A Short History of the Jewish Fixed Calendar

The Origin of the Molad

Still today most of the educated Jewish people have no idea about the origin and the evolution of the Jewish calendar. The most aware of them believe that the Jewish calendar was introduced, in its definitive form, in 359 C.E. by Hillel II the patriarch.
In the present paper we examine again the evolution of the Jewish calendar during the period 359 – 924 when the Jewish calendar reached its definitive character.
We review the positions of Bornstein and Jaffe, the two great founders of the evolution theory of the Jewish calendar and we try to determine which part of their theory is still valid and everlasting.
In fact Bornstein had studied punctual problems, his conclusions had a more general character and he did not always enter into practical details. For this reasons his conclusions hold better out against criticism. By contrast, Jaffe had built a much more detailed and elaborated synthesis resting on detailed and precise assumptions. His synthesis had the ambition to outline in detail the complete history of the Jewish calendar. It was of course a marvelous construction but today we acknowledge that some of his assumptions do not hold out against criticism.
In the present paper we outline the history of the Jewish calendar. We sort out the different assumptions underlying Jaffe's synthesis, we make their critical analysis and we behold only those assumptions which make still sense today.
It appears finally that the broad outline of the history of the Jewish calendar proposed by Bornstein and Jaffe is still valid. It represents an exceptional contribution to the understanding of one of the most fascinating aspects of Jewish history and culture.

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I. Introduction.

It was always believed that the transition from the observation to the fixed calendar was clear cut, with the fixed calendar immediately adopting its definitive form in 358/59, at the date of the inception of the fixed calendar.
The only divergent element was the fact that we find already in the Talmud that the postponement of Rosh ha-Shanah from Sunday was a later enactment.[1] The later character of this postponement was already recognized by some rare rabbinic authorities.[2]
However a passage of the epistle of R' Sherira Gaon implying that Rosh Ha-Shanah of the year 505 C.E. was still on Sunday was generally considered as the result of a copyist mistake.[3]
It is only at the turning point of the twentieth century and in its first decades that new evidence appeared after the discovery of new documents in the Cairo Geniza.
The former conviction that the Jewish calendar took immediately its definitive shape at the moment of its inception was shaken by two major discoveries:

- The discovery of letters attesting the existence of an important dispute between the Babylonian community lead by R' Sa'adia Gaon and the Palestinian community lead by (Aaron?) ben Meir about the *keviyah* of the years 4682, 4683 and 4684.[4]
- The discovery and the publication in 1922 of a document from the Cairo geniza: a letter from a Babylonian *Resh Galutah*[5] showing that the *keviyah* of the year 4596 or 835/836 C.E. was different than in our present-day calendar and that the Babylonian community received its calendric information from Palestine.

This last discovery was especially important; it proved beyond any doubt that almost five hundred years after the inception of the fixed calendar of Hillel, the fixed calendar in its present-day form had still not yet been instituted.
These two important discoveries were at the origin of much speculation about the history of the Jewish calendar. This history remains mostly conjectural because of the little quantity of the available pieces of evidence. But one thing is certain: our modern calendar

[1] See B. Niddah 67b. It appears that in the time of R' Yemar (427-432) Rosh ha-Shanah could fall on
[2] See Tossefot Rid on B. Megila 4b by R' Isaiah de Trani the Elder, south of Italy, 13th century.
[3] See Iggeret Rav Sherira Gaon part III, chap.4, p. 85 in the edition of R' Aaron Heyman, London 1910.
[4] See Bornstein, H. J: Mahaloket Rav Sa'adia Gaon u Ben Meir, Warsaw 1904.
[5] For the text of the letter of the *Resh Galuta* see note 83.

in its definitive form was not definitively instituted before the tenth century, exactly 922-924, after the end of the R. Sa'adia-Ben Meir controversy.
Hayim Jehiel Bornstein[6] (1845-1928) played a major role in the analysis of these documents and in their correct interpretation. Tsvi Hirsh Jaffe [7] (1853-1927) made also important contributions in this field. In general he appears more as the associate of Bornstein but some of his conclusions are more elaborated and more definitive than those of Bornstein. Akavya (Avraham Aryeh Leib Yakobovits) (1882-1964) devoted many years of research to the Hebrew calendar. He edited *Korot Heshbon ha-Ibbur*, the book of Jaffe and studied the tombstones of Zoar which were discovered from about 1940 onwards and revealed the great diversity of the Jewish calendar even after the institution of the rabbinic calendar and even in Palestine in places not removed from the rabbinic centers. Stern, a historian, surveyed again all the available historical elements and put them in perspective in his book "Calendar and Community"[8]. He put especially the emphasis on the lack of unity of the Jewish calendar and its great diversity through all the Jewish communities of the Diaspora. Furthermore, when later, after the sixth century, the rabbinic calendar asserts itself, all the removed communities, except the Babylonian and other neighboring communities, probably remained unaware of the *keviyah* adopted by the Palestinian academy and had to live according to parallel approximate calendars of their own. It is probably only after the end of the R. Sa'adia Gaon-Ben Meir dispute that the rules of the calendar and the four gates table became known in all the Diaspora.
In the present paper we try to outline the history of the Jewish calendar from the time of its inception onwards until the tenth century, when it reached its definitive form.
At this stage, when mentioning influential scholars who made significant contributions to the field of the Jewish calendar, we must also mention the place of pioneer of Hayim Selig Slonimski[9] (1810-1904). Before the discovery of the documents of the Cairo geniza, he had already discovered that the Jewish Molad is derived from the table of mean conjunctions of Ptolemy's Almagest.[10] Similarly he was the first to state the late

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[6] He is the author of the following papers, in connection with the problems of the Jewish calendar:
מחלקת רב סעדיה גאון ובן מאיר בקביעת שנות ד"א תרפ"ב תרפ"ד, ספר היובל לכבוד מו"ה נחום סוקאלאוו, וורשא תרס"ד
משפט הסמיכה וקורותיה, התקופה, ספר רביעי, תרע"ט
סדרי זמנים והתפתחותם בישראל, התקופה, ספר שישי, תר"ף
תאריכי ישראל, התקופה, ספר שמיני, תרפ"א
תאריכי ישראל, התקופה, ספר תשיעי, תרפ"א
חשבון שמיטין ויובלות, התקופה, ספר האחד עשר, תרפ"א
דברי ימי העיבור האחרונים, התקופה, ספר ארבע עשר וחמישה עשר, תרפ"ב
דברי ימי העיבור האחרונים, התקופה, ספר ששה עשר, תרפ"ג
עיבורים ומחזורים, התקופה, ספר עשרים, תרפ"ד

[7] Tsvi Hirsh Jaffe was born in Russia on 11 Sivan 5613. He had a thorough Talmudic education. He was an autodidact mathematician and talented engineer and inventor of a calculating machine. He was the editor of Azaria de Rossi's book מאור עניים Warsaw 1899. He wrote explanatory notes to the Hebrew translation by Shaeffer of the History of the Jews of Graetz. He wrote the article Ben Meir in the American Encyclopedia Otsar Israel. But his opus magnum is his book קורות חשבון העיבור that was edited by Akavia in Tel Aviv 1931.

