesh al pi ha-Rambam*, p. 179. The shift was thus more than 1.5 days.

49 In his *Sefer ha-Ibbur*, p. 9b and 10a, he notes that the *tekufa* of Adda has shifted in his time, i.e. 1147, by about 2 days. Therefore, the distance between the true spring equinox and the *tekufa* of Nissan is 4 days, in Tishri the distance is nearly non-existent, and in Tamuz and Tevet it is about 2 days plus (Tamuz) or minus (Tevet) a few hours. A correct calculation shows that the shift was only 1.42 days. Indeed, (2007 – 1146) x 6.66 m = 3.98 days. The shift was thus 5.4d – 3.98d = 1.42 days.

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on 30 May. Similarly, Tishri 21 will fall at the earliest on 31 October and at the latest on 28 November.

Taking into consideration the value of a shift of one day every 216.2898 years, we see that the answer to the problem would be to find a harmonious and smooth solution that would allow us to suppress one lunation in about 336 cycles.

d. Comparison between the Jewish Lunation and the Lunation of Brown

The Jewish lunation is:	29. 530 594 135 803
The lunation of Brown is:	29. 530 587 731 481
Excess of the Jewish lunation:	0. 000 006 404 322

This excess will amount to one day after a delay of 156144.5 lunations, corresponding to 156,144.5 x (19/235) = 12624.45 years. During this period, the Jewish years will exceed the Gregorian years by 12624.45/231.36334 = 54.56 days, and the tropical years by 12624.45/216.3634 = 58.37 days. It appears that the lunar problem is negligible with regard to the difference that will appear between the Jewish calendar and the solar calendar. It is a matter of fact that despite the great irregularity of the moon's movement, the ancients knew it very well. From Maimonides' *Hilkhot Kiddush ha-Hodesh*, it appears clearly that he knew and quantified the difference between the *molad* and the mean equinox. In Nissan 4938 the *molad* was (3) – 1 – 721 or 7h 40m p.m. while the astronomical mean equinox was at 6h 45m p.m; thus a delay of 55 minutes of the *molad*. In July 1953, the *molad* of Av was at (0) – 22 – 513 or in July at 14h 07m 46s + 17.57m[50] = 14h 25m UT, while the mean equinox was at 12h 13m. The *molad* was thus delayed by 2h 12m. This span of time increases slowly, and the danger is that there is an increased probability of seeing the new moon before the 1st of a month, more specifically before Tishri 1. Based on the precedent of 4536,[51] when the *molad* was increased by more than three hours to fit the Almagest, we will have to diminish the *molad* by three hours when the difference reaches this value, although we could probably wait until the difference reaches six hours. Of course, we will lose the periodicity that was supposed to exist with the *molad* but which existed in fact only from 4599 AMI on, and more precisely from 4683 on, when R. Saadia Gaon imposed the Babylonian *molad*, which was 642 ch less than the Palestinian *molad*.

50 Mean Time of Almagest + 17.57 m = modern mean time.

51 See the *baraita* of Samuel, chap V, 1.

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C. IMPROVEMENT OF THE JEWISH CALENDAR: FIRST SOLUTION

Let us refer to Table 5, representing all nineteen possible orders of intercalation. Let us further suppose that for all of the 17 x 19 = 323 years we adopt a different order of intercalation. Thus, the current order of intercalation corresponding to i = 1 will be replaced by the order of intercalation i = 2 and so on. We will be faced with a problem at the transition from the order i = 2 to the order i = 3 because, without the implementation of a special disposition against it, we would have two consecutive leap years, which is unacceptable. We will then be obliged to make the last year of the last cycle, i = 2, an ordinary year instead of a leap year.

Of course, this will have no noticeable consequence, because the following year will be a leap year. At the end of the process, after 19 x 323 = 6137 years, the number of elapsed months, which should normally be 323 x 235 = 75905 months, will now be: 75905 – 1 = 75904 months because of the suppressed month in the 19th year of the last cycle, i = 2. Finally, we will have suppressed one lunation in 6137 years instead of in 6387 years, as we initially expected.

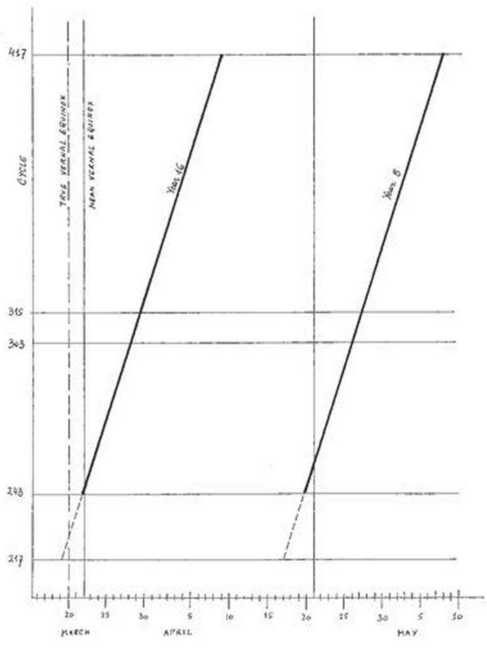

Figure 1: The shift of the Jewish calendar with regard to the sun; the limits of the date of Nissan 16 as a function of the serial number of the 19-year cycles. Thus, for each cycle we find the extreme dates of Nissan 16, the earliest for the 16th year and the latest for the 8th year of the cycle. The two straight lines of the figure are in fact an approximation and a general tendency.

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If this process had been implemented at the assumed beginning of the definitive calendar in 4599 AMI (autumn 838), then we would have had the following succession:
i = 1 in the years 4599–4921; i = 2 in the years 4922–5244; i = 3 in the years 5245 –5567; i = 4 in the years 5568–5890; i = 5 in the years 5891–6213. The cycle of intercalation i = 4 corresponds to אגו״ט בד״ז.[52]

Table 5: The 19 Types of Intercalation Cycles (first part)

For the signification of the coefficients K, i, and a_i, see below.

K = 8 i = 1		K = 7 i = 2		K = 6 i = 3		K = 5 i = 4		K = 4 i = 5	
N	a_1	N	a_2	N	a_3	N	a_4	N	a_5
1	17	1	18	1*	0*	1*	1*	1*	2*
2	10	2	11	2	12	2	13	2	14
3*	3*	3*	4*	3*	5*	3*	6*	3	7
4	15	4	16	4	17	4	18	4*	0*
5	8	5	9	5	10	5	11	5	12
6*	1*	6*	2*	6*	3*	6*	4*	6*	5*
7	13	7	14	7	15	7	16	7	17
8*	6*	8	7	8	8	8	9	8	10
9	18	9*	0*	9*	1*	9*	2*	9*	3*
10	11	10	12	10	13	10	14	10	15
11*	4*	11*	5*	11*	6*	11	7	11	8
12	16	12	17	12	18	12*	0*	12*	1*
13	9	13	10	13	11	13	12	13	13
14*	2*	14*	3*	14*	4*	14*	5*	14*	6*
15	14	15	15	15	16	15	17	15	18
16	7	16	8	16	9	16	10	16	11
17*	0*	17*	1*	17*	2*	17*	3*	17*	4*
18	12	18	13	18	14	18	15	18	16
19*	5*	19*	6*	19	7	19	8	19*	9*

52 Prof. A. Fraenkel was thus right in his article in *Sinai*, 17, pp. 176–81, and Y. Loewinger was mistaken in *Al ha-Sheminit*, p. 123. It is particularly interesting to note that the current cycle of intercalation of the ecclesiastical Gregorian calendar transposed in the corresponding Jewish years gives exactly the same cycle of intercalation, i = 4.

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Table 5: The 19 Types of Intercalation Cycles (first part continued)

K = 3 i = 6		K = 2 i = 7		K = 1 i = 8		K = 0 i = 9		K = 18 i = 10	
N	a_6	N	a_7	N	a_8	N	a_9	N	a_{10}
1*	3*	1*	4*	1*	5*	1*	6*	1	7
2	15	2	16	2	17	2	18	2*	0*
3	8	3	9	3	10	3	11	3	12
4*	1*	4*	2*	4*	3*	4*	4*	4*	5*
5	13	5	14	5	15	5	16	5	17
6*	6*	6	7	6	8	6	9	6	10
7	18	7*	0*	7*	1*	7*	2*	7*	3*
8	11	8	12	8	13	8	14	8	15
9*	4*	9*	5*	9*	6*	9	7	9	8
10	16	10	17	10	18	10*	0*	10*	1*
11	9	11	10	11	11	11	12	11	13
12*	2*	12*	3*	12*	4*	12*	5*	12*	6*
13	14	13	15	13	16	13	17	13	18
14	7	14	8	14	9	14	10	14	11
15*	0*	15*	1*	15*	2*	15*	3*	15*	4*
16	12	16	13	16	14	16	15	16	16
17*	5*	17*	6*	17	7	17	8	17	9
18	17	18	18	18*	0*	18*	1*	18*	2*
19	10	19	11	19	12	19	13	19	14

The coefficients i and K define the considered intercalation cycle. The first cycle of 19 years corresponds to i = 1; it is the current cycle גו״ח אדז״ט. The coefficient i varies from 1 to 19, and corresponds to the natural succession of the intercalation cycles. The coefficient K plays a similar role as i and is bound to K by the equation $[i + K]_{19} = 9$; thus $i = [28 - K]_{19}$ and $K = [28 - i]_{19}$. The coefficients a_i of the cycle of intercalation i define the characteristics of the corresponding year: when a <= 6* we have a leap year, when a > 6* we have an ordinary year. Furthermore, a = 6* corresponds to the leap year ending the latest, its Pesah is the latest. a = 7 is the common year which begins the earliest and Pesah is the earliest. These two years are the critical years of the cycle.

This type of cycle should therefore fit our period. In this project, we propose to introduce this solution in 6214 with a cycle of the type i = 6.

It seems impossible to think about this revolution before 6001 AMI. We should then introduce a sudden transition from the order i = 1 to the order i = 6 with the transformation of the 19th year of the last cycle i = 1, the leap year 6213, into an ordinary year. This solution was briefly outlined by Prof. Abraham Fraenkel.[53] It was also considered by Jaffe.[54]

53 Prof. Abraham ha-Levi Fraenkel, *Sinai*, 17: 176-81.

54 *Korot Heshbon ha-Ibbur*, pp. 119-20. See also Rahamim Sar Shalom, *Shearim le-Luah ha-Ivri*, p. 151.

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Table 6: The 19 Types of Intercalation Cycles (second part)

K = 17 i = 11		K = 16 i = 12		K = 15 i = 13		K = 14 i = 14		K = 13 i = 15	
N	a_{11}	N	a_{12}	N	a_{13}	N	a_{14}	N	a_{15}
1	8	1	9	1	10	1	11	1	12
2*	1*	2*	2*	2*	3*	2*	4*	2*	5*
3	13	3	14	3	15	3	16	3	17
4*	6*	4	7	4	8	4	9	4	10
5	18	5*	0*	5*	1*	5*	2*	5*	3*
6	11	6	12	6	13	6	14	6	15
7*	4*	7*	5*	7*	6*	7	7	7	8
8	16	8	17	8	18	8*	0*	8*	1*
9	9	9	10	9	11	9	12	9	13
10*	2*	10*	3*	10*	4*	10*	5*	10*	6*
11	14	11	15	11	16	11	17	11	18
12	7	12	8	12	9	12	10	12	11
13*	0*	13*	1*	13*	2*	13*	3*	13*	4*
14	12	14	13	14	14	14	15	14	16
15*	5*	15	6*	15	7	15	8	15	9
16	17	16	18	16*	0*	16*	1*	16*	2*
17	10	17	11	17	12	17	13	17	14
18*	3*	18*	4*	18*	5*	18*	6*	18	7
19	15	19	16	19	17	19	18	19*	0*
K = 12 i = 16		K = 11 i = 17		K = 10 i = 18		K = 9 i = 19		K = 8 i = 1	
N	a_{16}	N	a_{17}	N	a_{18}	N	a_{19}	N	a_{1}
1	13	1	14	1	15	1	16	1	17
2*	6*	2	7	2	8	2	9	2	10
3	18	3*	0*	3*	1*	3*	2*	3*	3*
4	11	4	12	4	13	4	14	4	15
5*	4*	5*	5*	5*	6*	5	7	5	8
6	16	6	17	6	18	6*	0*	6*	1*
7	9	7	10	7	11	7	12	7	13
8*	2*	8*	3*	8*	4*	8*	5*	8*	6*
9	14	9	15	9	16	9	17	9	18
10	7	10	8	10	9	10	10	10	11
11*	0*	11*	1*	11*	2*	11*	3*	11*	4*
12	12	12	13	12	14	12	15	12	16
13*	5*	13*	6*	13	7	13	8	13	9
14	17	14	18	14*	0*	14*	1*	14*	2*
15	10	15	11	15	12	15	13	15	14
16*	3*	16*	4*	16*	5*	16*	6*	16	7
17	15	17	16	17	17	17	18	17*	0*
18	8	18	9	18	10	18	11	18	12
19*	1*	19*	2*	19*	3*	19*	4*	19*	5*

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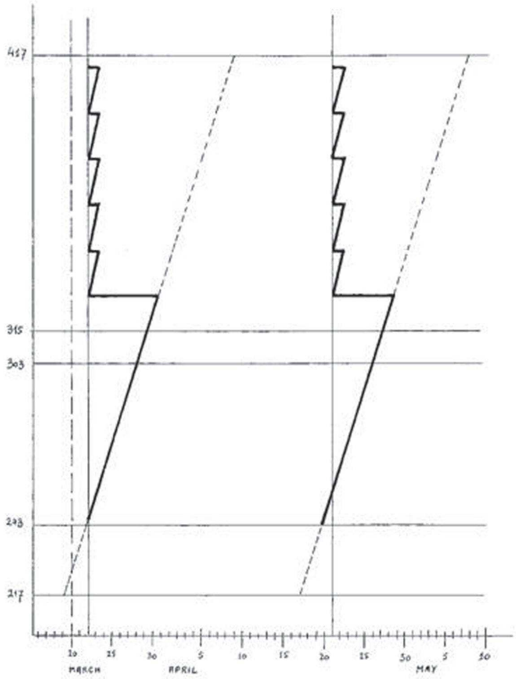

Figure 2: The principle of the improvement of the Jewish calendar according to Solution I: The shift of the Jewish calendar with regard to the sun; the limits of the date of Nissan 16 as a function of the serial number of the 19-year cycles. The figure can also apply to the improvement of the Jewish calendar according to Solution II, but the scale of the Y-axis must be adapted.

D. IMPROVEMENT OF THE JEWISH CALENDAR: SECOND SOLUTION

We refer to Tables 5 and 6. All 19-year cycles of intercalation are represented in these tables. Each year is characterized by a coefficient a_i. This coefficient gives us information about the civil date of Rosh ha-Shanah. More precisely, it indicates to us the order of succession of Rosh ha-Shanah during the 19 years of the cycle in the civil year. We know that the Jewish year is too long, resulting in the festivals shifting ahead, with Passover moving toward the summer and the Tishri festivals toward the winter. Therefore, we can imagine that after a certain number of cycles of type i, when the shift has reached an unacceptable level, one jumps to the cycle i + 1. The modality of the jump would be the following. We begin for the last time the cycle i, for example i = 1, but when we reach the leap year 8, characterized by the coefficient a = 6*, which begins the latest and has the latest Passover of the

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cycle, we decide that because this year ends too late to transform it into an ordinary year of index a = 7, this year will end now the earliest, and we will let it be followed by a leap year that begins the earliest and will have an index a = 0*. A more detailed mathematical approach will be given in a mathematical supplement.