[8] Stern, Sacha: Calendar and Community, History of the Jewish Calendar, Second Century B.C.E.-Tenth Century C.E. Oxford University Press, 2001.

[9] Hebrew popular science writer, popularize and inventor (he was awarded a prize by the Russian academy of Science in 1844 for a calculating machine.

[10] *Yessodei ha-Ibbur*, Zitomir 1865, pp. 49-51.

character of the *tekufah* of R' Adda bar Ahava. This concept seems to be a Spanish invention of the tenth century.[11]
In order to describe the evolution of the Jewish fixed calendar we will examine thoroughly the tables constructed by Jaffe in order to reconstruct the Jewish calendar in its different stages of development and make the critical analysis of the assumptions on which they are built.
Jaffe was a brilliant autodidact mathematician and he trusted probably exaggeratedly some of his mathematical achievements. The aim of this paper is to show how Jaffe constructed his tables for the different stages of development of the Jewish calendar and to discuss what can be considered as established and what must be considered with reservation.
On this manner, the main achievements of Jaffe in his book *Korot Heshbon ha-Ibbur* will be made available to the modern reader who has no access to both the papers of Bornstein and the more systematic but difficult book of Jaffe. Even if some of their conclusions may be contested, these works remain authoritative in many respects. This paper aims at paying them homage, especially to Jaffe, Talmudist, mathematician and historian of great value, closely bound to all the researches and discoveries of Bornstein but forgotten and neglected. He was even forgotten in the Encyclopedia Judaïca.

II. The calendar of Hillel[12] from 359 until the beginning of the seventh century (about 648).

According to a tradition[13] quoted in the name of R' Hai Gaon,[14] the present Jewish calendar was introduced by the patriarch Hillel II in 670 Era of the Seleucids = 4119 Era of Beharad = 358/359.
I have already demonstrated[15] that a pre-calculated calendar was already established by the Court of Tiberias and sent to Babylonia from about 325 onwards. However this calendar was still a semi-empirical calendar replicating a calendar based on the first visibility of the new moon. By contrast the calendar instituted in 359 seems to be a completely calculated calendar based on a mean conjunction called Molad. The basic assumptions of this calendar, according to Jaffe, were probably the following:

- The Molad of Nissan 4119 was chosen near to the moment of the maximum of the solar eclipse which occurred on 15 March 359 C.E. exactly the day of the inception of the new calendar.

[11] *Yessodei ha-Ibbur*, Zitomir 1865, pp. 43-45.
[12] The name of Hillel II the Patriarch is associated to the calendar instituted in 358 C.E. according to the tradition reported in the responsum of R' Hai Gaon . However the name of Hillel II is not mentioned in the Talmud and it is not certain at all that he had a direct part in this calendar. Maimonides does not mention him and is probably not aware of the tradition reported by R' Hai Gaon. It is clear that Rabbi Yose or Yousa, always mentioned in the Jerusalem Talmud in connection with the rules of the calendar, must have had a preponderant part in the foundation of this calendar.
[13] *Sefer ha-Ibbur* by R' Abraham bar Hiya edited by Filipowski, London 1846, p. 97.
[14] 939-1038
[15] See Ajdler (1996):" Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam" Sifriati 1966, p. 693-697 and Ajdler (2004): Rav Safra and the Second Festival day, Tradition, vol 38, n°4, pp.3 – 28.

- The lunation adopted in the new calendar was 29d– 12h–792*hal* or 29d 12h 44m.[16] At this epoch they did not use the *helek* and did not divide the hour in 1080 halakim. They could suffice themselves with the division of the hour in 15 *hayil*[17] a *hayil* representing 4 minutes or 72 *hal*. The length of the month was thus noted 29 – 12 – 11. The remainder of a month was 1 – 12 – 11, the remainder of 6 months was 2 – 4 – 6, the remainder of 12 months was 4 – 8 – 12 and the remainder of 13 months was 5 – 21 – 8.
- The rules of the calendar were about the same as today except that the first day of Rosh ha-Shanah may fall on Sunday. The rules were thus the following:
- The postponements were DU (Wednesday and Friday) and יח or 18 hours (noon).
- The length of the year was:

  for an ordinary year 353 days for a defective year. Shift of R.H: 3 days.
  354 days for a regular year. Shift of R.H: 4 days.
  355 days for an abundant year. Shift of R.H: 5 days.
  for a leap year 383 days for a defective year. Shift of R.H: 5 days.
  384 days for a regular year. Shift of R.H: 6 days.
  385 days for an abundant year. Shift of R.H: 7 = 0 days.

  Indeed when the number of days of the year is a multiple of 7, the day of R.H has no shift and remains unchanged.
- The derivate postponements (resulting from the former rules) were then

  1 – 9 – 3 in an ordinary year or א ט ג בפשוטה

  2 – 15 – 8 in a year following a leap year or ב טו ח אחר עיבור
- The number of types of year was 18.[18] There were 9 types of ordinary years and 9 types of leap years.
  Ordinary years: אכג, בשה, גכח, גשו, החו, הכז, השא, זחא, זשג.
  Leap years:אכה, בחה, בכו, בשז, גכז, החה, השג, זחג, זשה
- The cycle of intercalation of 19 years did not yet exist. The rule of intercalation or the rule of the equinox is that Pessah cannot fall before 19 March.[19] However the years were classified in table א of Jaffe,[20] in groups of 19 years, according to the principle of a fictitious cycle of intercalation of 19 years allowing an easy examination of the leap years with regard to our today cycle of intercalation.

The table א of Jaffe for the years 4119 (358/359) until 4408 (647/648) was constructed on the preceding assumptions. It gives for each year the *keviyah* and the date of the first day of Passover. The leap years were chosen in such a way that Nissan 15 is never before March 19.
Let us come back on these different assumptions. The eclipse of 15 March 359 on 1582256.14547 JD at 15h 29h 29s ET.[21] The difference $\Delta T = ET - UT \sim 1h\ 40\ m$.

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[16] According to the statement of Ravina in B. Arakhim 9b.
[17] See Braïta de Shemuel chap 2 and 3. 1 *Hayil* = 1° of the equator and therefore also 4 minutes. We have also1 *hayil* = 72 *halakim* and 1 minute = 18 *halakim*
[18] Instead of 14 types of *keviyot* in our present-day calendar.
[19] This questionable assumption of Jaffe will be discussed beneath.
[20] See Appendix.
[21] See Mucke, H. and Meeus, J. Canon of Solar eclipses – 2003 to + 2526. Astronomisches Büro, Wien. 1983.

Therefore the time of the eclipse was 15m 29m 29s – 1h 40m + 2h 21m = 16h 10m 29s Jerusalem modern mean time,[22]and 15h 54m Al-Battani Jerusalem mean time,[23] slightly less than the time calculated by Jaffe of about 18h, probably using the tables of the Canon of Oppolzer.[24] Jaffe assumed that the Court fixed the epoch of the Molad at 18h. The epoch of the Molad was thus 3 – 0 – 0. Jaffe believed that the conjunction was much nearer to 18h, the beginning of the night and therefore his assumption was genuine. This assumption is thus acceptable but 3 – 22 – 0 would have been more precise. We will see that Jaffe's assumption allows explaining justifying different pieces of evidence which could not be explained otherwise.

1. As the conjunction was on 15 March 359 Pessah was certainly after the equinox and therefore the rule of the equinox was respected for an ordinary year. The year 4119, corresponding to the fifteenth year of a fictitious cycle of 19 years was thus an ordinary year. It is thus easy to calculate the modern Molad of this month.
   The fundamental formula[25] of the modern calendar allows calculating the number of lunations elapsed from Beharad until the molad of the year 4119.
   4119 is the 15th year of the fictitious cycle of 19 years; the preceding year was probably a leap year.
   Ft = Int[(235 * 4118 + 1) / 19] = 50933.
   The number of lunations before the Molad of Nissan 4119 is then 50939.
   The molad of Nissan 4119 is thus
   Mol = $[31524 + 50939 * 39673]_{181440}$ = 55751 *hal* = 2d + 3h + 671 *hal* =
   3 – 3 – 671 thus 3h 671 *hal* later than the epoch.
2. In order to make later calculations easier, we will calculate the modern Molad for the year 4124 representing the first year of the fictitious 218th cycle of intercalation (of 19 years). The number of lunations between *Beharad* and Tishri 4124 is:
   Ft = Int [(235 * 4123 + 1) / 19] = 50995.
   Mol = $[31524 + 50995 * 39673]_{181440}$ = 100159 = 3d+20 h+799 = 4 – 20 – 799.
   The Molad of Hillel is 4 – 17 – 1 (*hayil*) = 4 – 17 – 72 *hal*
   The difference is thus 3h 727 *hal*[26] between our modern molad and the assumed molad of Hillel.

3. The most problematic aspect of the table א is Jaffe's assumption about the rule of the equinox that Pessah cannot fall before March 19. During the fourth century the true equinox was on 20 March[27] and the mean equinox was on 22 March. According to the rule of the equinox of Rabbi Huna bar Abin[28] Nissan 16 may fall on the day of the mean equinox, according to the understanding of R' Hananel[29]

[22] To be precise: Jerusalem modern mean time.
[23] See Ajdler (2005) : The Equation of Time in Ancient Jewish Astronomy, BDD 16, p. 14.
[24] Theodor Ritter von Oppolzer (Prague 1841 – Vienna 1886) : Canon Der Finsternisse, Vienna 1887.
[25] This formula is true for our modern calendar in which the intercalated years are 3-6-8-11-14-17-19. If the order of intercalation is different, it must be used very carefully. Indeed a leap year adds a 13th month.
[26] 3h 671hal + 50995 – 50939 = 3h 727hal.
[27] See Meeus, J: Astronomical Tables of Sun, Moon and Planets, Willmann-Bell, 2nd edition 1995, pp. 109-110.
[28] B. Rosh ha-Shanah 21a.

and R' Abraham bar Hiya,[30] it may fall on the day following the mean equinox according to Rashi[31] and Rambam.[32] Thus according to the rule of the equinox, with the understanding of R' Abraham bar Hiya, Nissan 16 could fall on March 22 and the first day of Passover could be on March 21, as much as the Court of the calendar considered effectively the *tekufa* on March 22.[33]
However the Christians considered that the true equinox is on March 21 and therefore, according to the rules adopted at the Council of Nicaea, Easter could fall the earliest on Sunday 22 March. Indeed the rule of intercalation adopted by the council of Nicaea said: Easter is on Sunday following the fourteenth day of the moon which reaches this age on March 21 or slightly later.[34]

At many occasions the Christians complained during the period of the second – fourth century and even later that the Jews did not respect the rule of the equinox and celebrated their festival of Passover to early. One must however be very cautious in the appreciation of these accusations. As noted by Stern there was a great diversity among the Jewish communities, some following the rabbinic calendar, others not. Furthermore removed communities, far from Palestine and Jewish rabbinic centers were not aware of the rabbinic calendar and could not follow it. It is important to note that when the Christians reproached to the Jews their early celebration of Passover they didn't take into account that the beginning of the festival, the night of the Seder, belongs to the next day; for them, in the Julian calendar, it belongs to the day before. Furthermore, the Christian writers confuse the ostentatious preparations of the feast on Nissan 14 and the public burning of the leaven, by the people disposing of it, with the actual festival, more intimate and less spectacular. In all Christian sources the Jewish "Pascha" referred to Nissan 14, the day when the Passover sacrifice, if applicable, would have been

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[29] Commentary on B. Rosh ha-Shanah 21a.
[30] Sefer ha-Ibbur, Maamar III, chap. 5 ; edition Filipowski 1846, p. 92.
[31] Rashi on B. Sanhedrin 13b: and B. Rosh ha-Shanah 21a: In fact Rashi understands that R' Huna bar Abin requires that the *tekufat* Nissan falls the latest on Nissan 14. But if it were on Nissan 15 he would make the month of Adar full and the year would remain an ordinary year. If we transpose this in the modern fixed calendar, this could be understood as the possibility of having the *tekufah* on Nissan 15. The reasoning of Tossafot is similar to that of Rashi but they require that the *tekufah* falls the latest on Nissan 15. If the *tekufah* were to fall on Nissan 16 they would make Adar full and they would behold an ordinary year. Therefore I consider that the position of Rashi can be compared to that of Rambam while the position of Tossafot could be compared to that of R. Abraham bar Hiya and R. Hananel despite the formal differences.
[32] *Hilkhot Kiddush ha-Hodesh* IV: 2.
[33] From a piece of evidence mentioned beneath it appears that effectively Passover i.e. Nissan 15 could begin as early as March 21 and the eve of Passover, which the Christians called the "Pascha" could fall as soon as March 20. This was considered too early by the Christians, for whom Easter could not occur before March 22, the day following March 21 which they considered as the day of the true equinox.
[34] Thus according to the rule of Nicaea, Nissan 14 was the earliest on March 21 and Easter is the earliest on Sunday March 22. By contrast, if the Jews considered the rule of the equinox according to the understanding of R' Abraham bar Hiya and consider the *tekufah* on March 22, Nissan 16 is the earliest on March 22 and Pessah which begins on Nissan 15, begins the earliest on March 21. In fact the Seder evening was even on March 20. The Christians considered that the Jews began Pessah too early and did not respect the rule of the equinox. We have a piece of evidence relating that the Jews began Pessah in 387 C.E. on March 21.