The above explanation explains clearly and intuitively how the successive orders of intercalation are generated. We see now that a cycle i will begin with a year of index a = 7 and it will end with a year of index 13. The period P_i will thus contain p cycles of 19 years + 11years, and the number of lunar months will be 235 x p + 136.

We can then write: (19 x p + 11) x A_t = (235 x p + 136) x M. A_t represents the length of the tropical year.

Thus $p = \frac{11\,A_t - 136\,M}{235\,M - 19\,A_t}$

with M = 29.5305941358 and A_t = 365.24219878.
We find p = 17.1140
If we adopt: p = 17 then the cycle p will contain 334 years and
(19p + 11) x 12 +7p + 4 = 4131 lunar months.
Now: 334 tropical years contain 121,990.8944 days
4131 Jewish months contain 121,990.8844 days.
At the end of 19 periods of 334 years, 6346 years and 19 x 4131 = 78489 Jewish months have elapsed. If we had worked with the traditional calendar, we would have had 334 x 235 = 78490 elapsed months. We have thus suppressed one month in 6346 years, or one day in about 214.9 years. This is approximately what we wanted to end up with.

If this process had been implemented at the beginning of the definitive calendar in 4599 AMI (autumn 838), then we would have had the following succession:
i = 1 in the years 4599–4947; i = 2 in the years 4948–5281; i = 3 in the years 5282 –5615: i = 4 in the years 5616–5949; i = 5 in the years 5950–6283: i = 6 in the years 6284–6617.

The first cycle must last 15 years more, because the cycle begins normally with the sixteenth year of index a = 6 of the cycle type i – 1.

The cycle of intercalation i = 4 corresponds to אגו"ט בד"ז. It is thus this type of cycle that should fit for our period. In this project, we propose to introduce this solution in 6284. It doesn't seem possible to think about this revolution before 6001 AMI.

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Table 7: Improvement of the Jewish Calendar: Second Solution

[The new cycle begins with a year of index 7 (dark gray) and ends with a year of index 13 (light gray)]

K = 8 i = 1		K = 7 i = 2		K = 6 i = 3		K = 5 i = 4		K = 4 i = 5	
N	a_1	N	a_2	N	a_3	N	a_4	N	a_5
1	17	1	18	1*	0*	1*	1*	1*	2*
2	10	2	11	2	12	2	13	2	14
3*	3*	3*	4*	3*	5*	3*	6*	3	7
4	15	4	16	4	17	4	18	4*	0*
5	8	5	9	5	10	5	11	5	12
6*	1*	6*	2*	6*	3*	6*	4*	6*	5*
7	13	7	14	7	15	7	16	7	17
8*	6*	8	7	8	8	8	9	8	10
9	18	9*	0*	9*	1*	9*	2*	9*	3*
10	11	10	12	10	13	10	14	10	15
11*	4*	11*	5*	11*	6*	11	7	11	8
12	16	12	17	12	18	12*	0*	12*	1*
13	9	13	10	13	11	13	12	13	13
14*	2*	14*	3*	14*	4*	14*	5*	14*	6*
15	14	15	15	15	16	15	17	15	18
16	7	16	8	16	9	16	10	16	11
17*	0*	17*	1*	17*	2*	17*	3*	17*	4*
18	12	18	13	18	14	18	15	18	16
19*	5*	19*	6*	19	7	19	8	19*	9*
K = 3 i = 6		K = 2 i = 7		K = 1 i = 8		K = 0 i = 9		K = 18 i = 10	
N	a_6	N	a_7	N	a_8	N	a_9	N	a_{10}
1*	3*	1*	4*	1*	5*	1*	6*	1	7
2	15	2	16	2	17	2	18	2*	0*
3	8	3	9	3	10	3	11	3	12
4*	1*	4*	2*	4*	3*	4*	4*	4*	5*
5	13	5	14	5	15	5	16	5	17
6*	6*	6	7	6	8	6	9	6	10
7	18	7*	0*	7*	1*	7*	2*	7*	3*
8	11	8	12	8	13	8	14	8	15
9*	4*	9*	5*	9*	6*	9	7	9	8
10	16	10	17	10	18	10*	0*	10*	1*
11	9	11	10	11	11	11	12	11	13
12*	2*	12*	3*	12*	4*	12*	5*	12*	6*
13	14	13	15	13	16	13	17	13	18
14	7	14	8	14	9	14	10	14	11
15*	0*	15*	1*	15*	2*	15*	3*	15*	4*
16	12	16	13	16	14	16	15	16	16
17*	5*	17*	6*	17	7	17	8	17	9
18	17	18	18	18*	0*	18*	1*	18*	2*
19	10	19	11	19	12	19	13	19	14

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Table 8: Improvement of the Jewish Calendar: Second Solution (continued)

K = 17 i = 11		K = 16 i = 12		K = 15 i = 13		K = 14 i = 14		K = 13 i = 15	
N	a_{11}	N	a_{12}	N	a_{13}	N	a_{14}	N	a_{15}
1	8	1	9	1	10	1	11	1	12
2*	1*	2*	2*	2*	3*	2*	4*	2*	5*
3	13	3	14	3	15	3	16	3	17
4*	6*	4	7	4	8	4	9	4	10
5	18	5*	0*	5*	1*	5*	2*	5*	3*
6	11	6	12	6	13	6	14	6	15
7*	4*	7*	5*	7*	6*	7	7	7	8
8	16	8	17	8	18	8*	0*	8*	1*
9	9	9	10	9	11	9	12	9	13
10*	2*	10*	3*	10*	4*	10*	5*	10*	6*
11	14	11	15	11	16	11	17	11	18
12	7	12	8	12	9	12	10	12	11
13*	0*	13*	1*	13*	2*	13*	3*	13*	4*
14	12	14	13	14	14	14	15	14	16
15*	5*	15	6*	15	7	15	8	15	9
16	17	16	18	16*	0*	16*	1*	16*	2*
17	10	17	11	17	12	17	13	17	14
18*	3*	18*	4*	18*	5*	18*	6*	18	7
19	15	19	16	19	17	19	18	19*	0*
K = 12 i = 16		K = 11 i = 17		K = 10 i = 18		K = 9 i = 19		K = 8 i = 1	
N	a_{16}	N	a_{17}	N	a_{18}	N	a_{19}	N	a_{1}
1	13	1	14	1	15	1	16	1	17
2*	6*	2	7	2	8	2	9	2	10
3	18	3*	0*	3*	1*	3*	2*	3*	3*
4	11	4	12	4	13	4	14	4	15
5*	4*	5*	5*	5*	6*	5	7	5	8
6	16	6	17	6	18	6*	0*	6*	1*
7	9	7	10	7	11	7	12	7	13
8*	2*	8*	3*	8*	4*	8*	5*	8*	6*
9	14	9	15	9	16	9	17	9	18
10	7	10	8	10	9	10	10	10	11
11*	0*	11*	1*	11*	2*	11*	3*	11*	4*
12	12	12	13	12	14	12	15	12	16
13*	5*	13*	6*	13	7	13	8	13	9
14	17	14	18	14*	0*	14*	1*	14*	2*
15	10	15	11	15	12	15	13	15	14
16*	3*	16*	4*	16*	5*	16*	6*	16	7
17	15	17	16	17	17	17	18	17*	0*
18	8	18	9	18	10	18	11	18	12
19*	1*	19*	2*	19*	3*	19*	4*	19*	5*

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E. IMPROVEMENT OF THE JEWISH CALENDAR: THIRD SOLUTION

We have already seen that the difference between the Jewish year and the tropical year amounts to 1 day in 216.2898 years. The cycle of 19 years is actually made up of two cycles: a cycle of 11 years comprising 136 months and a cycle of 8 years comprising 99 months. The eleven Jewish years are shorter than eleven tropical years by about 1.5 days and the eight Jewish years are shorter than the eight tropical years by about 1.5 days.
136M = 4016.16080247 days and 11A = 4017.664187 days.
Difference: – 1.50338453 d.
99M = 2923.52881944 days and 8 A = 2921.937590 days.
Difference: 1.59122944 d.
19 Jewish years exceed 235 Jewish months by 0.08784509 d.

In order to compensate for this difference, one can thus consider introducing a small cycle of Jaffe of 11 years after a number of c cycles of 19 years, when the excess of the Jewish years compared to the tropical years reaches about 1.50 days. The number c of cycles after which we must introduce a cycle of Jaffe is given by c = (1.503384) / (0.08784509) = 17.1140.

We observe that c is identical to p in the second method. We will adopt: c = 17. Thus, after 17 cycles of 19 years or 323 years, always with the traditional order of intercalation of 3, 6, 8, 11, 14, 17 and 19, we will introduce a cycle of 11 years, whose leap years are the years 3, 6, 8 and 11. Then we begin another series of 17 cycles of 19 years, and so on. 323 Jewish years exceed 323 tropical years by 1.49297247 d, but 11 tropical years exceed 136 Jewish months by 1.503384 d. There is nearly perfect compensation. If we consider now 19 cycles of 334 years, we have:
6346 tropical years = 2317826.9935 d
6346 Jewish years = 2317826.8031 d
The 19 cycles of 334 years are shorter than the 6346 tropical years by 0.1903 d.

The knowledge of the length of the tropical year and its evolution over time is insufficient to bother with this difference.

Again, in a cycle of 334 years we have 17 x 235 + 136 = 4131 Jewish months, and in 6346 years we have 78489 Jewish months. In the traditional calendar we would have 334 x 235 = 78490 months.

If we compare this with the second method, we see that we got exactly the same results by means of interventions presenting exactly the same periodicity.

If this process had been implemented at the beginning of the definitive calendar in 4599 AMI (autumn 838), then we would have had the following succession:
4599 – 4921: 1st cycle of 323 years

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4922 – 4932: 1st cycle of 11 years
4933 – 5255: 2nd cycle of 323 years
5256 – 5266: 2nd cycle of 11 years
5267 – 5589: 3rd cycle of 323 years
5590 – 5600: 3rd cycle of 11 years
5601 – 5923: 4th cycle of 323 years
5924 – 5934: 4th cycle of 11 years
5935 – 6257: 5th cycle of 323 years
6258 – 6268: 5th cycle of 11 years
6269 – 6591: 6th cycle of 323 years

This third solution was briefly mentioned by Jaffe[55] and briefly outlined by Feldman.[56]

In this project, we propose to introduce this solution in 6233 after the completion of 328 cycles of 19 years. We then introduce five small cycles of eleven years that were omitted from 6233 until 6287, and 6288 will be the first year of the cycle of 323 years.

F. COMPARISON BETWEEN THESE SOLUTIONS

The three solutions give similar results; they re-establish the calendar in the same position with regard to the sun as it was during the first cycles after 4599, when all the elements of the Jewish calendar were fixed. The first solution seems less precise than the two others; the cycle of 334 years fits better than the cycle of 323 years.

The two last methods, although essentially different – the second is based on the principle of regularly changing the order of intercalations while the third always works with the traditional order of intercalation – have many similarities. In both methods, there appears a regular cycle of 17 x 19 + 11 = 334 years. In this cycle, we have 7 x 19 + 4 = 123 leap years. The excess of the tropical year compared to 12 Jewish months is about 10.8751 days, or 15660.1013 minutes, corresponding to 0.36827 of the Jewish month. We observe that the traditional solution of 7 leap years in 19 years gives the ratio 7 / 19 = 0.36842.

The ratio of our solution 123 / 334 = 0.36826 is nearly equal to the former ratio of 0.36827, proving the quality of the proposed solution.

We have just written that the first solution seems less precise than the other two with regard to the length of the tropical year. Indeed, the difference between the

55 *Korot Heshbon ha-Ibbur*, p. 119-20. See also *Shearim le-Luah ha-Ivri*, p. 151.
56 *Rabbinical Mathematics and Astronomy*, p. 208.

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length of 19 Jewish years and 19 tropical years amounts to about 126.49693 minutes, corresponding to 1 day in 216.2898 years and one Jewish month in 336.1667 cycles of 19 Jewish years.

Indeed, according to Tables 10 to 12, we observe that Nissan 16, at the horizon of 10000 CE will fall, at the earliest, on 19 March, according to the first solution, and on 20 March according to the second and third solutions. Today, the true equinox is on 20–21 March and the mean equinox is on 22–23 March. But the Gregorian year of 365.2425 days is too long compared to the tropical year of 365.24219878 by 0.000302 d, and, after 8100 years,[57] this difference will amount to 2.45 days. The date of the mean equinox will then be 20 March. Therefore, the date of 19 March for the earliest 16 Nissan, given by the first solution seems too early.

In fact, the reality is more complex. As the earth's rotation slows, the day becomes longer and the tropical year shortens. This phenomenon increases the difference between the tropical year and the Gregorian year. According to G. Moyer,[58] the true equinox in 10000 CE would be on 16 March. According to Parisot and Suager,[59] the length of the tropical year is given by:

365.24219878 – (616 x 10^-8) x t, where t is measured in centuries counted from 1900.

Therefore, the length of the tropical year diminishes by 5.32s per millenary.

The length of the day is given by 1 + (1.74 x 10^-8) x t.

Therefore, the length of the day increases by 1.5 milliseconds after a century.

The second effect is negligible, but the first has an important effect in 10000 CE.

The shrinkage of the span of time between 1900 and 10000 CE is then given by the integration of the function (616 x 10^-8) x 100 x t dt from 0 to 81 centuries; this gives (616 x 10^-6) x (1/2) x (81)^2 = 2.02 days.

Thus, at the horizon of 10000 CE, the effect of the diminution of the tropical year is nearly as important as the consequences of the approximation of the length of the Gregorian year. The true equinox would then fall on 16 March and the mean equinox would be on 18 March, at least if we can admit that the span of time between the true and the mean equinox is still two days.

Jean Meeus[60] has indicated that the true equinox in 10000 CE, according to the theory VSOP87 of Bretagnon, is 19 March 10000 CE, 7h 46m Dynamic Time. The

57 Counted from 1900 CE.
58 "The Gregorian Calendar," *Scientific American* (1982).
59 *Calendriers et Chronologie* (Paris: Masson, 1996).
60 Personal communication of 14 August, 2006.

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dynamic time is the uniform time used today in astronomy. The time used in civil life is Universal Time, or Greenwich Mean Time; it depends on the rotation of the earth and it is difficult to predict with precision. The difference $\Delta T = DT - UT$ is approximated by different formulas. Meeus has used the formula:
ΔT (in seconds) = 32 x u^^{2}, with u = (year – 1820) / 100. If year = 10000 CE, then ΔT = 214120 sec = 2d 11h 29m.
This brings us to the date of 16 March, around 20h UT. Meeus insists, however, on the fact that this result is conjectural and uncertain because we are outside the confidence interval of the theory of Bretagnon, and because of the uncertainty of the formula of ΔT depending on the regularity of the earth's rotation.

Thus, at first glance, the first solution corrects the Jewish calendar slightly too much and makes the Jewish calendar retrograde faster than the Gregorian calendar, and even a little too fast with respect to the length tropical year of 1900, contrary to the second and third solutions, which are in accordance with the tropical year. But, when we take into consideration the shrinkage of the tropical year, we observe that the first solution appears to be the best.

In fact, it appears that it is ambitious to define with precision a solution that works perfectly until 10000 CE. It is probable that the solution adopted from among the three proposed solutions will require a slight adaptation at a certain moment in order to take into consideration the diminution of the length of the tropical year, the lengthening of the day, and the irregularities of the earth's rotation. The date of the true equinox in 10000 CE is still conjectural. In any case, the three solutions proposed in this paper are versatile and can be adapted to these new challenges by diminishing c, for example from 17 to 16 if necessary, using periods of 304 years in the first solution and 315 years in the second and third solutions.

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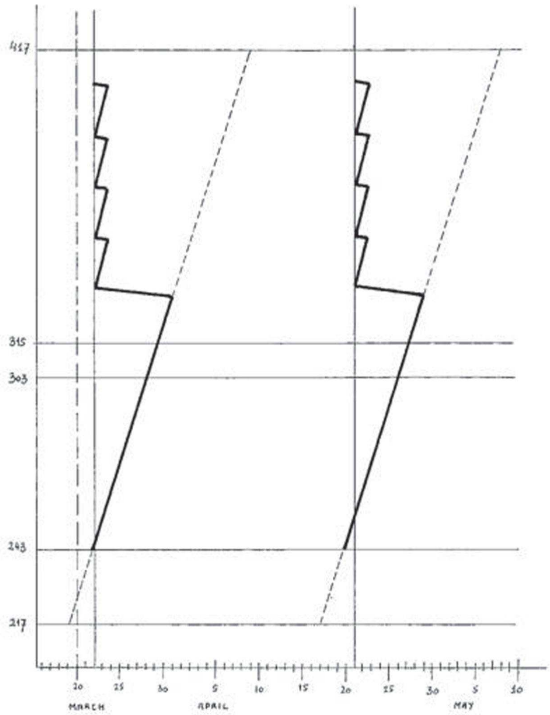

Figure 3: The principle of the improvement of the Jewish calendar according to Solution III. The limits of the date of Nissan 16 in a function of the Jewish year divided by 19.

G. CONCLUSIONS

The reader could have perceived the title of this paper as provocative. In fact, the title was intended to emphasize the similarity between the Gregorian revolution of the Julian calendar and the future revolution of the so-called "Hillel calendar." The similarity between these two revolutions turns on the following points:

1. The necessity of finding a solution that is as simple as possible, and that continues the former rules.
2. The changes should be imperceptible to the common people.
3. This solution should be versatile in order to be long lasting.
4. The difficulty of imposing a solution without provoking a schism.

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Table 9: Traditional Jewish Calendar for the Years 13757–13775

N	Year	Jewish Year	Nissan 16 Gregorian	Tishri 21 Gregorian
19	9996	13756–57	29 May	27 Nov.
1	9997	13757–58	18 May	16 Nov.
2	9998	13758–59	6 May	4 Nov.
3	9999	13759–60	26 May	24 Nov.
4	10000	13760–61	14 May	12 Nov.
5	10001	13761–62	4 May	2 Nov.
6	10002	13762–63	22 May	20 Nov.
7	10003	13763–64	11 May	9 Nov.
8	10004	13764–65	30 May	28 Nov.
9	10005	13765–66	18 May	16 Nov.
10	10006	13766–67	8 May	6 Nov.
11	10007	13767–68	27 May	25 Nov.
12	10008	13768–69	16 May	14 Nov.
13	10009	13769–70	4 May	2 Nov.
14	10010	13770–71	23 May	21 Nov.
15	10011	13771–72	13 May	11 Nov.
16	10012	13772–73	2 May	31 Oct.
17	10013	13773–74	20 May	10 Nov.
18	10014	13774–75	9 May	7 Nov.
19	10015	13775–76	29 May	27 Nov.

Table 10: Improved Jewish Calendar I for the Years 13757–13775

N	Year	Jewish Year	Nissan 16 Gregorian	Tishri 21 Gregorian
19	9996	13756–57	29 March	27 Sept.
1	9997	13757–58	19 March	17 Sept.
2	9998	13758–59	8 April	7 Oct.
3	9999	13759–60	28 March	26 Sept.
4	10000	13760–61	14 April	13 Oct.
5	10001	13761–62	4 April	3 Oct.
6	10002	13762–63	24 March	22 Sept.
7	10003	13763–64	11 April	10 Oct.
8	10004	13764–65	31 March	29 Sept.
9	10005	13765–66	20 March	18 Sept.
10	10006	13766–67	9 April	8 Oct.
11	10007	13767–68	28 March	26 Sept.
12	10008	13768–69	16 April	15 Oct.
13	10009	13769–70	5 April	4 Oct.
14	10010	13770–71	26 March	24 Sept.
15	10011	13771–72	13 April	12 Oct.
16	10012	13772–73	1 April	30 Sept.
17	10013	13773–74	22 March	20 Sept.
18	10014	13774–75	11 April	10 Oct.
19	10015	13775–76	30 March	28 Sept.

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Table 11: Improved Jewish Calendar II for the Years 13757–13775

N	Year	Jewish Year	Nissan 16 Gregorian	Tishri 21 Gregorian
19	9996	13756–57	29 March	27 Sept.
1	9997	13757–58	18 April	17 Oct.
2	9998	13758–59	8 April	7 Oct.
3	9999	13759–60	28 March	26 Sept.
4	10000	13760–61	14 April	13 Oct.
5	10001	13761–62	4 April	3 Oct.
6	10002	13762–63	24 March	22 Sept.
7	10003	13763–64	11 April	10 Oct.
8	10004	13764–65	31 March	29 Sept.
9	10005	13765–66	20 March	18 Sept.
10	10006	13766–67	9 April	8 Oct.
11	10007	13767–68	28 March	26 Sept.
12	10008	13768–69	16 April	15 Oct.
13	10009	13769–70	5 April	4 Oct.
14	10010	13770–71	26 March	24 Sept.
15	10011	13771–72	13 April	12 Oct.
16	10012	13772–73	1 April	30 Sept.
17	10013	13773–74	22 March	20 Sept.
18	10014	13774–75	11 April	10 Oct.
19	10015	13775–76	30 March	28 Sept.

Table 12: Improved Jewish Calendar III for the Years 13757–13775

N	Year	Jewish Year	Nissan 16 Gregorian	Tishri 21 Gregorian
19	9996	13756–57	29 March	27 Sept.
1	9997	13757–58	18 April	17 Oct.
2	9998	13758–59	8 April	7 Oct.
3	9999	13759–60	28 March	26 Sept.
4	10000	13760–61	14 April	13 Oct.
5	10001	13761–62	4 April	3 Oct.
6	10002	13762–63	March 24	22 Sept.
7	10003	13763–64	11 April	10 Oct.
8	10004	13764–65	31 March	29 Sept.
9	10005	13765–66	20 March	18 Sept.
10	10006	13766–67	9 April	8 Oct.
11	10007	13767–68	28 March	26 Sept.
12	10008	13768–69	16 April	15 Oct.
13	10009	13769–70	5 April	4 Oct.
14	10010	13770–71	26 March	24 Sept.
15	10011	13771–72	13 April	12 Oct.
16	10012	13772–73	1 April	30 Sept.
17	10013	13773–74	22 March	20 Sept.
18	10014	13774–75	11 April	10 Oct.
19	10015	13775–76	30 March	28 Sept.

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Furthermore, in both cases, the former Julian calendar and the present Jewish calendar could each continue to work and fulfill their respective roles despite a shift with regard to the tropical year; the Egyptian calendar also played this role despite a much greater shift.

In both cases, however, religious reasons are the primary cause making it necessary to re-establish the correct correspondence with the solar year. In the case of the Julian calendar, it was Easter, which was connected to the equinox. It was supposed to occur on 21 March, but the spring equinox was shifting toward winter and Easter was shifting toward summer. In our Jewish calendar, it is Passover that is shifting toward summer at the rate of one day in 216.2898 tropical years. More precisely, Nissan 16 should fall in the span of time between the Gregorian dates of 22 March and 21 April,[61] i.e. in the month following the mean vernal equinox. If

61 It has been objected that this final limit, the "*terminus ad quem*" is not universally accepted. Furthermore, Professor Merzbach had objected that Maimonides in *Hilkhot Kiddush ha-Hodesh* (H.K.H.) IV: 2 mentions an initial limit, "*terminus a quo*," but mentions no final limit. I think that these objections are not significant.

1. In Deut. 16:1: .....שמור את חדש האביב ועשית פסח
   In Exod. 13:4: ....היום אתם יצאים בחדש האביב
   In Exod. 13:4: ועבדת את העבודה הזאת בחדש הזה
   The biblical text indicates, without any doubt, that Pesah must belong to the month of the spring, the month that Rashi calls ניסן של חמה and also ניסן של תקופה.
2. Maimonides in H.K.H. IV:1 writes that Nissan must fall during the month of spring:
   כדי שיהיה הפסח באותו זמן שנאמר שמור את חדש האביב שיהיה חדש זה בזמן האביב. ולולא הוספת החדש הזה הפסח בא פעמים בימות החמה ופעמים בימות הגשמים.
   Pesah in summer is thus as disturbing as in winter.
3. Nissan 15 (according to Maimonides, but Nissan 16 according to R. Abraham bar Hiya) may not occur too early, before 22 March, because we need the maturity of the barley. Nowever, it also cannot occur too late, after 21 April, after the barley has grown, when the population is waiting for its harvesting, and when the maturity of the wheat is approaching, announcing the time of Shavuot.
4. Why did Maimonides not mention a final limit in H.K.H. IV: 2?
   a. Maimonides uses the text of the dictum of Rabbi Huna bar Abin in B. Rosh ha-Shanah 21a.
   b. We are in the section of the empirical calendar or vision calendar. The year has 12 months except if we decide to give it a 13th month in order to make it a leap year. The fear is that the year will be too short and that Nissan 16 will arrive too early, before the mean spring equinox. The regular application of the rule of Shitsar prevents this occurrence. But there is no danger and no possibility that Nissan 16 will occur after the month of the spring. The mechanism prevents Nissan 16 from moving from the equinox and another lunation intercalating itself between the equinox and Nissan 16. Indeed, the years always have 12 months unless the month of Nissan would arrive too early and Nissan 16 would occur before the day of the mean equinox.

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we accept that the Jewish calendar was perfectly centered in the 243rd cycle, it has shifted since then by 5.4 days.[62] This already represents half the shift reached by the Julian calendar at the time of the Gregorian revolution at the end of the 16th century.

Among the reasons that make it urgent[63] to solve the problem of the shift of the Jewish calendar, we can enumerate:

1. The Jews have always been proud of the exactitude of their calendar and they referred with pride to the verse "because it is your wisdom and your understanding among the people who will hear these rules..." Deut. 4:6. It would be difficult to face a situation where our calendar is wrong.
2. If we remember that Jews and Muslims mocked the incoherence of the Julian calendar and the Christian festivals, we can imagine the reaction of the Gentile world should the shift of the Jewish calendar become generally known.
3. It is well known in the Talmud that the date of Jewish festivals is fixed by the Jewish people and imposed upon God and His servants. This is clearly the significance of the introduction of the Sephardi and Hassidic *kedusha* of the *mussaf* of Shabbat and festivals:

   כתר יתנו לך ... מלאכים המוני מעלה עם עמך ישראל קבוצי מטה.

   Therefore, the heavy divine penalties (*karet*) imposed on the violator of such infractions as the violation of the fast of Yom Kippur or of the interdiction of eating leaven on Passover, depend on the human fixation of the year. Similarly, the dates of the annual "judicial appearances" of Tishri before the Celestial Court are in fact fixed by the accused. In the Talmud it says:[64]

   מלמד שאין ב"ד של מעלה נכנסין לדין אא"כ קידשו ב"ד של מטה את החודש...

   All these elements are sufficient to convince us that it would be courteous to work with a correct calendar and to summon the Celestial Court at the proper dates without giving the impression that we are trying to win time and delay the case.

Therefore, the lack of a reference to a "*terminus ad quem*" cannot be understood as meaning that there is no final limit for Nissan 16.

5. In *Sefer Yessod Olam*, Book 4, chap. 13, p. 26a column 2:
...למדת שדין תורה הוא שלא לעשות הפסח אלא בפרק האמור הנקרא חודש האביב הן בתחילתו כמו שכן הוא בכל שנות י"ו למחזור או באמצעיתו כמו שכן הוא ברוב שנות המחזור או בקרוב מכדי סופו כמו שכן הוא בשנת ח' למחזור.

62 R. Abraham ibn Ezra had already noted a shift of 2 days in 1147. See *Sefer ha-Ibbur*, p. 9b and 10a.

63 Anything is relative: we propose the implementation of a solution in about 450 years, or if not possible, 334 years later.

64 B. Rosh ha-Shanah 8b.

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4. The respect of our calendar by Jews. When the discrepancy of the Jewish calendar increases and becomes widely known, I am afraid that the observance of the Jewish holidays will dramatically diminish. Today, there is still a general respect among nearly all Jews for the fast of Yom Kippur and the *seder* of Passover. What will the situation be after a shift of two weeks and after a shift of one month, when the Jewish calendar will have lost its credibility?
5. A draft of an article, of which I was co-reviewer, wrote that there is no other solution for improving the Jewish calendar than waiting until the shift reaches the length of a lunar month, i.e. the beginning of the cycle 579 or 10983 AMI. At this moment, one month should be suppressed. This solution would be similar to the jump of ten days made in 1582 on the occasion of the Gregorian revolution according to the solution proposed by Lilio. Such a solution would be a brutal and traumatizing one. Imagine in 10983, at the end of Tishri, that one announces in the synagogues at the benediction of the month that the next month will be Kislev! Such a solution would be unacceptable!

It seems urgent to react in a timely manner, and implement a solution while we still have command of the situation, and not to wait until it becomes too late, when the problem is known by all and debated in the international press.

The object of the present paper is to present three acceptable, smooth solutions, without any brutal jump. The common layman wouldn't even notice anything.

The first solution probably seems to correspond to what the old council of intercalation would have enacted if it were confronted with this problem. Maybe this was already their initial project, since we see that they debated the order of intercalation at the moment of its implementation. It is likely that they were aware of the influence of this order on the position of the Jewish calendar with regard to the solar year.

The second solution is similar to the first; it seems more accurate at the price of greater complication.

The third solution seems to me the simplest, and has the same accuracy as the second. I would champion this last solution. Indeed, this solution will be understood easily. The solutions based on the permutation of the intercalation cycles are conceptually difficult. Furthermore, although it seems that the Jewish council of intercalation discovered experimentally that the shift of the Jewish calendar with regard to the solar equinox involves a permutation of the order of intercalation, such solutions could be perceived as being influenced by the Gregorian computation. This third method has one weak point, however: the correction, when it is implemented, is not instantaneous and requires a period of transition, which will

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be all the longer as we delay its implementation.

In light of the difficulties met in the implementation of the Gregorian revolution and taking into consideration the pusillanimity and the ideological division of the Jewish people, I think that 450 years is not too long a period to convince the different influential circles of the necessity of this revolution. If really necessary, the revolution could even be delayed by another 334 years; but I am persuaded that when the time comes, the adopted solution will be one of the three solutions presented above. It is clear that the implementation of this calendar revolution requires the existence of a central and authoritative rabbinical council. The Jewish people cannot afford itself a new schism. Hopefully, this important delay will allow the emergence of an authoritative and respected chief rabbinate, independent from the political streams, in accordance with the hopes raised by the greatness of the first chief rabbis of Israel.