prepared.[35] They may have considered that this day is the beginning of the festival. Jaffe mentioned two pieces of evidence for justifying his assumption. The first[36] is related to the year 387; it states that the Church of Alexandria, which considered that Easter cannot fall before Sunday 22 March because of the rule of the equinox adopted at the Council of Nicaea, reproached to the Church of Roma, that they celebrated Easter on March 21, before the limit accepted by the ecclesiastical rules, together with the Jews.
The second piece of evidence[37] mentions that the Christian Church had adopted a cycle of intercalation of 84 years in 298 C.E. This cycle departed from the assumption that the true vernal equinox falls on March 18. Jaffe assumes that the rabbinical Court in Palestine accepted this true equinox and considered that March 20 is the mean equinox and accepted therefore that the first day of Passover falls on March 19.[38] Only if the first day of Passover falls on March 18 would they intercalate the year. This is the justification of the date of March, 19 adopted by Jaffe in his tables, as the limit of Passover and this is the basis of the calculation of the leap years in his reconstructed calendar. It leads to fictitious cycles of intercalation 3-6-8-11-14-16-19[39] or 3-6-8-11-14-17-19.[40] We will see that this assumption is very problematic and must be considered with much reservation.

4. The first piece of evidence mentioned by Jaffe concerns the year 387. Stern (2001) mentions other sources from which it appears that the year 387 was a very special year; it was the subject of many intense polemical debates. In the West it opposed the Alexandrians versus the Romans, in the East it opposed the Alexandrians versus the early Easter observers who followed the Eastern tradition of observing Easter "with the Jews". Besides the piece of evidence mentioned by Jaffe we know the third homily of John Chrysostom "against the Jews" which was delivered in Antioch early in 387 C.E. against the Jews observing Passover before the equinox and against the Christians following them. Similarly the letter of Ambrose, bishop of Milan, from 387 C.E. was a pro-Alexandrian document and an attack against the Roman Church.
   Let us examine the year 4147 corresponding to 386/387. 4147 is the fifth year of a fictitious cycle of intercalation. We will assume that it was not a leap year[41] and we will prove that this was really the case, without any doubt.

   ***Keviyah* of the year 4147 (386/387 C.E.).**
   The number of lunations preceding Tishri 4147 is:
   Ft = Int[(235*4146 + 1) / 19] = 51279.

[35] See Leviticus XXIII; 5.
[36] Ideler, L: Handbuch der mathem. Und technischen Chronologie. 2 vol. Berlin 1825. Vol II, p. 255
[37] Ideler. L, Vol II p. 232.
[38] According to the rule of the equinox of Rabbi Huna bar Abin as understood by R' Abraham bar Hiya.
[39] This is the order ascribed to *Hakhamim* in the Braïta of the order of intercalation.
[40] This is the present order of intercalation; it is ascribed to Rabban Gamliel in the Brïata of the order of intercalation.
[41] 4145, the third year of the fictitious cycle and 4148, the sixth year of the fictitious cycle, were leap years. 4146 was the fourth year of the fictitious cycle of 19 years and was an ordinary year.
We can thus apply the fundamental formula of the Jewish calendar.

The Molad in the modern calendar is:
Mol = $[31524 + 51279 * 39673]_{181440}$ =118011= 4d+13h+291*hal* = 5 – 13 – 291
In the calendar of Hillel the Molad was thus 5 – 13 – 291
– 3 – 727 difference in 4124
– 284 = (51279 – 50995)

5 – 9 – 360

It corresponds perfectly to the Molad of Jaffe: 5 – 9 – 5 in his table א.
The *keviyah* of the year 4147 was thus in the calendar of Hillel as it is also the case in our modern calendar: השא.

Molad Nissan 4147.
The year 4147 is assumed to be an ordinary year, the number of lunations preceding Nissan is thus 51285.
The molad in the modern calendar is:
Mol = $[31524 + 51285 * 39673]_{181440}$ =174609=6d+17h+729 *hal* = 7 – 17 – 729
In the calendar of Hillel the Molad was thus:
7 – 17 – 729
– 3 – 727
– 290 = (51285 – 50995)

Molad in the calendar of Hillel 7 – 13 – 792

Now if we write the modern Molad in terms of the Julian Period, we get:
Mol = 347997.466203703 + 29.530594135804 * 51285 = 1862473.98645 JD
Thus our modern Molad falls slightly before the beginning of the day 1862474.
It corresponds to Saturday 6 March 387. But Nissan 1 was a Sunday; the Molad Nissan 387 was thus on Saturday 6 March 387, 1 Nissan was Sunday 7 March and 15 Nissan, the first day of Passover was on Sunday 21 March 387. The preparation of the festival and the public burning of the leaven was exceptionally early, on Friday 13 Nissan or 19 March.
Our assumption that the year 4147, the fifth year of the fictitious cycle of 19 years was an ordinary year is thus perfectly justified as we see that Nissan 15 of this ordinary year fell on Sunday 21 March and satisfied the rule of the equinox.[42]
This historical piece of evidence gives us precious indications about the practical rule of the equinox used by the Court of Tiberias at the end of the fourth century, during the first decades of the Jewish calendar.
However, according to the Christian rules adopted at the Council of Nicaea, Easter must be on the Sunday following the 14th day of the moon which reaches this age on March 21 or immediately after.
In 387 Nissan 14 was on Saturday 20 March, and for the Church, this lunation was not paschal because it fell before March 21. The year 387 was thus a limit case for the Christians. In fact it appears even that according to the Christian lunar tables the 14th day of the moon was even a day before on 19 March.[43] Therefore

[42] The *tekufah* or mean equinox was on March 22 and the rule of the equinox of rabbi Huna bar Abin must be understood according to the understanding of R' Abraham bar Hiya and R' Hananel.
[43] Stern (2001) p. 144.

this year must be intercalated in the Ecclesiastic calendar. The full moon of March 387 was not paschal and Easter must be delayed to the next lunation. Now the 14th day of the next lunation, according to the Christian tables was on Sunday 18 April and Easter must then be celebrated on Sunday 25 April.[44] The Roman Church could not accept such a late celebration of Easter. We see now that the year 387 was really exceptional. It is because of the exceptional lateness of the Alexandrian Easter that the date of Easter became in that year the object of such intense polemical debates.

Anyhow we see that the Jews celebrated Passover on March 21 in accordance with the Jewish rule of the Equinox, according to the understanding of R' Abraham bar Hiya and R' Hananel of the rule of Shitsar given by Rabbi Huna bar Abin. Indeed Nissan Nissan 16 was on March 22, the day of the *tekufah* or mean equinox.