Mathematical Supplement

A. THE CLASSICAL CALENDAR

1. Notations and Definitions

M = 29d 12h 793ch = 29. 530594136 d = (765,433 / 25,920) d.
M / 19 = 1.55424179662118 d = (1 + 272953 / 492480) d.
m = 1 h 485 ch.
h = m / 19
B = 0d 9h 642ch = 0.399768519 d.
M is the length of the Jewish lunation.
m is the excess of 19 Julian years with regard to 235 Jewish lunations.
Jewish years are counted from the *molad Beharad*, (2) – 5 – 204; this is the common style AMI; the ancient chronologists used to count the years from *molad Veyad*, (6) – 14 twelve months later than *Beharad*; this was the style AMII.
F is the number of elapsed months from *molad* Nissan year 1 until the considered *molad* Nissan.
Ft is the number of elapsed months from *Beharad* until the considered *molad* Tishri.
G is the number of leap years (and the number of leap months) from *molad* Nissan year 1 until the considered *molad* Nissan.
Gt is the number of leap years (and the number of leap months) from *molad* Tishri year 1 until the considered *molad* Tishri.
Q is the number of years between *molad* Nissan of year 1 until *molad* Nissan of the examined year.

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N is the Jewish year in which the considered month of Nissan finds itself; N = Q + 1. A is a Jewish year beginning after the considered month of Nissan; it is preceded by N years and A = N + 1 = Q + 2. Without the use of Q and A, N can also represent the beginning of a Jewish year.

The total number of months from Nissan of the first year of the era until the *molad* of Nissan Q years later, i.e. the ***molad*** **Nissan** of the year N = Q +1 is given by:[65]

(1) F = INT [(235Q + 8) / 19]

The total number of leap years and leap months from Nissan of the first year of the era until the *molad* of Nissan Q years later, i.e. the *molad* Nissan of the year N = Q + 1 is given[66] by:

(2) G = INT [(7Q + 8) / 19]

The total number of months from *Beharad* until the *molad* of Tishri N years later, i.e. the *molad* of Tishri of the Jewish year A = N + 1 is given by Ft = 12 + F with N = Q + 1 in (1).

Thus Ft = 12 + INT [(235(N – 1) + 8) / 19] = INT [(228 + 235N – 235 + 8) /19] = INT [(235N + 1) /19]

(3) Ft = INT [(235N + 1) / 19]

The total number of leap years and leap months from *Beharad* until the *molad* Tishri N years later, i.e. the *molad* of Tishri of the Jewish year A = N + 1 is given by:

(4) Gt = INT [(7N + 1) / 19]

All these formulas can be checked in Table 13.

65 See Y. Loewinger, *Al ha-Sheminit*, p. 55.

66 Ibid., p. 56.

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Table 13: Table of the Computed Values of Different Functions

N	A	B	C	D	E	F	G	H	I
1	12	12	0	8	7	7	17	10	0
2	24	25	0	15	14	14	10	3	1
3	37	37	1	3	2	2	3	15	1
4	49	49	1	10	9	9	15	8	1
5	61	62	1	17	16	16	8	1	2
6	74	74	2	5	4	4	1	13	2
7	86	87	2	12	11	11	13	6	3
8	99	99	3	0	18	-1	6	18	3
9	111	111	3	7	6	6	18	11	3
10	123	124	3	14	13	13	11	4	4
11	136	136	4	2	1	1	4	16	4
12	148	148	4	9	8	8	16	9	4
13	160	161	4	16	15	15	9	2	5
14	173	173	5	4	3	3	2	14	5
15	185	185	5	11	10	10	14	7	5
16	197	198	5	18	17	17	7	0	6
17	210	210	6	6	5	5	0	12	6
18	222	223	6	13	12	12	12	5	7
19	235	235	7	1	0	0	5	17	7
20	247	247	7	8	7	7	17	10	7

A = INT [(235N+1)/19]
B = INT [(235Q+8)/19]
C = INT [(7N+1)/19]
D = [7N+1] mod 19
E = [7N] mod 19
F = {[7N+1] mod 19}-1
G = [12N+5] mod 19
H = [12N+17] mod 19
I = INT [(7Q+8)/19]

2. The *Molad* Expressed in the Julian Calendar

The *molad* of *Beharad* occurred on Sunday, 6 October –3760 or 6 October 3761 BCE, at 11 h 204 ch, p.m. civil time of Jerusalem corresponding to 5 h 204 ch in the second Jewish weekday. This represents the epoch of the Jewish calendar; this epoch will be noted as *e*. This epoch was thus at the end of a bissextile Julian year.

If the length of twelve Jewish lunations, i.e. of twelve synodical months, were equal to the length of a Julian year, i.e. 365.25 days, then after N years counted from the epoch, the *molad* of year N + 1 would be:

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Mol = e + 0.25 x (N) mod 4

(N) mod 4 or $[N]_4$ or also in Matlab language, mod (N, 4), read "N, modulo 4" represent the remainder of the division of N by 4.

In reality, the mean Jewish year is shorter than the Julian year by h = m/19 and therefore, after N years, we have the relation:

Mol = e + 0.25 x $[N]_4$ – N x h.

We must now compare the mean Jewish year with the true Jewish years of 12 or 13 lunar months.

Except for year 8 of the cycle, the real Jewish year always ends before the end of the mean Jewish year and, therefore, we must subtract a new term in the former relation. The mean Jewish year has a length of 235 x M/19, while the real year has a length of 12 M for a regular year or 13 M for a leap year.

One ascertains that the difference between the mean years and the real years is expressed by $[7N]_{19}$ x M/19, except at the end of the eighth year, when the real Jewish year ends M/19 after the mean Jewish year.

In order to consider this case, one must introduce the term $\{[7N + 1]_{19} - 1\}$ x M/19, which expresses the fact that the *molad* Tishri of the ninth year is exceptionally delayed with regard to the beginning of the mean Jewish year.

The final formula is now:

(5) Mol = e + 0.25 x $[N]_4$ – N x h – $\{[7N + 1]_{19} - 1\}$ x M/19

It gives the *molad* of Jewish year A = N + 1 expressed in the Julian calendar, i.e. in October of Julian year N – 3761.

3. The *Molad* of Jewish Year A Expressed in Days and Fraction Counted from Noon of Julian September

a. The Expression of the Molad of Year A

In formula (5): the *molad* of *Beharad* occurred on 6 October, at 23h 204ch, or on 36 September at 11h 204ch, counted from noon.

e = 36.466203703703

h = 0.00317779402

M/19 = 1.55424179662118

N = A – 1

(6) Mol = 38.02362329434697 – 0.003177794022A + 0.25 $[A - 1]_4$
– 1.554241796621 $[7A - 6]_{19}$

If we consider the identity: $[12A + 5]_{19} + [7A - 6]_{19} = 18$, we can transform (6) into (7).

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Mol can be replaced by M + m, in which M represents the day of Julian September at noon and m the fraction of the Julian day counted from noon.

(7) $M + m = 10.04727095516575 - 0.003177794022A + 0.25\,[A-1]_4 + 1.554241796621\,[12A+5]_{19}$

M is the date of Julian September at noon preceding the *molad* of Tishri or coinciding exceptionally with it, and m is the fraction of day between noon and the *molad* of Tishri.

This is the equivalent for Tishri of the celebrated formula of Gauss for Nissan.

b. The Coefficient $a = [12A+5]_{19}$

The formula (10) shows that each Jewish year of the intercalation cycle is characterized by a coefficient a, depending on the order of that year in the intercalation cycle of 19 years.

Table 14: Table of the Coefficient $a = [12A+5]_{19}$

A = 0	a = 5	A = 5	a = 8	A = 10	a = 11	A = 15	a = 14
1	17	6*	1	11*	4	16	7
2	10	7	13	12	16	17*	0
3*	3	8*	6	13	9	18	12
4	15	9	18	14*	2	0 or 19*	5

If we classify these coefficients according to a numerical order, we get:

Table 15: Table of the Coefficient a Classified According to the Numerical Order

Years L	Years L	Years L – 1	Years L – 1	Years L + 1, L – 1	Years L+1, L–1	Years L+1	Years L+1
a	A	a	A	a	A	a	A
0	17*	7	16	12	18	14	15
1	6*	8	5	13	7	15	4
2	14*	9	13			16	12
3	3*	10	2			17	1
4	11*	11	10			18	9
5	19*						
6	8*						

We see thus that a > 6 corresponds to regular years while a <= 6 corresponds to leap years. a > 11 corresponds to years L + 1 (year following a leap year) or to years L +–1 (year following and preceding a leap year).

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c. *Implementation of the Rules of the Jewish Calendar*

J is the weekday of September M. The convention of the numbering of the weekdays is the following:

J = 0 corresponds to Sunday,
J = 1 corresponds to Monday,
J = 2 corresponds to Tuesday,
J = 3 corresponds to Wednesday,
J = 4 corresponds to Thursday,
J = 5 corresponds to Friday,
J = 6 corresponds to Saturday.

t = 0 corresponds to noon, t = 0.25 corresponds to 6 p.m., i.e. the beginning of the next Jewish day. The Julian day begins 12 hours after the civil day and the next Jewish day begins six hours, or 0.25d, after the beginning of the former Julian day. In other words, the Julian Sunday begins on Sunday at noon, but six hours later, the Jewish Monday begins.

Rule 1. If the *molad* falls at or after 18 hours, measured by Jewish hours (noon), Rosh ha-Shanah is postponed to the following day. The day of Rosh ha-Shanah will then be J + 1.

Rule 2. If the *molad* falls on ADU, Sunday, Wednesday or Friday, Rosh ha-Shanah is postponed to the day after. Thus if J = 0, 3 or 5, Rosh ha-Shanah will be on day J + 1.

Rule 3. Rules 1 and 2 are cumulative. If the *molad* falls on Saturday, Tuesday or Thursday at 18 Jewish hours or later, Rosh ha-Shanah is postponed by two days and will be on day J + 2.

Rule 4. If the *molad* of a common year falls on Tuesday at 9h 204 ch, Jewish time, Rosh ha-Shanah is postponed to Thursday. In other words, if J = 1, a > 6 and t >= 0.632870370370 or (311676 / 492480), Rosh ha-Shanah will be on day J + 3 (Thursday).

Rule 5. If the *molad* of a common year of the type L + 1 (following a leap year) falls on Monday at 15h 589 ch. Jewish time, Rosh ha-Shanah will be postponed until Tuesday. In other words, if J = 0, a > 11 and t > = 0.897723765435 or (442111 / 492480), Rosh ha-Shanah will be on Tuesday, J + 2.

We know that 6 October, –3760 was a Sunday and therefore 0 September, – 3760 was a Saturday with J = 6 and September M, –3760 was the weekday J = M + 6. We are now examining the weekday of September M of the current civil year B = A – 3761. The number of elapsed days between September M, –3760 and September M, A – 1 years later is 365.25 (A – 1) – 0.25 $[A - 1]_4$. The weekday of

September M of the current civil year B = A – 1 will then be:

$J = [M + 6 + 365.25\ (A – 1) – 0.25\ [A – 1]_4]_7$

$J = [M + 6 + 365A + A/4 – 364 – 1 – 1/4 – 0.25[A – 1]_4]_7$

$J = [M + 5 + A + 8A/4 – 8/4 – (8/4)*[A – 1]_4]_7$

$J = [M + 5 + 3A – 2 – 2\ [A – 1]_4]_7$

(8) J = [M + 3A + 3 + 5 [A – 1] mod 4] mod 7

It is also possible to calculate the shift of the weekday between September M, –3760 and September M, of the civil year B = A – 3761.

$J = [M + 6 + (A – 1) + INT\ [(A – 1)/4]\ ]_7 = [5 + M + A + (A – 1)/4 – 0.25[A – 1]_4]_7$

$J = [5 + M + A + 8A/4 – 8/4 – (8/4)*[A – 1]_4]_7$

(8) J = [M + 3A + 3 + 5 [A – 1] mod 4] mod 7

Conclusions:

If J = 2, 4 or 6, Rosh ha-Shanah is on the day J + 2, Julian September M + 2.

If J = 1, a > 6 and t >= 0.632870370370, Rosh ha-Shanah is the day J + 3, Julian September M + 3.

If J = 0, a > 11 and t >= 0.897723765435, Rosh ha-Shanah is the day J + 2, Julian September M + 2.

In all other cases, Rosh ha-Shanah is the day J + 1, Julian September M + 1.

4. The Formula of Gauss for Nissan of the Jewish Year A[67]

If we consider the fictitious moment corresponding to *molad* Tishri of the next Jewish year A + 1, minus 162 days, and we note this moment as P + p, P being the date in Julian March of noon preceding this fictitious moment or coinciding exceptionally with it, and p being the fraction of day counted from this noon until this fictitious moment, we will deduce P + p from the formula (7) if we say that September M minus 162 days = March (M + 22) = March P.

Thus P = M + 22. In (7) we replace A by A + 1:

$M + m = 10.04727095516575 – 0.003177794022\ (A + 1) + 0.25[A]_{19} + 1.554241796621\ [12A + 17]_{19}$

Introducing P + p = M + m + 22:

(9) $P + p = 32.044093161144 + 1.554241796621\ [12A + 17]_{19} + 0.25\ [A]_4 – 0.003177794022A$

67 Gauss, *Werke* VI Bd. 1874, pp. 80-81. *Berechnung des Judischen Osterfestes, Zach's Monatliche Correspondenz zur Beforderung der Erd- und Himmelskunde* (May 1802), p. 435. Note: 162 days after March 23 is September 1, and 163 days after Nissan 15 (1st day of Pesah) is Tishri 1 (1st day of Rosh ha-Shanah).

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P is a date in March at noon preceding the *molad* of next Tishri minus 162 or coinciding with it, and p is the fraction of day counted from that noon until the *molad* of the next Tishri minus 162 days. This is the formula of Gauss. We must now implement the rules of the Jewish calendar in this formula.

0 March, –3760 was a Thursday, J = 4, thus March P, –3760 was the weekday $J = P + 4$ and September M, –3760 was the weekday $J = P + 4 + 162 = P + 5$.

Now September M, Civil year $B = A + 1 - 3761 = A - 3760$, A years later will be the weekday: $J = [P + 5 + A + \text{INT}(A/4)]_7 = [P + 5 + A/4 - (1/4)\,[A]_4]_7$

$J = [P + 5 + A + 8A/4 - (8/4)*[A]_4]_7$

(10) J = [5 + P + 3A + 5[A] mod 4] mod 7

The coefficient a characterizing the different years of the intercalation cycle has now become: $a = [12A + 17]_{19}$.

We know that Pesah is 163 days before Rosh ha-Shanah, therefore the weekday of Pesah will be two days before the weekday of the following Rosh ha-Shanah. Therefore J will be replaced by J – 2 and M will be replaced by $P = M - [162]_7 = M - 1$.

If J = 2, 4 or 6, Pesah is the day J, Julian March P + 1.

If J = 1, a > 6 and p >= 0.632870370, Pesah is the day J + 1, Julian March P + 2.

If J = 0, a > 11 and p >= 0.897723765, Pesah is the day J, Julian March P + 1.

In all other cases, Pesah is the day J – 1, Julian March P.

To the best of my knowledge, this is the shortest and the most elegant demonstration of the formula of Gauss for Nissan ever written.

5. The Formula of Schram[68]

a. Expression of the Molad of Jewish Year A

The Julian Period was defined by Scaliger; its origin was Monday, 1 January –4712 at noon. This moment is the epoch of the Julian period; at this moment, the elapsed time of the Julian period was 0 and, according to the convention of this paper, the weekday was $[0 + 1]_7 = 1$, corresponding to Monday.

The era of the creation of the world was Sunday, 6 October –3760 at 23 h 204 ch. At this moment: 347,997.466203703703 days had elapsed from the epoch of the Julian period.

Indeed, on 1 January –3760, 4712 – 3760 = 952 years had elapsed corresponding to 952 x 365.25 = 347,718 days of the Julian period. Year –3760 is bissextile and

68 Schram, *R. Kalendariographische und Chronologische Tafeln* (Leipzig, 1908).

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therefore, on 7 October –3760 Julian, at noon, 347,718 + 280 = 347,998 days had elapsed from the beginning of the period. This day was $[347,998 + 1]_7 = 1$ or Monday.

The mean Jewish year has a length of 235M/19 = 365.246822205978 d.

If the Jewish years were identical to the mean year, then, after N years, the *molad* expressed in days and hours of the Julian period would be:
Mol = e + (235M/19) x N.

In reality, the real Jewish years are made up of 12 or 13 months. Ordinary years have a length of 12 months while leap years have a length of 13 months. One ascertains that the difference between mean Jewish years and real Jewish years is expressed by: $[7N]_{19}$ x M/19, except at the end of the eighth year, when the real Jewish year ends M/19 after the mean Jewish year. In order to consider this case, we must introduce the term $\{[7N + 1]_{19} – 1\}$ x M/19, which expresses the fact that the *molad* of the ninth year of the cycle is exceptionally delayed with regard to the beginning of the mean Jewish year. We can then write:

(11) Mol = T + t = e + (235M/19) x N – $\{[7N + 1]_{19} – 1\}$ x M/19

This formula can also be deduced from formula (3).
Ft = INT[(235N + 1)/19] = 12N + INT [(7N + 1)/19] = 12N + (7N + 1)/19 – $[7N + 1]_{19}$ x 1/19.
Therefore Mol = e + M x Ft (**11 bis**)
Mol = e + 12N x M + M x (7N + 1)/19 – M x $[7N + 1]_{19}$ x 1/19.
Mol = e + 12N x M + 7N x M/19 + M/19 – M x $[7N + 1]_{19}$ x 1/19.
Mol = e + 235N x M/19 – $\{[7N + 1]_{19} – 1\}$ x M/19.
T represents the number of whole days elapsed from the epoch of the Julian period and t the last fraction of a day counted from noon until the moment of the *molad*. If we replace N by A – 1, A being the Jewish year that begins:
T + t = e + (235M/19) x N – $\{[7A – 6]_{19} – 1\}$ x M/19.
e = 347,997.466203703703
235M/19 = 365.246822205978
M/19 = 1.554241796621
Making use of the identity: $[12A + 5]_{19} + [7A – 6]_{19} = 18$, we find:
(12) T + t = e + (235M/19) x N + $[12A + 5]_{19}$ x M/19 – 18M/19 + M/19

After calculations we find:
(13) T + t = 347605.797270955166 + 365.246822205978A + 1.554241796621 $[12A + 5]_{19}$

T is the number of whole days elapsed of the Julian period, and t is the fractional part of a day counted from noon until the *molad* Tishri of year A. Thus, at the moment of the *molad*, T whole days and t parts of the T + 1 day of the Julian period have elapsed. At noon, at the end of Julian day T and at the beginning of Julian day T + 1, according to the rule of Scaliger, the weekday is J = $[T + 1]_7$; J = 0 corresponding to Sunday.

b. The Implementation of the Rules of the Jewish Calendar in the Formula of Schram

Rule 1. If the *molad* falls at or after 18 hours, measured by Jewish hours (noon), Rosh ha-Shanah is postponed to the following day. The day of Rosh ha-Shanah will then be J + 1.

Rule 2. If the *molad* falls on ADU, Sunday, Wednesday or Friday, Rosh ha-Shanah is postponed to the day after. Thus if J = 0, 3 or 5, Rosh ha-Shanah will be the day J + 1.

Rule 3. Rules 1 and 2 are cumulative. If the *molad* falls on Saturday, Tuesday or Thursday at 18 Jewish hours or later, Rosh ha-Shanah is postponed by two days and will be the day J + 2.

Rule 4. If the *molad* of a common year falls on Tuesday at 9h 204ch, Jewish time, Rosh ha-Shanah is postponed until Thursday. In other words, if J = 1, a > 6 and t >= 0.632870370370 or (311676 / 492480), Rosh ha-Shanah will be the day J + 3 (Thursday).

Rule 5. If the *molad* of a common year of the type L + 1 (following a leap year) falls on Monday at 15h 589ch Jewish time, Rosh ha-Shanah will be postponed to Tuesday. In other words, if J = 0, a > 11 and t >= 0.897723765435 or (442111/492480), Rosh ha-Shanah will be on Tuesday, J + 2.

Conclusion:
If J = 2, 4 or 6, Rosh ha-Shanah is on the day J + 2.
If J = 1, a > 6 and t >= 0.632870370370, Rosh ha-Shanah is the day J + 3.
If J = 0, a > 11 and t >= 0.897723765435, Rosh ha-Shanah is the day J + 2.
In all the other cases, Rosh ha-Shanah is the day J + 1.

6. The Formula of Gauss for Tishri of Jewish Year A, Deduced from the Formula of Schram

On 1 January –3760 at noon, 347,718 days of the Julian period had elapsed. On 0 January –3760 at noon, 347,717 days of the Julian period had elapsed. On 0 September –3760 at noon, 244 more days of the Julian period have elapsed and on September M, –3760 at noon, M days more had elapsed, i.e. 347961 + M.

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At the beginning of the Jewish year A, A – 1 years later, September M will be on the day of the Julian period given by the following relationship:
$T = 347961 + M + 365.25 \times (A - 1) - 0.25[A - 1]_4$
(14) $\mathbf{T = 347595.75 + M + 365.25A - 0.25[A - 1]_4}$

Now, if T is the integer of the *molad* of Tishri, expressed in the Julian period, which is equal to T + t, then:

(13) $\mathbf{T + t = 347605.797270955166 + 365.246822205978A + 1.554241796621 \times [12A + 5]_{19}}$

(14) $\mathbf{T = 347595.75 + M + 365.25A + 0.25[A - 1]_4}$

(15) $\mathbf{t = m}$

Equation (13) is the formula of Schram, and equation (14) has just been established above. The equation (15) expresses the fact that we consider that the *molad* Tishri of Jewish year A is T + t, expressed in the Julian period, and M + m, expressed according to the notation of Gauss. M is the date in Julian September at noon preceding the *molad* and m is the fraction of day counted from noon preceding the *molad*. Subtracting (14) and (15) from (13) we get:
(7) $\mathbf{M + m = 10.047270955166 - 0.003177794022A + 1.554241796621\ [12A + 5]_{19} + 0.25[A - 1]_4}$

M is the date of Julian September at noon preceding the *molad* Tishri or coinciding exceptionally with it, and m is the day fraction counted from noon until the *molad* of Tishri of Jewish year A.

The weekday of this *molad* is given by $J = [T + 1]_7$. We can write:
$J = [T + 1]_7 = [347961 + M + 365.25(A - 1) - 0.25[A - 1]_4 + 1]_7$
$= [347962 + M + 365A + A/4 - 365 - 1/4 - 0.25[A - 1]_4]_7$
$= [6 + M + A + (8A)/4 - 1 + 20/4 + (20/4) \times [A - 1]_4]_7$
$= [5 + M + 3A + 5 + 5[A - 1]_4]_7$

(8) $\mathbf{J = [M + 3A + 3 + 5[A - 1] \bmod 4] \bmod 7}$

Conclusions:
If J = 2, 4 or 6, Rosh ha-Shanah is on the day J + 2, Julian September M + 2.
If J = 1, a > 6 and t >= 0.632870370370, Rosh ha-Shanah is the day J + 3, Julian September M + 3.
If J = 0, a > 11 and t >= 0.897723765435, Rosh ha-Shanah is the day J + 2, Julian September M + 2.
In all the other cases, Rosh ha-Shanah is the day J + 1, Julian September M + 1.

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7. The Formula of Gauss for Nissan of Jewish Year A, Deduced from the Formula of Schram

On 0 January –3760, a number of 347717 days had elapsed from the beginning of the Julian period. Now the year –3760 is bissextile and on 0 March, at noon, 60 more days had elapsed, i.e. 347717 + 60 = 347777 days and on March P, at noon, of the same year, the number of elapsed days of the Julian period was 347777 + P.

If we consider the same March P in the Jewish year A, A years later, we will have the following day of the Julian period:

$T = 347777 + P + 365.25A - 0.25[A]_4$

Let us consider that this March P was 162 days before the day T and, more precisely, let us consider the moment 162 days before the *molad* of Jewish year A + 1; this moment is defined in the Julian period by T + t – 162 and, with regard to March, by P + p, P being the date of Julian March at noon preceding this fictitious moment and p the fraction of day counted from noon until this moment.

Replacing A by A + 1 in (13), we can write the following relations:

$T + t = 347605.797270955166 + 365.246822205978\ (A + 1) + [12\ (A + 1) + 5]_{19}$ x 1.554241796621

$T - 162 = 347777 + P + 365.25 \text{ x } A - 0.25 \text{ x } [A]_4$

$t = p$

Subtracting the two last relations from the first, we find:

(9) $P + p = 32.044093161144 + 1.554241796621\ [12A + 17]_{19} + 0.25[A]_4 - 0.003177794022A$

P is a date in March at noon preceding the *molad* of next Tishri minus 162 and p is the fraction of day counted from that noon until the *molad* of next Tishri minus 162 days. This is the formula of Gauss. We must now implement the rules of the Jewish calendar in this formula. The basic weekday to consider is $J = [T + 1]_7$, where T is the integer of T + t, the *molad* of Tishri of next year, Jewish year A + 1. Thus:

$J = [347777 + P + 365.25 \text{ x } A - 0.25 \text{ x } [A - 1]_4 + 162 + 1]_7$.

$J = [3 + P + A + A/4 - [A]_4/4 + 1 + 1]_7$

(10) J = [5 + P + 3A + 5[A] mod 4] mod 7

The coefficient *a* characterizing the different years in the intercalation cycle has now become: $a = [12A + 17]_{19}$. We can now implement the rules of the Jewish calendar.

We know that Pesah is 163 days before Rosh ha-Shanah, therefore the weekday of Pesah will be two days before the weekday of the following Rosh ha-Shanah. Therefore J will be replaced by J – 2 and M will be replaced by $P = M - [162]_7 = M - 1$.

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If J = 2, 4 or 6, Pesah is the day J, Julian March P + 1.
If J = 1, a > 6 and p >= 0.632870370, Pesah is the day J + 1, Julian March P + 2.
If J = 0, a > 11 and p >= 0.897723765, Pesah is the day J, Julian March P + 1.
In all other cases, Pesah is the day J – 1, Julian March P.

8. Numerical Examples

Let us consider the Jewish years 5767 and 5768.
Year 5767: *Molad* (0) – 1 – 672. Rosh ha-Shanah: Saturday, 23 September 2006.
The *molad* is on Friday, 22 September at 7 h 672 ch p.m., or, according to the formalism of Gauss, at M + m = 22.31759259259.
On Friday, 22 September at noon, the number of elapsed days of the Julian period was:

2000	2451544
2006, September	2435
22	22
JP	2454001
Molad in JP	2454001.31759259259

Relationship of Schram

T + t =	347605.797270955166
	2104955
	1423.42366189
	17.0966597
T + t =	2454001.31759

$J = [T + 1]_7 = [2454002]_7 = 5$. Rosh ha-Shanah is on the weekday J + 1 = 6, Saturday, the day 2454002 of the Julian Period, 23 September 2006.

Relationship of Gauss for Tishri

M + m =	10.047270955166
	– 18.3263381138
	+ 17.0966597628
	+ 0.5
M + m =	9.31759
Julian delay	+ 13
In Gregorian calendar	22.31759

$J = [9 + 3 \times 5767 + 3 + (5 \times 2)]_7 = 5$
a = 11
Rosh ha-Shanah falls on the weekday J + 1 = 6 Saturday, September M + 1 = 23.

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Relationship of Gauss for Nissan: Pesah 5766

P + p =	32.044093161144
	17.0966597628
	–18.3231603193
	0.25
P + p =	31.31759 March
Julian delay	13
In Gregorian Calendar	13.31759 April

$J = [31 + 3 \times 5766 + 5 \times [5766]_4 + 5]_7 = [17344]_7 = 5$

Pesah falls on day J – 1 = 4 thus Thursday, Julian March P = 31 or Gregorian 13 April 2006.

Year 5768: *Molad* (4) – 10 – 468. Rosh ha-Shanah: Thursday, 13 September 2007. The *molad* is on Wednesday, 12 September at 4 h 468 ch a.m., or according to the formalism of Gauss at M + m = 11.6847222222.

The number of days elapsed from the origin of the Julian period until the *molad* of 5768 is 2454355.68472222.

Relationship of Schram

T + t =	347605.797270955166
	2105320
	1423.67048409
	6.21696718648
T + t =	2454355.68472

$J = [T + 1]_7 = [2454356]_7 = 2$. Rosh ha-Shanah is on the weekday J + 2 = 4: Thursday, the day 2454357 of the Julian Period, September 13, 2007.

Relationship of Gauss for Tishri

M + m =	10.047270955166
	– 18.3295159074
	6.21696718648
	0.75
M + m =	– 1.31527776582 Julian September = – 2 + 0.6847222342
	31
M + m =	29.6847222342 Julian August
M + m =	11.6847222342 Gregorian September

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$J = [-2 + 3 \times 5768 + 3 + 5 \times 3]_7$
$J = [17320]_7 = 2$
$A = [12A + 5]_7 = 4$
Rosh ha-Shanah falls on day J + 2 = 4 Thursday, M + 2 = 13 September.

Relationship of Gauss for Nissan: Pesah 5767

P + p =	32.044093161144
	6.21696718648
	0.75
	–18.3263381133
P + p =	20.6847222343

$J = [20 + 3 \times 5767 + 5 \times 3 + 5]_{19} = [17341]_{19} = 2$
Pesah is on the weekday J = 2, Tuesday, Julian March P + 1 = 21 or Tuesday, 3 April 2006 Gregorian.

B. THE NINETEEN JEWISH FUNDAMENTAL CALENDARS

Engineer Jacob Loewinger[69] has studied the properties of a family of 19 Jewish calendars, corresponding to all the possible combinations of the seven leap years in a 19-year cycle of intercalation. Of course, these 19 cycles of intercalation, the traditional intercalation cycle in use in the Jewish calendar and the 18 other virtual calendars must have a common element in order to be compared to one another. In the first year of the chronology, 1 AMI, we assume that all these intercalation cycles had the same *molad* Nissan (4) – 9 – 642 and the same *tekufa* of Adda (4) – 0 – 0, preceding the *molad* Nissan by 9 h 642 ch. The *molad* Tishri of the first year of these cycles was *Beharad* (2) – 5 – 204, except for the seven cycles beginning with leap years whose *molad* Tishri was a month before *Beharad* (0) –16 – 491, on Saturday, 7 September –3760 at 10 h 491 ch a.m. We call these 19 calendars the 19 fundamental calendars. Other calendars, derived from them by a shift of a certain number of Jewish lunar months, will be called derived calendars.

Formula (1) now becomes:
(16) F = INT [(235Q + K)/19]

It gives the number of elapsed months between the *molad* Nissan of year 1 AMI and the *molad* Nissan Q years later, i.e. the *molad* Nissan of the year N = Q + 1.