5. The second piece of evidence given by Jaffe is related to the fact that the Church of Roma considered in its intercalation cycle that the true equinox is on March 18. It is likely that this data could influence the local Jewish community and its calculation of the intercalated years but there is no reason that the Palestinian Court would have been influenced by the data used by the removed Church of Roma. The only undisputable data is that the Jews in the East celebrated Passover in 387 on March 21. If they celebrated Passover even on 19 March, we would certainly have more polemical material extant. Apparently their early celebration of Passover on 21 March[45] was already sufficient to create intense disputes because it was a sufficient reason for the Christians to intercalate their ecclesiastic year. Now if the Court of Tiberias accepted an early Passover on March 19, in contradiction with the rule of the equinox of Rabbi Huna bar Abin and the other rules of the equinox defined in the Talmud,[46] the number of disputes would certainly have been much greater and the year 387 would not have been the most exemplary case of the Jewish deviation. In conclusion this second piece of evidence could apply to the Jews of Roma, removed from Palestine and the Court, but not to the Court of Tiberias.

6. In conclusion the table א corresponding to the calendar of Hillel during the period 4119- 4408 with the *moladot* and the *keviyot* of the different years is a precious device representing a tremendous work. However it was built on the basis of a problematic[47] assumption that the limit of Passover was March 19. Therefore the sequence of the leap years is problematic and in consequence also the *moladot* and the *keviyot* of the years following the problematic years. At the inception of the calendar of Hillel the limit of March 21 for Passover seems more likely. It would be generally associated to the orders of intercalation 3-5-8-11-14-16-19[48] and 3-6-

[44] Thus 35 days later!
[45] Preceded by the burning of the leaven on March 19.
[46] See B. Sanhedrin 13b-14a.
[47] And probably erroneous.
[48] This is the order of intercalation of Rabbi Eliezer in the Braita of the order of the leap years in the cycle of 19 years quoted in *Sefer Yessod Olam*, book IV chap 2. It would correspond to the oldest order of intercalation.

8-11-14-16-19.[49] However we know that the Julian calendar has an excess of 1 day all the 128 years with regard to the length of the tropical year and it is therefore likely that the accepted limit of Passover of March 21 moved back with the time to March 20, March 19 and probably March 18 at the end of the eighth century. It appears therefore that it is impossible to establish a fixed table reconstituting the Jewish calendar because there remain too many unknowns. These considerations are also valid for all the tables of Jaffe whose purpose is the reconstitution of the Jewish calendar between 359 C.E. and 838 C.E.

7. Another factor of incertitude in the tables of Jaffe is the following problem which was raised by Bornstein[50] and Jaffe[51]: did the ancient masters of the calendar take into account, at a moment of history, the *Molad Zaken* in other months than Tishri?
   The origin of this problematic is the discovery, by these scholars, in *Sefer ha-Pardes*[52] of the school of Rashi and in the tractate Soferim, of elements about an unknown *keviyah* גשא, for a leap year. It could only be the reminiscence of an ancient *keviyah* not more in use.

   Jaffe has noted in his tables the years which would have been affected by this problem. The problem of *Molad Zaken* in Shevat and possibly in Kislev is an intricate problem which will be examined in appendix.

In order to examine the merits of the Table of Jaffe let us examine other pieces of evidence mentioned by Jaffe.

- The date of the death of R. Ahai bar R. Huna on Sunday 4 Adar 4266 AMI.[53] It implies that the next year 4267 began on Sunday. This year was the eleventh year of the fictitious cycle 224 of 19 years. It is likely that it was a leap year.
  In our modern calendar the Molad of 4267 is 1 – 22 – 983. We can deduce the Molad of Hillel: – 3 – 727
  – 1768 = (52763 – 50995)

Molad of Hillel of year 4267: 1 – 17 – 648
It corresponds to the Molad of Jaffe 1 – 17 – 9.
We see that the modern Molad could not have fitted because it introduce a *Molad Zaken* and Rosh ha-Shanah would have been postponed to Tuesday. The Molad calculated according to the assumptions of Jaffe explains that we just escaped the postponement of *Molad Zaken*[54], Rosh ha-Shannah and Adar 4 were on Sunday.

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[49] This is the order of intercalation of *Hakhamim* in the same Braita.
[50] Ha-Tekufah vol 16, 1923, pp. 270-273
[51] *Korot Heshbon ha-Ibbur*, Tel Aviv 1931, pp. 168 -172
[52] *Sefer ha-Pardes*, edited by R' H.L. Ehrenreich, Budapest 1924, p. 340 lines 33-34:
אם יבוא סוכות יום שלישי ויהיו מרחשון וכסליו שלימין, יהיו כ"ט שבתות ולא נצטרך לכפול
[53] See note 3. AMI refers to *Beharad* and AMII refers to *Weyad*.
[54] Stern (2001) p. 182 note 113 wants to prove that the Molad was already the modern Molad but *Molad Zaken* was not yet observed. He ascertains even that in 836 C.E. (see the letter of the *Resh Galuta*) the *Molad Zaken* was not yet applied. This position seems indefensible. It seems unconceivable that the rules

- Jaffe mentioned a reference[55] from the Christian writer Victorius according which in 590 C.E. Passover, Nissan 15, fell on Sunday[56] together with the Christian Easter and so did indeed some churches feast Easter.[57] However, the Alexandrian Church, to which belonged the writer, decided to celebrate Easter on the 22nd day of the lunar month, on the next Sunday,[58]in order not to celebrate Easter together with the Jews. Let us check this situation and check if it was indeed an exceptional case. We saw already in other examples how the calculations must be performed, allowing the checking of Jaffe's tables.[59] We can calculate the following table for the year 4350 and for following years which seem to have also Pessah beginning on Sunday. We note that in our modern calendar Pessah falls on the Sunday of Easter in 4350, 4354 and 4374. However when we check the situation according to the Calendar of Hillel, there is a coincidence only in 4350, Pessah occurring on Saturday in the two other years. Similarly if we examine the calendar of Hillel, we note that Pessah falls on Sunday in the years 4350, 4353, 4357 and 4377. However Pessah coincided with Easter only in 4350. The piece of evidence of the Christian writer Victorius, seems to indicate that the year 590 was an exceptional year with the coincidence of Passover and Easter. Our modern calendar cannot explain this exceptional character because the same coincidence should have occurred also in 4354 and 4374. The calendar of Hillel, based on the assumptions of Jaffe, gives a satisfactory explanation: there was a coincidence only in 4530. In 4353, 4357 and 4377 Nissan 14 was on a Saturday later than March 21 and Easter could have been on the next Sunday, together with the Jews. However it seems that the Ecclesiastic lunar calendar was slightly different than the Jewish lunar calendar and, in these three cases, the fourteenth day of the moon was a day later, on the Sunday,

years	Modern Calendar			Calendar of Hillel			Easter
	Molad	Keviyah	Pessah	Molad	Keviyah	Pessah	Sunday
4350	5-20-1074	**זחא**	26/3/590	5-14-11	**השא**	26/3/590	26/3/590
4353	6-12-175	זשג	24/3/593	6-5-13	**זחא**	22/3/593	29/3/593
4354*	3-20-1051	**החא**	11/4/594	3-14-10	גכז	10/4/594	11/4/594
4357*	4-12-152	השג	9/4/597	4-5-12	**החא**	7/4/597	14/4/597
4374	5-11-75	**השא**	31/3/614	5-4-8	הכז	30/3/614	31/3/614
4377	6-2-256	**זחא**	27/3/617	5-19-10	**זחא**	27/3/617	3/4/617

Table 1: Pessah and Easter on Sunday in 590 CE and following years.