K is a coefficient that was introduced by Loewinger; it takes 19 values from 0

69 See *Al ha-Sheminit*, pp. 53-58 and pp. 118-19.

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until 18, and defines the different intercalation orders. The coefficient K of the current intercalation order 3, 6, 8, 11, 14, 17, 19 is thus 8 because of formula (1).

The generalization of formula (3) requires some attention. Fb is the number of elapsed months from *molad Beharad* until the *molad* of Tishri N years later, i.e. until the *molad* of Tishri of the Jewish year A = N + 1; it is given by:
Fb = 12 + F with N = Q + 1 in (16). Therefore, Fb = 12 + F = 12 + INT [(235(N – 1) + K) / 19]
Fb = INT [(228 + 235N – 235 + K) / 19] = INT [235N + K – 7) / 19].

Ft is the number of elapsed months from *molad* Tishri of year 1 of the Jewish era of AMI until the *molad* Tishri N years later. Normally, this moment is *Beharad*, but in the case of the seven cycles of intercalation K = 6 until K = 0, K – 7 is negative and the first year of the cycle is a leap year; therefore the *molad* of Tishri is one month before *Beharad*.
Ft = 13 + INT [(235(N – 1) + K) / 19] = INT [235N + 12 + K) / 19].
Thus, if K – 7 > = 0

(17) Fb = Ft = INT [(235N + K – 7) / 19]

If K – 7 < 0, the first year of the cycle is a leap year and we must make the distinction:

(18) Fb = INT [(235N + K – 7) / 19]

(19) Ft = INT [(235N + $[12 + K]_{19}$ / 19]

With Ft = Fb +1
Fb is counted from *Beharad* and Ft is counted from the *molad* of the first of Tishri of the era.

Normally, in the case of the 19 fundamental calendars, we will need Fb in order to find the *molad* of a year of this particular calendar, but we will see that Ft will also be useful. Indeed, in the case of a derived calendar, *Beharad* no longer has any significance and we must refer to the epoch of this calendar, i.e. the *molad* of the first Tishri of this calendar.

The number of elapsed months from the epoch until the *molad* Tishri N years later is then in this case:
(19) Ft = INT [(235N + $[12 + K]_{19}$ / 19]

The order of the leap years in an intercalation cycle characterized by the coefficient K is given by the following formula:
(20) N = INT [(19G + 13 – K) / 7]

Thus, for each value of K, we find the order number N of the different leap years corresponding to the order number G of the seven leap years of the intercalation cycle; G taking the successive values 1, 2, 3, 4, 5, 6, and 7.

Table 16: Table of the Seven Leap Years in the Different Types of Intercalation Cycles

K	8	7	6	5	4	3	2	1	0	18	17	16	15	14	13	12	11	10	9
i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	3	3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3
2	6	6	3	3	4	4	4	4	4	4	4	5	5	5	5	5	5	5	6
3	8	9	6	6	6	6	7	7	7	7	7	7	7	8	8	8	8	8	8
4	11	11	9	9	9	9	9	9	10	10	10	10	10	10	10	11	11	11	11
5	14	14	11	12	12	12	12	12	12	12	13	13	13	13	13	13	13	14	14
6	17	17	14	14	14	15	15	15	15	15	15	15	16	16	16	16	16	16	16
7	19	19	17	17	17	17	17	18	18	18	18	18	18	18	19	19	19	19	19

N = INT [(19G + 13 – K)/7]. In the left column we have under K and i the seven values 1, 2, 3, 4, 5, 6 and 7 of G. The table then gives the leap years in 19-year cycles. For example, for i = 1 or K = 8 we find the leap years 3, 6, 8, 11, 14, 17 and 19. The exactness of this table can be checked on pages 38-39, where the 19 calendars were naturally generated.

The leap years in the different intercalation cycles correspond to the successive values: 1, 2, 3, 4, 5, 6, and 7 of G.

K is the figure characterizing the cycle defined by Loewinger; is the natural order number of these cycles.

Generalization of the Formula of Schram

The generalization of formula (11) gives:

Mol = e + Fb x M = e + INT [(235N + K – 7) / 19] x M.

Considering the identity (7N + K – 7)/19 = INT [(7N + K – 7) / 19] + $[7N + K - 7]_{19}$ x 1/19, we get:

Mol = e + 12N x M + M x (7N + K – 7)/19 – $[7N + K - 7]_{19}$ x M/19

(21) Mol = e + 235N x M/19 – {$[7N + K - 7]_{19}$ – (K – 7)} x M/19

If we write that N = A – 1, referring to the *molad* of the coming year, we will get the generalization of formula (12), the formula of Schram.

Mol = e + 235M x (A – 1)/19 – {$[7A - 7 + K - 7]_{19}$ – (K – 7)} x M/19

Considering the identity: $[7A + K - 14]_{19}$ + $[12A + 13 - K]_{19}$ = 18,

Mol = e + 235M x (A – 1)/19 + {$[12A + 13 - K]_{19}$– 18 + (K – 7)} x M/19.

We see thus that the coefficient a = $[12A + 5]_{19}$, which allows us to characterize the different years of the intercalation cycle, now becomes:

(22) a = $[12A + 13 - K]_{19}$

Or, more generally,

(23) a = $[12A + [32 - K]_{19}]_{19}$

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C. THE POSITION OF THE 19 FUNDAMENTAL CALENDARS AND THE *TEKUFA* OF ADDA

If we consider the traditional intercalation cycle of our traditional calendar, corresponding to the coefficient K = 8, we observe that the coefficient $a = [12A + 5]_{19}$ gives us indications about the Jewish year A. Values of "a" from 0 until 6 correspond to leap years, while values from 7 until 18 correspond to ordinary years.

Furthermore, the formulas (12) and (13) show that $a = [12A + [32 - K]_{19}]_{19}$ informs us whether the Jewish year begins early or late in the civil year.

Table 17: The Correlation between a = [12A + [32 – K] mod 19] mod 19 when K = 8 (Our Current Calendar) and the Date of Rosh ha-Shanah in the Civil Calendar

a	Rank in the Cycle	Date of RH 303rd Cycle	a	Rank in the Cycle	Date of RH 303rd Cycle
0	17*	6 September	10	2	22 September
1	6*	8 September	11	10	24 September
2	14*	9 September	12	18	25 September
3	3*	11 September	13	7	27 September
4	11*	12 September	14	15	28 September
5	19*	14 September	15	4	29 September
6	8*	16 September	16	12	30 September
7	16	16 September	17	1	2 October
8	5	18 September	18	9	4 October
9	13	20 September	19	17*	6 September

The leap year 17, whose coefficient *a* is 0, begins the earliest and, consequently, the year 16 ends the earliest. Pesah of the 16th year of the cycle is thus the earliest of the cycle in the season.

The year 8 has the coefficient 6. Among the leap years, it is the year that begins the latest and, because it has thirteen months, it is the year that ends the latest. Pesah of the eighth year is the latest of the cycle in the season.

We know that in the first year of the era, the *tekufa* of Adda was on Wednesday at (4) – 0 – 0 while the *molad* Nissan was at (4) – 9 – 642. This delay of 9h 642ch is noted as B = 0.399769d. M = 29.530594136 is the Jewish lunation, and A = 235M / 19 = 365.246822d is the mean Jewish year.

In the eighth year, the *tekufa* occurs after *molad* Nissan by:
$-B + 7A - (7 \times 12 + 3) \times M = -12.83370d$ (The *tekufa* precedes the *molad* by 12.8337d).

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In the 16th year, the *tekufa* occurs after *molad* Nissan by:
–B + 15A – (15 x 12 + 5) x M = 15.142650d.

Thus, in the course of the 19 years' cycle, the *molad* of Nissan can move from 15.14265d before the *tekufa* until 12.8337d after the *tekufa*.[70] This is the rule for our current calendar. It corresponds to an adaptation of the rule of Shitsar, applied to the distance between the *molad* and the *tekufa*. This is thus the situation for the intercalation cycle i = 1 or K = 8. In the intercalation cycle i = 2 or K = 7, the leap year 19, with a_2 = 6* ends the latest and has the latest Pesah. The *tekufa* occurs after the *molad* Nissan by: B + 18A – (18 x 12 + 7) x M = –11.27946d.

The leap year 9, with a_2=0* begins the earliest and, consequently, Pesah of the eighth year will be the earliest. The *tekufa* will occur after *molad* Nissan by: –B +7A – (7 x 12 + 2) x M = 16.69686d.

Now, in our current cycle i = 1, when Pesah of the eighth year, the latest in the cycle will be considered as occurring too late after the month of spring, we could make this year ordinary and decide to pass to cycle i = 2. The coefficient a_2 of the eighth year would become 6* + 1 = 7 and now Pesah of the regular eighth year would become the earliest year of the cycle. By contrast, the year 19 would have a coefficient a_2= 5* + 1 = 6*. The leap year 19 would end the latest and its Pesah would be the latest in the cycle. The process explains how the different cycles can be successively generated and how it is possible to correct a shift of 1.5542 days by the passage from the cycle of intercalation i to the cycle i + 1. The *tekufa* of Adda shifts by one day in 216.2898 years, or 1.5542 days in about 336 years.

Conversely, we can deduce that during about three centuries, during the sixth, seventh and eighth centuries, the cycle of intercalation that fitted the application of the rule of Shitsar was the cycle i = 19. Similarly, during the third, fourth and fifth centuries, the cycle that fitted was the cycle i = 18. We observe that this was exactly the cycle of intercalation that the church introduced at the Council of Nicaea in about 325. It appears that the church adopted the empirical order of intercalation used by the rabbis (and corresponding to the order of intercalation championed by

70 This mathematical model was in fact the great achievement of Engineer Yakov Loewinger in his book *Al ha-Sheminit* (Tel Aviv, 1985), pp. 116–23. There is a direct correlation between the order of intercalation and the position of the *tekufa* of Adda with regard to the extreme positions of the *molad* of Nissan during the cycle. In other words, the choice of an order of intercalation during the 19-year cycle automatically imposes a rule of intercalation, i.e. "*hok ha-ibbur*" or the extreme positions of the *molad* of Nissan in the cycle with regard to the *tekufa* of Adda. Z. H. Jaffe had already described this property in *Korot Heshbon ha-Ibbur* (Tel Aviv, 1931), pp. 21, 28–29, 71, 118–20, 230–31. Prof. Abraham Fraenkel mentioned it in his paper in *Sinai*, 17: 176–81.

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R. Eliezer, see note 44) as a definitive rule for their intercalation cycle, while the latter fixed the order of intercalation much later and got a much better result.

Table 18: The Position of the Critical Years of the Different Intercalation Cycles with Regard to the *Tekufa*

[The 19 first rows relate to the 19 fundamental calendars. The two last rows relate to two derived calendars and will be examined later.]

K	N critical	*Molad* Nissan before *Tekufa*	N critical	*Molad* Nissan after *Tekufa*
18	12	-0.3998	1	28.3761
17	4	1.1545	12	26.8219
16	15	2.7087	4	25.2676
15	7	4.2630	15	23.7134
14	18	5.8172	7	22.1592
13	10	7.3714	18	20.6049
12	2	8.9257	10	19.0507
11	13	10.4799	2	17.4964
10	5	12.0342	13	15.9422
9	16	13.5884	5	14.3871
8	8	15.1427	16	12.8337
7	19	16.6969	8	11.2795
6	11	18.2511	19	9.7252
5	3	19.8054	11	8.1710
4	14	21.3596	3	6.6117
3	6	22.9139	14	5.0625
2	17	24.4681	6	3.5083
1	9	26.0223	17	1.9540
0	1	27.5766	9	-0.3998
18	12	29.1308	1	-1.9540
17	4	30.6851	12	-3.5083

D. IMPROVED JEWISH CALENDAR I

1. Another Point of View on this Method

We have seen above that each fundamental calendar, the traditional Jewish calendar and the eighteen other fictitious calendars, has its own position with regard to the *tekufa* of Adda. When K diminishes from K = 18 until K = 0, the distance between the *tekufa* of Adda and the preceding earliest *molad* of Nissan of the cycle, belonging to the year of coefficient a = 7, increases by 1.5542 days each time that K diminishes with one unity. Now we know that the mean Jewish year is longer than the tropical year by 6.6559 minutes and therefore, after 17 x 19 = 323 years, the difference reaches 1.4930 days.

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If at the end of 323 years we jump to the next calendar, we observe that the earliest Pesah of this new calendar will occur 1.5542 days in advance of the *tekufa* of Adda. This will thus compensate and even slightly over-compensate for the delay in the mean Jewish year and the shift of Pesah toward the summer. Now, when we jump from calendar K = 7 to calendar K = 6, we must transform the last year of the last cycle K = 7 into a regular year. This last cycle K = 7 will end, and the new cycle K = 6 will begin a month earlier than expected. This is normal because we have already seen that the fundamental calendars K = 6 until 0 begin with a leap year, and their *molad* Tishri of year 1 AMI was one month before *Beharad*. Now, when K jumps from 0 to –1 or 18, then the cycle K = 18 begins again with an ordinary year but it happens one month earlier. That means that this calendar K = 18 is no longer a fundamental calendar but a derived calendar. Its *molad* of Tishri I AMI is no more on *Beharad* but one month before and, therefore, the earliest *molad* Nissan of this cycle doesn't begin –0.3991 days before the *tekufa* of Adda but –0.3991 + 29.5306 = 29.1308 days before the *tekufa*.

Thus, the solution works, and at each jump of the calendar, the shift of 1.4930 days of the spring *tekufah* of Adda toward the summer is slightly over-compensated for by the advance of 1.5542 days of the new calendar with regard to the same *tekufa*. After 19 jumps, when K will jump again from 0 to 18, the beginning of this cycle will begin two months in advance and the earliest *molad* Nissan of the cycle with coefficient a = 7 will precede the spring *tekufa* of Adda by
–0.3991 + 2 x 29.5306 = 58.6614 days.

2. Algorithm of this Method

The process begins in 6214. The year 6213, which is normally a leap year, becomes an ordinary year. Then, at the beginning of 6214 we jump from the calendar type K = 8 into the calendar K = 3 and i = 6 (see page 33).

The number of elapsed months at the beginning of 6214 is then
Em_0 = INT [((235 x 6213) + 1) / 19] – 1 = 76844.

We define: I_1 = INT [(A – 6214)/323]
$I_2 = [A - 6214]_{323}$

I_1 is the number of elapsed cycles of 323 years and I_2 is the number of elapsed years in the (I_1 + 1) and not completed cycle, before the Jewish year A.

In 6214, we begin the process with K = 3. At each change of calendar type, K diminishes by one unity, but K must remain positive.

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The number of elapsed months is: $Em = Em_0 + I_1 \times 17 \times 235 + Em_1$. With Em_1 equal to the number of elapsed months in the non-completed cycle $I_1 + 1$ of 323 years.

(19) $Em_1 = INT\ [((235 \times I_2) + [K + 12]_{19}) / 19]$

Indeed, the calendar beginning in Tishri 6214 is not a fundamental calendar; we must use (19) and not (17) or (18).

For the implementation of the rules of the calendar, we must consider the coefficient $a = [12 \times I_2 + [32 - K]_{19}]_{19} = [12 \times A + [32 - K]_{19}]_{19}$, because $[A]_{19} = [I_2]_{19}$.

Now, we must take into account that each time K jumps from K = 7 to K = 6, the last leap year of the former calendar becomes an ordinary year. This will happen for the first time after 15 cycles of 323 years and 322 years of the 16th cycle counted from 6214 when we will introduce the first calendar K = 3 and i = 6. This condition can be expressed in the following way:

If $[I_1]_{19} = 15$ and if $I_2 = 322$, then a = a + 1 (a = 6* becomes a = 7).