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of the calendar would still have changed in 836 C.E. and that a new postponement, would have been introduced. I have always championed the principle that the rules of the calendar were introduced at its inception; only the postponement A was introduced later but it was already debated at the origin. Only technical elements subject to new observation or measurement could be adapted: the Molad, the length of the Jewish lunation or the date of the *tekufah*.

[55] Ideler II, p. 264.

[56] Sunday 26 March 590.

[57] Together with the Jews.

[58] Sunday 2 April 590.

[59] The *keviyah* is deduced from the Molad, using the four gates table (see appendix). The date of Easter was calculated using the algorithm of the Julian Easter by Spencer Jones p. 69 in Astronomical Algorithms Jean Meeus; Willmann-Bell 1991.

delaying automatically Easter to the next Sunday.[60] It is of course funny to note that the Christians created scandals when the Jews celebrated their festival before them but they felt obliged to delay Easter when it happened that both festivals coincided.
We see again that the modern calendar and the modern Molad cannot explain why the coincidence of Passover and Easter in 590 C.E. was such a particular event. By contrast the assumptions of Jaffe allow explaining that this coincidence was unique.

Conclusion.

The table א of Jaffe is related to the period 359 – 648 C.E. The number of pieces of evidence related to this period is not large but it is however not negligible and greater than for any other period. The assumptions of Jaffe, about the limit of Passover, are questionable and, even untenable; therefore the order of the leap years is questionable.
Therefore in the present paper we always tried verifying any data and not relying on Jaffe's table. The examination of different pieces of evidence shows that the assumptions of Jaffe about the epoch of the molad and the length of the Jewish month give interesting results and allow explaining many historical facts otherwise not understandable. It is however necessary to be cautious and question the order of intercalation. We can finally say that his table is reliable except for years with Pessah (Nissan 15) before March 21 or 20 which pose a problem. For such years we must delay Pessah with a month and make the next year a leap year beginning a month earlier. The *keviyah* of both years must be adapted using the four gates table. This makes it necessary to adopt a likely date for the limit of Passover and then adapt the table of Jaffe for the litigious years.
In function of the date of the true equinox, the theoretical limit date for the beginning of Passover should be:
From about 300 until about 430: the limit of Passover should be 21 March.
From about 430 until about 560: the limit of Passover should be 20 March.
From about 560 until about 690: the limit of Passover should be 19 March
From about 690 until about 820: the limit of Passover should be 18 March.
This table is of course purely theoretical. However the ancients did not know the length of the tropical year and the date of the equinox with precision and we don't know at which rate they moved back the limit of Passover.
The Tables of Jaffe inform the reader about the civil date of Passover and allows changing the order of intercalation without too much difficulty.

III. The Jewish Calendar from about 648 until 776.
The Introduction of the Postponement lo **A**DU in the seventh Century.

We have seen that Rosh ha-Shannah could fall on Sunday in the calendar of Hillel.
We found evidence in the Talmud that in the beginning of the fifth century under the reign of Rav Yeimar, Rosh ha-Shannah could still fall on Sunday.
In the epistle of Rav Sherira Gaon it mentions that R' Ahai bar R' Huna died on Sunday 4 Adar 817 of the era of the contracts[61] or 4266 AMI of *Beharad*.[62] This implies that 14

[60] The algorithm of Spencer Jones takes automatically these situations into account.

Adar (Purim) would have been on Wednesday, the following Passover on Friday and the following Rosh ha-Shanah on Sunday.
In the *Sheiltot*[63] of R' Ahai Gaon[64] the postponement A seems already old history and is presented at the same level as the two former postponements DU. For this reason Jaffe and Bornstein considered that the postponement A must have been introduced during the first half of the seventh century. Stern (2001) refers also to an additional reference: the *Sefer ha-Ma'asim*.[65] In this work reference is still made to Rosh ha-Shannah occurring on Sunday.
Jaffe constructed the table א until 4408 and the table ב, related to the second period with the postponement A from 4390 onwards. This places the introduction of this postponement between 629 and 648 C.E. This last date seems to fit all the extant pieces of evidence.
The rules of the calendar were thus the same as before except the additional postponement A. There was probably not yet a regular cycle of intercalation; the intercalations were probably calculated on the basis of an adopted limit for Passover which was adapted according to the acquired knowledge about the length of the solar year and the date of the equinox.
The basic assumptions of Jaffe for the calendar were thus the following:

- The Molad has been chosen near to the moment of the maximum of the solar eclipse which occurred on 15 March 359 C.E. exactly the day of the inception of the calendar. This Molad was still valid
- The lunation was still 29 d– 12 h– 792 *hal* or 29d 12h 44m. At this epoch they did not yet use the *helek*[66] and did not divide the hour in 1080 *halakim*. They could suffice themselves with the division of the hour in 15 *hayil*[67] a *hayil* representing 4 minutes or 72 *hal*. The length of the month was thus noted 29 – 12 – 11. The remainder of a month was 1 – 12 – 11, the remainder of 6 months was 2 – 4 – 6, the remainder of 12 months was 4 – 8 – 12 and the remainder of 13 months was 5 – 21 – 8.
- The rules of the calendar were about the same as today and the postponements were from now on the same as today. The rules were thus the following:
- The postponements were ADU (Sunday, Wednesday and Friday) and יח or 18 hours (noon).
- The length of the year was for an ordinary year 353 days for a defective year
  354 days for a regular year

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[61] Also the Seleucid era.
[62] The relation between these two eras is : 1 SE = 3450 AMI
[63] Sheiltot of Rav Ahai, chapter 79. This work was completed after R' Ahai Gaon settled in Palestine, in about 750 C.E.
[64] R' Ahai of Shabha (680-752) is generally called R' Ahai Gaon although he never was Gaon. When a vacancy occurred in the geonate of Pumbedita in 748, the exilarch named a pupil of R' Ahai as Gaon. Incensed at this slight R' Ahai left Babylonia and settled in Palestine were he ended his masterpiece the Sheiltot.
[65] The *Sefer ha-Ma'asim li-benei Yisrael*, Hillel Newman, Yad Ben Tsvi, 201 is a book of *halakhot* of Palestinian composition; the date of composition is uncertain but the first half of the seventh century is likely. See Stern (2001) p. 184.
[66] 1 minute = 18 *halakim*.
[67] 1 *helek* = 4 minutes and 1 *Hayil* = 72 *halakim*.

355 days for an abundant year
for a leap year 383 days for a defective year
384 days for a regular year
385 days for an abundant year.

- The derivate postponements (resulting from the former rules) are then
  3 – 9 – 3 in an ordinary year or ג ט ג בפשוטה
  2 – 15 – 8 in a year following a leap year or ב טו ח אחר עיבור
- Because of the introduction of the postponement A, the number of possible keviyot was reduced to 14 as today and the possible *keviyot* were the same as today: for ordinary years בחג, בשה, גכה, הכז, השא, זחא, זשג.
  and for leap years בחה, בשז, גכז, החא, השג, זחג זשה .

The weak point of the table ב of Jaffe is again the list of the intercalated years. However the limit of Passover of March 19 seems suitable during the period 560 – 690, it appears that from about 690 onwards the limit of Passover should have been March 18.

By contrast with the first period, no piece of evidence could be produced.

IV. The Observation of September 776 C.E and the adaptation of the Molad.

The chapter V of the Braita of Samuel, in our printed version,[68] begins as follows:

בשנת ארבעת אלפים וחמש מאות ושלשים ושש שוו חמה ולבנה שמטות ותקופות ולא נשתיר לחמה אלא שעה אחת בלבד[69]. מן ארבעת אלפים ותקל"ו שנה[70] ואילך שנה ראשונה מולד לבנה בתשרי בתחילת ליל ד', בניסן בליל ו' בשתי שעות גדולות. הרוצה לידע מולד לבנה יחשוב משששו חמה ולבנה כמה שנים ויתן ד' ימים וארבע שעות גדולות לכל שנה ושנה.....

This text did not exist in the version of the Braita of Samuel quoted by R' Abraham ibn Ezra[71] and R' Abraham bar Hiya;[72] their chapter V began with:

הרוצה לידע בכמה בשבת מולד לבנה נופלת יחשוב משנברא העולם עד עכשיו כמה שנים ויתן ארבעה ימים וארבע שעות גדולות לכל שנה ושנה[73].....

[68] Based on the edition by R' Nathan Amram, Salonika 1861.
[69] It speaks of a "great hour" equal to two hours.
[70] The year 4536 mentioned in the Braita of Samuel is counted according to the style AMII (*Weyad*) and it corresponds to 4537 AMI (*Beharad*). This year is the first year of *shemitah* and the first year of the great cycle of 28 years. We can assume that the members of the council of intercalation had made an observation of the equinox on Thursday September 19, 776 and deduced from it the mean equinox or *tekufah* on Tuesday, September 17, 776 at 16h Jerusalem time. This was a fairly good observation with a precision of about 7 hours with regard of modern calculations. We see that the original text which assumes a lunation of 29 – 12 – 720 was adapted in the manuscript used by R' Nathan Amram, in order to perpetuate this observation and the decisions of the Council of intercalation.
[71] Commentary on Shemot XII: 2.
[72] *Sefer ha-Ibbur*, p. 36 edition Filipowski, London 1851.
[73] The remainder of 12 months is 4 – 8 – 876 in the modern calendar, the remainder 4 – 8 – 0 corresponds to a month of 29 – 12 – 720. The text of the Braita of Samuel seems to consider the more primitive value of the Jewish month of 29 – 12 – 720. See the dictum of Ravina in B. Arakhin 9b. See also Ajdler (2004):

It appears that the Molad was fixed on Tuesday 17 September 776 at 6 p.m. or 4 – 0 – 0 and the *tekufah* which occurred at 3 – 22 – 0 was apparently delayed to 4 – 0 – 0 in order to create an epoch similar to the current situation, according to the Palestinian calendar, at the creation when the first Molad Nissan and the *tekufah* of Nissan of the year AMI were also 4 – 0 – 0.[74] This coincidence fitted perfectly the biblical narrative of the creation of the luminaries on the fourth day.

The year 4537 is the 15th year of a fictitious cycle of 19 years; it is assumed to be an ordinary year, the number of lunations preceding Tishri 4537 is given by the formula

Ft = Int [(235 * 4536+1) / 19] = 56103. The modern Molad is given by:

Mol = $[31524 + 56103 * 39673]_{181440}$ =81363=3d+3h+363*hal* = 4 – 3 – 363

In the calendar of Hillel the Molad was thus – (3 –727)

– = (56103 – 50995)= 5108

3 – 18 – 1008 = 3 – 18 – 14 of Jaffe.

Thus the Molad of Hillel of Tishri 4537 was 3 – 18 – 1008. It was corrected after the observation of September 776 to 4 – 0 – 0 by the addition of 5 – 72, thus 5 hours and 1/15. The modern value of the corresponding Molad is 4 – 3 – 363.

We ascertain that the consequence of the use of a lunation of 29 – 12 – 792 from the inception of the calendar on brought an accumulated difference of 5108 *hal* = 4h 788 *hal*. It is not far from the correction of 5h 1/15 that was made by adopting the Molad of 4 – 0 – 0. We don't know how they found the new value of the mean equinox. Did they find it from a number of eclipses like Ptolemy or did they simply consider that the lunation of 29 – 12 – 793 is more correct and they simply added the accumulated difference and rounded the result off? Anyhow it seems that they adopted a new epoch for the Molad on Tishri 4537: 4 – 0 – 0.

By contrast it is certain that the moment of the autumnal equinox must have been determined experimentally. They found apparently a true equinox on Thursday 19 September 776 at about 4 p.m. and deduced from it the mean equinox on Tuesday 17 September 776 at about 4 p.m. in Jerusalem. The date of the equinox given in the Braita of Samuel is the mean equinox. This is a proof that the equinox generally considered in the study of the Jewish calendar for the rule of the equinox is always the mean equinox. This confirmed them that Nissan 16 could fall on March 19 and Pessah could then be on March 18.

We observe also, from the text of the Braita, that their counting of the Sabbatical years was the same as today, according to the counting of the Geonim mentioned by Rambam in his *Hibbur*, *Hilkhot Shemitah ve-Yovel* X;6 the year 4536 AMI was a sabbatical year.

Jaffe has constructed table ה of Moladot from 4542 onwards, based on the results of the observation of September 776. He adopted the following assumptions:

Rav Safra and the Second Festival Day: Lesson about the evolution of the Jewish calendar. p.17. Tradition Vol 38, N° 4, Winter 2004.

[74] According to the calendar of the Palestinians and Ben Meir. In the modern Babylonian calendar, i.e. our modern calendar the Molad and the Tekufah are not at the same moment: in our modern calendar the Molad of Nissan AMI was 4 – 9 – 642 and the *tekufah* was 4 – 9 – 0. There were thus two styles: The Molad was 4 – 9 - 642 according to the Babylonian style and 4 – 9 – 0 according to the Palestinian style. In 776 all these figures were still unknown. But the purpose was the same: replicate the biblical narrative.