If $[I_1]_{19} = 15$ can correspond to $I_1 = 15$ and i therefore increases from 6 to 6 + 15 = 21 = 2, K diminishes from 3 to 3 – 15 = –12 = 7.

Or it can correspond to I_1=34,[71] and then i increases from 6 to 6 + 34 = 40 = 2 and K diminishes from 3 to 3 – 34 = –31 = 7 and so on. Further, if $[I_1]_{19} = 16$, we must subtract another month from Em; when it reaches 35 we must subtract one month more and so on.

Thus Em = Em – 1 – INT $(I_1 - 16)/19)$.

E. IMPROVED JEWISH CALENDAR II

Algorithm of this Method

In this method, we enter the new calendar type by a year of coefficient a = 7 and we exit through a year of coefficient 6, which becomes 7 in the new calendar.

The process begins in year 6284, which is the 14th year of the cycle 331. If the process had begun in 4599, at the beginning of the definitive calendar, we would be in the 14th year of a cycle of the calendar type K = 4, with the coefficient a = 6* and, by the jump in 6284 to K = 3, the coefficient a = 6* becomes a = 7. The calendar K = 3 will jump to the calendar K = 2, 17 x 19 + 11 = 334 years later with the year 6618, year 6 of a cycle K = 3 with coefficient a = 6* becoming year 6 of the first cycle K = 2 with the coefficient a = 7. Thus, each calendar of a certain K works during 334 years.

71 19 x 323 = 6137 years later.

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We define: I_1 = INT [(A – 6284) / 334]

$I_2 = [A – 6284]_{334}$

I_1 is the number of elapsed cycles of 334 years and I_2 is the number of elapsed years in the I_1 + 1, not completed cycle, before the Jewish year A.

The number of elapsed months at the beginning of Tishri 6284, the beginning of the cycle of 334 years characterized by K = 3, is given by:

Em_0 = INT [(235 x 6283) + 1)/19] = 77710.

The number of elapsed months from the *molad* Tishri 6284 until the beginning of the Jewish year A, is given by:

Em = I_1 x 4131 + INT [(235 x (A – 1) + $[K – 7]_{19}$) / 19] – INT [(235 x (A – 1) – I_2 + $[K – 7]_{19}$) / 19].

The first term, I_1 x 4131, represents the number of months of the elapsed cycles of 334 years, each of them comprising 17 x 235 + 11 x 12 + 4 = 4131 months.

We must now calculate the number of elapsed months in the current and uncompleted cycle I_1 + 1. This cycle begins with a Jewish year of coefficient a = 6, which is not a multiple of 19, making the problem harder. In the calendar type K corresponding to the Cycle I_1 + 1, i.e. K = 3 – I_1, the number of elapsed months is the difference between the number of elapsed months from *Beharad* until the beginning of year A, and the number of elapsed months between the beginning of *Beharad* and the beginning of the cycle I_1 + 1. Because of the difference, it doesn't matter if we use formula (19) or (17). Theoretically, we should use formula (17).

F. IMPROVED JEWISH CALENDAR III

Algorithm of this Method

The process begins in 6233. In this year, we introduce five consecutive short cycles of eleven years. By 6288, we are again at a good level and begin a cycle of 334 years, comprising 17 cycles of 19 years followed by a shorter cycle of 11 years. The order of intercalation is always the standard order 3, 6, 8, 11, 14, 17, and 19 corresponding to K = 8. For 6232 < A < 6288,

We define: I_1 = INT [(A – 6233) / 11]

$I_2 = [A – 6233]_{11}$

The number of elapsed months before the *molad* Tishri of year 6233 is em_0 = 77080. The number of elapsed months until Tishri of year A is:

Em = 77080 + I_1 x 136 + INT [((235 x I_2) + 1) / 19].

Each small cycle of 11 years contains 136 months, and the number of elapsed months in the uncompleted cycle I_1 + 1 is given by the formula (17) for K = 8.

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For A > 6288,
We define: I_1 = INT [(A – 6288) / 334]
I_2 = $[A – 6284]_{334}$

I_1 is the number of elapsed cycles of 334 years and I_2 is the number of elapsed years in the I_1 + 1, the not completed cycle, before the Jewish year A.

The number of elapsed months at the beginning of year 6288 is 77760. Each cycle of 334 years comprises 4131 months, and the number of elapsed months in the cycle I_1 + 1 is given by INT [(235 x I_2 + 1) / 19]. Therefore Em = 77760 + I_1 x 4131 + INT [(235 x I_2 + 1)/19]. What now about the coefficient a? The formula (22) with K = 8 becomes a = $[12 \times (I_2 + 1) + 5]_{19}$.

G. COMPARISON BETWEEN THESE METHODS

1. Numerical Example: The Year A = 13760

Traditional Calendar
(3) Em = INT [(235 x 13759 + 1) / 19] = 170177 months
The *molad* of Tishri in the Jewish week:

Mol = $[31{,}524 + 170{,}177 \times 765{,}433]_{181{,}440}$ = 81245 = (4) – 3 – 245

Thus, Tuesday at 9h; 13m 36s p.m. Rosh ha-Shanah will fall on Thursday.
The *molad* calculated in the Julian Period:

M = T + t = 347,997.466204... + (170,177 x 765,433) / 25,920 = 5,373,425.3845... JP.

It corresponds to Tuesday, 2 November; 9999 at 9h 13m p.m. By application of the rules of the Jewish calendar, we find that Rosh ha-Shanah is then on Thursday, 4 November 9999, and Tishri 21 is on Wednesday, 24 November 9999. The shift of the Jewish calendar with regard to the Gregorian calendar is (9999 – 839) / 231.3633 = 39.16 days. The shift of the Jewish calendar with regard to the tropical year 2000 is (9999 – 839) / 216.2898 = 42.35 days.

Improved Calendar: Solution I
I_1 = INT [(13760 – 6214) / 323] = 23
I_2 = $[13760 – 6214]_{323}$ = 117
I_3 = $[117]_{19}$ = 3
K = 3 – 23 + 19 + 19 = 18

Thus, A = 13760 is in calendar 24 of 334 years; it is the year 118 in this calendar. This calendar corresponds to K = 18. This year is also the fourth year in a cycle K = 18 of 19 years. The number of elapsed months from *Beharad* is:

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Em = 76844 + 23 x 3995 + INT [(235 x 117 + [18 +1_2]$_{19}$) / 19] = 76844 + 91885 + 1447 = 170176 months

I_1 = 23, thus 16 < I_1< 35 and Em = Em – 1 = 170175 months
a = [12A + (32 – K)$_{19}$]$_{19}$ = 5. A is a leap year 4*.

Improved Calendar: Solution II
I_1= INT [(13760 – 6284) / 334] = 22
I_2 = [13760 – 6284]$_{334}$ = 128
3760 is thus the year 129 of calendar 23
K = 3 – 22 = –19 = 0
13760 is the fourth year of a cycle K = 0 of 19 years. The number of elapsed months from *Beharad* is: Em = 77710 + 22 x 4131 + INT [(235 x 13759 – 7) / 19] – INT [(235 x (13759 – 128) – 7)/19] = 77710 + 90882 + 170176 – 168593 = 170175.
a = [12A + (32 – K)$_{19}$]$_{19}$ = 4. A is a leap year 4*.

Improved Calendar: Solution III
I_1 = INT [(13760 – 6288) / 334] = 22
I_2 = [13760 – 6288]$_{334}$ = 124
I_3 = [124]$_{19}$ = 10
K = 8
a = [12 x 124 + 5]$_{19}$ = 4. A is a leap year 11*.

Em = 77760 + 22 x 4131 + INT [(235 x 124 + 1) / 19] = 77760 + 90882 + 1533 = 170175 months.

13760 is the tenth year of a cycle of 19 years.

The number of elapsed months before Tishri 13760 is the same in the three calendars. This would not be true in 13758.

The *molad* in the Jewish week:
Mol = [31,524 + 170,175 x 765,433]$_{181.440}$ = 1899 = (1) – 1 – 819

Thus, Saturday at 7h 45m 30s p.m. Rosh ha-Shanah will fall on Monday.

The *molad* calculated in the Julian period:
M = T + t = 347,997.466204... + 170,175 x 765,433) / 25,920 = 5,373,366.3233... JP.

It corresponds to Saturday, 4 September 9999 at 7h 45m p.m. By application of the rules of the Jewish calendar, we find that Rosh ha-Shanah is then on Monday, 6 September 9999 and Tishri 21 is on Sunday, 26 September 9999.

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2. Conclusion

The calendar has been improved, Nissan 16 is now again in the month of the spring. In the traditional calendar, Nissan 16, 13772 falls on May 2 and Nissan 16, 13764 falls on May 30; the Jewish calendar has shifted by about 42 days with regard to the tropical year.

In the improved calendar I, Nissan 16 is found between March 19 and April 16. In the improved calendar II, Nissan 16 is found between March 20 and April 18. In the improved calendar III, Nissan 16 is found between March 20 and April 18.

The Jewish calendar is again correctly centered with regard to the solar year.

3. Comparison Between the Three Systems

Table 19: Table of Ordinary and Leap Years and Their Serial Numbers in the 19-Year Cycle

	Calendar I	Calendar II	Calendar III
13757	1 earliest	1* latest	8* latest
13758	2*	2	9
13759	3	3	10
13760	4*	4*	11*
13761	5	5	12
13762	6	6	13
13763	7*	7*	14*
13764	8	8	15
13765	9	9 earliest	16 earliest
13766	10*	10*	17*
13767	11	11	18
13768	12* latest	12*	19 *
13769	13	13	1
13770	14	14	2
13771	15*	15*	3*
13772	16	16	4
13773	17	17	5
13774	18*	18*	6*
13775	19	19	7
K and i	K = 18, i = 10	K = 0, i = 9	K = 8, i = 1

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Table 20: Dates of Nissan 16 in the Traditional and in the Three Improved Calendars

	Traditional Nissan 16	Calendar I Nissan 16	Calendar II Nissan 16	Calendar III Nissan 16
13757	18 May	19 March	18 April	18 April
13758	6 May	8 April	8 April	8 April
13759	26 May	28 March	28 March	28 March
13760	14 May	14 April	14 April	14 April
13761	4 May	4 April	4 April	4 April
13762	22 May	24 March	24 March	24 March
13763	11 May	11 April	11 April	11 April
13764	30 May	31 March	31 March	31 March
13765	18 May	20 March	20 March	20 March
13766	8 May	9 April	9 April	9 April
13767	27 May	28 March	28 March	28 March
13768	16 May	16 April	16 April	16 April
13769	4 May	5 April	5 April	5 April
13770	23 May	26 March	26 March	26 March
13771	13 May	13 April	13 April	13 April
13772	2 May	1 April	1 April	1 April
13773	20 May	22 March	22 March	22 March
13774	9 May	11 April	11 April	11 April
13775	29 May	30 March	30 March	30 March

Table 21: Dates of Tishri 21 in the Traditional and in the Three Improved Calendars

	Traditional Tishri 21	Calendar I Tishri 21	Calendar II Tishri 21	Calendar III Tishri 21
13757	27 November	27 September	27 September	27 September
13758	16 November	17 September	17 October	17 October
13759	4 November	7 October	7 October	7 October
13760	24 November	26 September	26 September	26 September
13761	12 November	13 October	13 October	13 October
13762	2 November	3 October	3 October	3 October
13763	20 November	22 September	22 September	22 September
13764	9 November	10 October	10 October	10 October
13765	28 November	29 September	29 September	29 September
13766	16 November	18 September	18 September	18 September
13767	6 November	8 October	8 October	8 October
13768	25 November	26 September	26 September	26 September
13769	14 November	15 October	15 October	15 October
13770	2 November	4 October	4 October	4 October
13771	21 November	24 September	24 September	24 September
13772	11 November	12 October	12 October	12 October
13773	31 October	30 September	30 September	30 September
13774	10 November	20 September	20 September	20 September
13775	7 November	10 October	10 October	10 October
13776	27 November	28 September	28 September	28 September

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בד"ד

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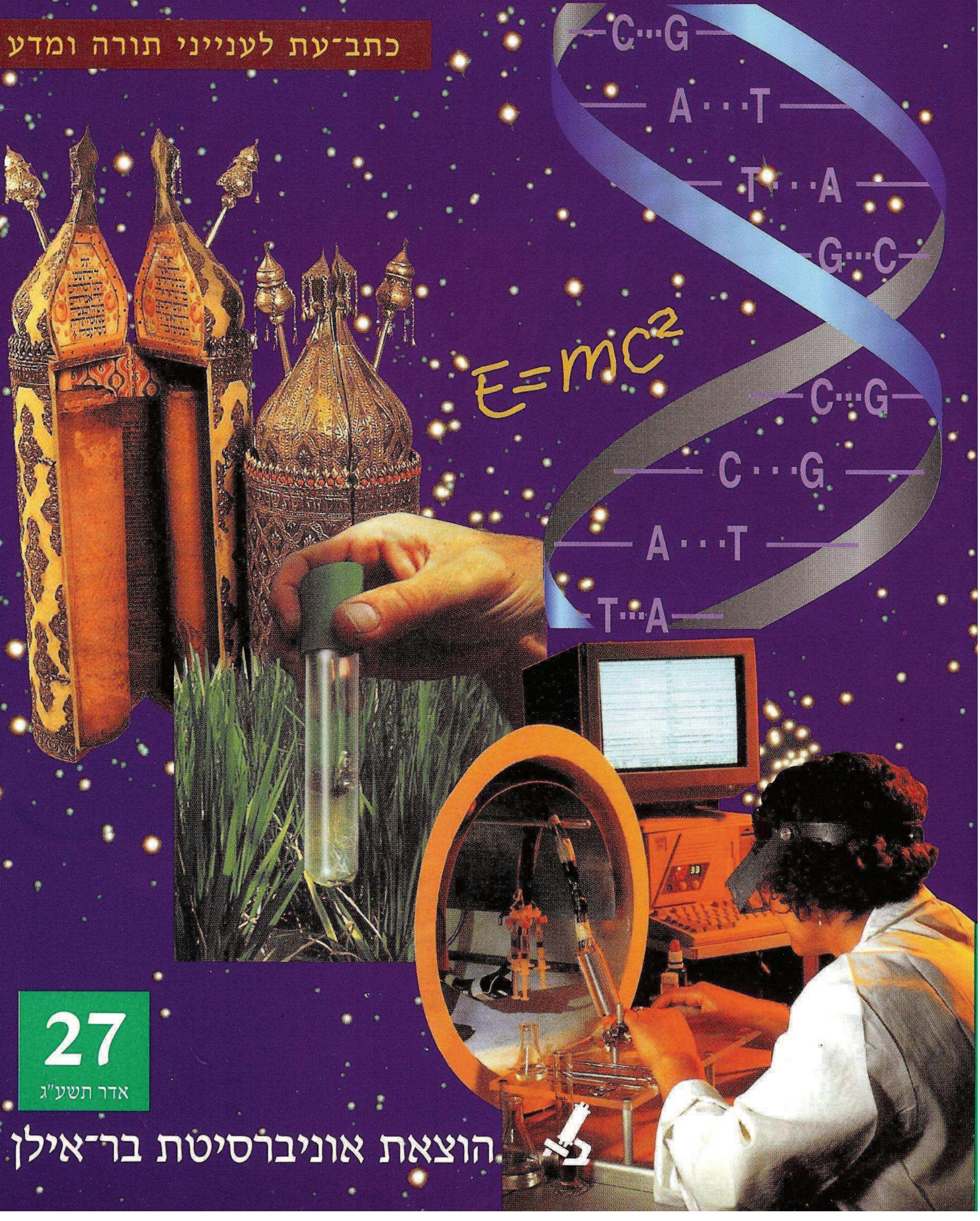
כתב־עת לענייני תורה ומדע



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אדר תשע"ג

הוצאת אוניברסיטת בר־אילן

בכל דרכיך דעהו

כתב־עת לענייני תורה ומדע

חוב׳ 27 - אדר תשע״ג

עורך
עלי מרצבך

הוצאת אוניברסיטת בר־אילן, רמת־גן

עורך: עלי מרצבך, המחלקה למתמטיקה, אוניברסיטת בר־אילן

עורכי משנה: דניאל שפרבר, המחלקה לתלמוד, אוניברסיטת בר־אילן
יהודה פרידלנדר, המחלקה לספרות עם־ישראל, אוניברסיטת בר־אילן

עורך קודם (גליונות 1-16): יחיאל דומב ז״ל

מערכת:

ישראל אומן	המרכז הבין־תחומי לחקר הרציונליות, האוניברסיטה העברית בירושלים
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ערן רביב, המחלקה למתמטיקה, אוניברסיטת בר־אילן

הוכחה מתמטית לכדאיות צמצום ההוצאות בימי חול על מנת לאפשר יותר בשבת

וקראת לשבת ענג לקדוש ה׳ מכבד

גרשון אהרונוב

אנו מוכיחים בצורה מתמטית — במודל כלכלי של עקומת הביקוש בשיפוע שלילי — כי טוב יותר לצרכנים לצמצם צריכת אוכל בימי חול כדי שתהיה אפשרות לצרוך יותר אוכל בשבת! בכך אנו מקיימים וקראת לשבת ענג לקדוש ה׳ מכבד (ישעיהו נ״ח, י״ג) וגם ״למי שהשעה דחוקה לו ביותר על כן צריך לצמצם בשאר ימים כדי לכבד השבת״ (שו״ע אורח חיים סימן רמ״ב).

התיזה של המאמר באה מכתבי כלכלן אמריקאי פרופ׳ ג׳ון מ. קלרק (1963–1884) שאומר שכלכלנים צריכים להמליץ על דרכים לגרום להגברת השיא של מחזורי עסקים.

במודל במאמר קיימות שתי קבוצות: ספקים (יצרנים) וצרכנים (משקי בית). הצרכנים קונים סלים תקניים של אוכל, סל לאדם ליום, אוכל טרי כמו בשר, דגים, לחם, גבינות, ירקות, פירות, ושתייה — מיום ראשון עד יום חמישי — וביום שישי מנה נוספת לשבת. זה דומה למן שירד מן השמים לעם ישראל ארבעים שנה במדבר, עומר לגולגולת, מיום ראשון עד יום ששי כולל תוספת מנה ביום שישי. ירד מן ליומים ביום שישי ולא ירד מן בשבת שכתוב ראו כי ה׳ נתן לכם השבת על כן הוא נתן לכם ביום השׁשי לחם יומים שבו איש תחתיו אל יצא איש ממקמו ביום השביעי (שמות ט״ז כ״ט).

במודל יש שתי עקומות הביקוש: של ימי חול ושל שבת. עקומת הביקוש מראה מחיר מקסימלי שהצרכנים מוכנים לשלם בעבור כל מיני מספר סלי קניות ליום. עקומת הביקוש של שבת יותר גבוהה (לימין) משל עקומת הביקוש של ימי חול היות ולא עובדים בשבת וכל בני המשפחה אוכלים שלוש סעודות ביחד בבית בשבת.

במודל הצרכנים קונים סלי אוכל בשוק ומשלמים מחיר אחיד ליום לסל, מיום ראשון עד יום חמישי. ביום שישי יש מחיר לסל ליום שישי ומספר סלי קניות ליום שישי וגם מחיר לסל לשבת ומספר סלי קניות לשבת. הצרכנים רגישים למחירים במובן שבמחיר גבוה קונים מספר סלים נמוך יותר, ובמחיר נמוך קונים מספר סלים גדול יותר.

אנו מניחים במודל, שמחיר השוק לשבת יותר גבוה ממחיר השוק בימי חול כי עקומת הביקוש לשבת יותר גבוהה מעקומת הביקוש לימי חול. במודל אנו מעלים את המחיר בימי חול

(כדי שיהיה צמצום קניות בימי חול) ומורידים את המחיר לשבת (כדי שתהיה תוספת קניות לשבת).

קיימות שתי הנחות בסיסיות להוכחת המודל: שלפני ואחרי השינויים במחירים הצרכנים משלמים אותן הוצאות ומקבלים אותו מספר סלים בסך הכול בשבוע. אנו מוכיחים שהשיטה של הוספה למחיר של ימי חול והפחת למחיר של שבת עדיפה לצרכנים, שהיא נותנת יותר מרווח לצרכן, יותר הפרש בין מחיר שוק ועקומת הביקוש.

הוכחנו במודל שצרכנים מעדיפים צמצום אוכל בימי חול שגורם לתוספת אוכל לשבת באותם התשלומים בסך הכול בשבוע. אספקת היתר בשבת, השיא של המחזור, קובע ולא חשוב ששיא המחזור יום בשבוע כמו שבת או חודש אחד בשנה או שנה אחת בעשר או יותר שנים. הפוקוס תמיד צריך להיות על האספקה בזמן השיא.

המהפכה הגריגוריאנית של הלוח העברי

יוסף יצחק איידלר

הלוח העברי הוא לוח ירחי-שמשי. החודשים הם חודשים ירחיים בני 29 ו-30 יום והשנים הן בנות 12 ו-13 חודשים כדי לקרב, עד כדי שאפשר, בממוצע, את אורך השנה הטרופית.

בכל זאת, השנה היהודית הממוצעת ארוכה מהשנה הטרופית ב-6.658 דקות. ההפרש הקטן הזה הוא הסיבה להזזה איטית של הלוח העברי וחגיו ביחס לשנה השמשית ועונותיה לכיוון הקיץ.

אנחנו מוצאים, על בסיס של חקירה היסטורית, שהזזתו של הלוח העברי כבר הגיע היום ל-5.4 ימים. ההזזה הזאת היא חצי הזזתו של הלוח היוליאני בזמן המהפכה הגריגוריאנית בשנת 1582. לכן הזזתו של הלוח העברי ההולכת ומחמירה יכולה לעורר דאגה.

מטרת המאמר הזה היא להגיש שלושה פתרונות מספיקים כדי לשפר את הלוח העברי, לחקור אותם ביסודיות ולהשוות אותם ביניהם.

בנספח המתמטי אנחנו בודקים ומוכיחים את הנוסחאות היותר מתקדמות של הלוח העברי. אחרי-כן אנחנו מכלילים אותן כדי להרחיב שיטה מתמטית המאפשרת לחשב את קביעות השנים והקבלתן עם השנים הגריגוריאניות.

אנחנו צריכים לראות במאמר הזה ניתוח עיוני ומתמטי שעשוי לסייע לסנהדרין, כאשר תתחדש לפתור את בעית הזזתו של הלוח העברי.

המחקר הזה תואם את שיטתו של הרב בעל ה"חזון איש" הכותב על דברי רמב"ם בהלכות קידוש החודש (ה, ב) שאין להבין מתוך הרמב"ם שהלוח הנוכחי, עם כל פרטותיו, הוא בגדר הלכה למשה מסיני. אבל לחכמים יש רשות לעשות חשבון קבוע שעל פיו יסודרו השנים ויתאימו שנות החמה ושנות הלבנה.

BDD

Bekhol Derakhekha Daehu

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Ely Merzbach

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ENGLISH ABSTRACTS

THE IDENTITY OF *TEKHELET* (BIBLICAL BLUE DYE): NEW FINDINGS

Roy Emanuel Hoffman

The secret of the *tekhelet* (biblical blue) dye was lost long ago. Attempts to rediscover it have led to mistakes in the past, so that religious authorities are very wary of accepting its reintroduction. In this work, new findings are reported that resolve the remaining major objection, i.e. that the dying process had not been reliably reproduced without resorting to modern chemicals, unknown in ancient times. A description is given of the dying process using chemicals used in ancient times, and comparisons are made with similar dying techniques. The resulting color is analyzed, and compared with ancient writings and archaeological artifacts. We discuss the ramifications for religious law pertaining to the new findings reported here.

IDENTIFYING THE BIBLICAL *ARNEVETH* WITH THE MUSK-DEER AND THE *SHAFAN* WITH THE MOUSE-DEER: A HYPOTHESIS

Zvi Weinberger

The Torah identifies three animals that chew the cud but do not have split hooves: the camel, the *arneveth* and the *shafan* (Leviticus 11:4, 5, 6; Deuteronomy 12:7). Accepted translations of the Torah identify the *arneveth* with the hare and the *shafan* with the hyrax. However, neither hare nor hyrax chew cud in the ordinary sense. We propose that the biblical *arneveth* and *shafan* are not the animals known from contemporary parlance and common to modern Israel, but are rather animals not found in the Middle East.

We suggest that the *arneveth* corresponds to the musk-deer (Family *Moschidae*), native to Central Asia, and the *shafan* to the mouse-deer (Family *Tragulidae*), two genera of which are common to Southeast and South-Central Asia and a third to Central and West Africa. These animals are ruminants, and their feet have well-developed digits culminating in small individual hooves at the extreme of each digit, and not single split hooves on each foot.

The shape of a musk-deer resembles a large hare, and so does its running pattern. For these reasons, we associate the *arneveth* with the musk-deer. The mouse-deer, genus *tragulus*, finds shelter in rock crevices during the day – as attributed to the *shafan* in Psalms 104:18. For this reason, we associate the *shafan* with the mouse-deer.

However, our proposal has its own difficulties. If the musk-deer and mouse-deer were common in ancient biblical Israel, and have since become extinct, why have their skeletal remains not been discovered? Both families have an uncommon distinctive feature, large upper canine teeth. Climatic considerations also cast doubt on the existence of these families in ancient Israel. If the *shafan* was not common in Israel, why would David and Solomon have referred to the *shafan* in their verses if the mouse-deer had not been familiar to their audience in Israel? In spite of these difficulties, we advance our proposal that conforms to a straightforward interpretation of the Torah's description.

SHAVING THE HEAD AS PART OF THE MOURNING RITES OF THE BETA YISRAEL

Yossi Ziv

In the Beta Yisrael community (the Ethiopian Jews), it was customary for the relatives of the departed to shave their heads during the days of mourning. This custom is contrary to the explicit prohibition, written in the Torah and accepted as Jewish Law (Halakhah) in the rabbinical literature, of not removing one's hair as part of the mourning process. Nonetheless, a thorough reading of the sources reveals that there is considerable literary and archeological evidence that cutting the hair, in the context of mourning, was practiced by Jews and gentiles alike. Moreover, in many books of the Bible, as a minority opinion in the literature of the *Tana'im*, and as written by the commentators on the Bible who wrote in the Middle Ages, one finds explicit references to the custom of head-shaving as something well-known, permitted, and accepted.

It may be assumed that the Jewish People observed two opposing customs. The precise plucking of every hair on the mourner's head was a permitted and acceptable custom. The tearing out of hair from the scalp to the point of bleeding, in a frenzy of sorrow, is the custom prohibited by the Torah. When a person is beside himself with grief, the precise, careful plucking of hair can get out of control and become

an uncontrolled ripping of hair, scalp, and blood; the act proscribed by the Torah. For this reason, the rabbis put an end to this custom of plucking out the mourner's hair, and determined that the removal of the mourner's hair be prohibited in every way.

However, at the same time, Beta Yisrael had already been cut off from the main body of the Jewish people. They continued, therefore, to follow their ancient tradition: the careful removal of every hair on the mourner's head. In summary, we learn that acquainting ourselves with the customs of Beta Yisrael gives us new ways of understanding the development of Jewish Halakhah.

213-ROW TABLE – A NEW TOOL TO DETERMINE TYPE PERCENTAGES IN THE HEBREW CALENDAR

Eran Raviv

This paper is a continuation of an article that appeared in *B.D.D.* 22 entitled "Tablets and Tablet Shards – On *Molad* and their Characteristics." In the previous paper, we presented a new understanding related to the possibility of the *molad* of Tishre occurring in each of the *hakalim* of the week, which differs from the previous assumption.

As an addendum to the paper, we are presenting a new 213-row table, which can be used to create a *siman* for each type of year similar to that in the "61-row table." The new table adds an additional letter that indicates the type of *dechiya*.

The importance of this table is that it can be used as a precise and very accurate tool to calculate the prevalence and type of each *dechiya*.

We will analyze the table; explain the source of the number 213, and present additional implications of this new table.

CLASSIFICATION OF TEXTUAL WITNESSES OF THE BABYLONIAN TALMUD – NEW STATISTICAL ASPECTS

Alex J. Tal

Research on the textual variants of a classical text aspires to find genealogical relationships between extant witnesses and represent them in a stemmatic tree. Because of the great complexity of its creation and transmission history, it seems that this is not a realistic aim with regard to Talmudic literature. A more realistic aim is the exposure of mutual relationships between the textual witnesses and the discovery of different families of textual traditions.

This study is based on the textual variants of tractate Beitza from the Babylonian Talmud. Seven complete medieval manuscripts are extant for this tractate, and an equal number of partial ones that include more than ten percent of the complete text. Based on more than 850 variants, and aided by dedicated software, a distance matrix was constructed. A two-dimensional distance map was produced from this matrix by the MDS program PROXSCAL.

Analysis of this map led to the identification of a geographical axis, whose extremes represent the medieval Ashkenazi (German and French) and the Eastern textual traditions. Manuscripts with Spanish characterization are located between these two extremes. Parallel lines (simplex) were used to divide this map. In addition, it was found that a circumplex division is possible, and that the more complex – and therefore more original – manuscripts occupy the centers of the unconcentric circles. Thus, two facets were found – geographical and degree of complexity. Utilizing these new methods in the field of Talmudic philology is exceptionally challenging, in the way that it leads to new insights into the history of the textual traditions of the Babylonian Talmud.

PREVENTING MODERN NUISANCES – CRITERIA AND STANDARDS

Shlomo E. Glicksberg

Detailed laws regarding nuisances and how they should be prevented are included in the Mishnah. Throughout the generations, our scholars and decisors often dealt with changes to those nuisances, and with nuisances that for various reasons did not appear in the original collection. This article will investigate the different methodologies used in the past for making halakhic decisions regarding nuisances as new situations arose. These methodologies may lead the way today when approaching modern ecological hazards such as pollution and global warming.

COMPUTATIONAL TOOLS FOR IDENTIFYING THE MOST ACCURATE TENACHIC (OLD TESTAMENT) VERSION

Joseph Klein

Which versions of the Old Testament now in existence are most similar to the original? At what period did scholars arrange the internal division of the Pentateuch into five books, and the rest of the Old Testament into two sections? Without the original form, the textual differences between early versions have given rise to confusion. Today, attempts to find the correct form of the text are based on an examination of the early texts and the Massorah. The version based on the Aleppo Codex (*Keter Aram Zova*) and the Massorah is considered to be the most accurate. The present work discusses an independent computational method for determination of the period(s) in which the *Tenach* was divided into sections, books, and verses.