- The cycle of intercalation is now fixed; it is the cycle 3 – 6 – 8 – 11 – 14 – 17 – 19

This assumption makes sense.[75] The earliest mentions of the19-year cycle is in the end of chapter 8 of Pirquei de-Rabbi Eliezer, a work generally dated to the eighth or the ninth century. It is also mentioned the liturgical poem *Kiddush Yerahim* of R' Pinhas which was written not earlier than the mid eighth century.[76]
The adoption of a fixed order of intercalation represents a considerable evolution in the solar regulation of the Jewish calendar. Instead of being obliged to be dependent on the *keviyot*, and compare Nissan 16 with a date of the *tekufah*, the rule of the equinox of Rabbi Huna bar Abin would depend now, after the introduction of a fixed order of intercalation, from the distance of the *tekufah* to the Molad of Nissan.[77] The rule of the equinox would simply imply that the vernal *tekufah* may not fall later than 16 days or 384 hours after the Molad of Nissan in the sixteenth year of the cycle, in which Pessah is the earliest.
This new procedure would be perfect if the length of the tropical year was exactly equal to the length of the mean Jewish year. In reality the Jewish year is longer than the tropical year and the Jewish year will shift toward the summer. In fact it appears that this cycle of intercalation was probably introduced several tens of years too early. Indeed the adopted *tekufah* on 17 September 776 at 18h corresponds to a true vernal equinox on 19 March 18 p.m. and a first day of Pessah or Nissan 15 on 18 March. We observe in table ה that the introduction of the cycle of intercalation 3 – 6 – 8 – 11 – 14 – 17 - 19 leads to a limit of Passover of 17 and 18 March. The date of March 17 is still too early for the first day of Passover. Although there is not a clear cut limit it seems that this order of intercalation would have better fitted during the period 838 – 1160.
The *tekufah* used at this stage is not yet the formal *tekufah* of Rabbi Adda bar Ahava but the mean equinox deduced from the observed astronomical true equinox.

- The Council adopted a cycle of 13 * 19 = 247 years corresponding to a synodical lunation of 29 – 12 – 793 + 905 / (13 * 235) = 29 – 12 – 793.2962.

This assumption rests on a very tiny argument: an allusion of Ibn Ezra about the relinquishment of the cycle of 247 years[78] also called עיגול דרב נחשון גאון, which convinced Jaffe that this cycle was ones in effective use. Indeed the *Iggul* of Rav Nahshon 13 * 235 months corresponds to a remainder of 6 – 23 – 175 = 7days – 905 *hal*.
The cycle of 247 years contains 3055 months. If a month had a length of 29 – 12 – 793 then the remainder of 3055 months would be 121,201,015 hal = [M(181440)] – 905 hal.

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[75] But it could also have been introduced a little later.
[76] Because there is mention in this liturgical poem of a fast commemorating the earthquake of January 748 C.E.
[77] This principle was already proposed by R' Isaac Yisraeli in *Yessod Olam*, Ma'amar IV, chap 2, p. 4a and chap 4, p. 6a. Jaffe, in *Korot* (1931) p.112 adopted the same principle to explain the evolution of the understanding of the rule of the equinox. Loewinger in *Al ha-Sheminit*, Tel Aviv 1986, pp. 25-26 proposed to understand Rambam H.K.H. IV, 2 on the basis of this principle but the argument is questionable.
[78] רמז לעגולת רמ"ז, הלוחות הראשונים אשר שברת, יישר כחך ששברת.
Ibn Ezra in *Sefer ha-Meorot*, Leiden 1496 and 1550; Roma 1544; Frankfort on the Main 1624. This reference was mentioned by Jaffe p. 159 and Bornstein *Makhaloket* p. 142. See also Jaffe p. 158 two references to *seder de rav Nahshon* and *iggul de rav Nahshon*.

Thus introducing a cycle of 247 years gives a supplement of 905 *hal* for 3055 months. Jaffe built the table ה with the following assumptions. The Jewish month is still considered as (29 – 12 – 11) = (29 – 12 – 792), but after the first year and then after successively all the 4 and 5 years, he adds 1 *hayil*. With this procedure he adds in the complete cycle 55 *hayil* or 3960 *hal* corresponding to 3055 months * 1 *hal* + 905 *hal*. The procedure proposed by Jaffe is thus rigorously correct, but it the fruit of his inventive spirit and his ingenuity. There is not the least piece of evidence that this cycle was **really in use** and, if this was the case, it is not sure at all that it was implanted on this way. It is also possible that this cycle was only a working hypothesis which was abandoned and never in effective use. The length of the lunation would have been fixed from 4542 onwards to 29 – 12 – 793.[79]

V. The Letter of the *Resh Galuta* of 836 C.E.

J. Mann discovered an exceptional document from the Cairo Geniza and published it in 1922.[80] This document was called the letter of the *Resh Galuta*,[81]because its author appeared to be a very important and authoritative personality.
This letters reveals that Passover (15 Nissan) of the year 836 C.E. was due to occur on a Tuesday whilst according to the present-day calendar, it should have occurred on Thursday. According to the Exilarch the year must be made defective because of a concern to avoid visibility of the new moon of Nissan before the first day of the month. Today, however, we don't care for this problem and the Talmud accepted the case of a

4596 AM1	835 C.E.	Tishri 1	Nissan 1
385 days		Saturday, August 28 Molad (6)-22-660	
	836 C.E.		Thursday, March 23 Molad (3)-15-811 *Molad Zaken* if Molad >=(3)-13-642
4597 AM1		Saturday, Sept. 16 Molad (5)-20-169 *Molad Zaken*	

Table 2: The situation according to our modern calendar. Rosh ha-Shannah is on Saturday in both 4596 and 4597. 4596 is the 17th year of a cycle; it is a leap year זשה of 385 days and Rosh hodesh Nissan is on Thursday

[79] It is generally accepted that only at the introduction of the Jewish month of 29 – 12 – 793 the necessity to introduce the *helek* (1/1080 of the hour) was felt. The first mention of the division of the hour in 1080 parts is made in a liturgical poem of Rabbi Pinhas. Similarly the earliest mention of the 19- year cycle of intercalation is made at the end of chap VIII of *Pirquei de-Rabbi Eliezer* (generally dated to the eighth or ninth century) and in the *Kiddush Yerahim* of Rabbi Pinhas. R' Pinhas is supposed to have lived in the late eighth or early ninth century. See Stern (2001) p. 197 and 204. R' Pinhas mentions in his *Kiddush Yerahim* the fast commemorating the earthquake of January 748 C.E. and wrote certainly after this date.
[80] Mann, J. (1920-1922); The Jews in Egypt and Palestine under the Fatimid Caliphs 2 vols, London. See vol 2. pp. 41-42
[81] The Babylonian exilarch. There is indeed at the end of the letter an allusion about the authority of the letter's author.

4596 AM1	835 C.E.	Tishri 1	Nissan 1
383 days		Saturday August 28	
	836 C.E.		Tuesday, March 21
4597 AM1		Thursday, Sept. 14	

Table 3: The data According to the Letter of the Resh Galuta.

first visibility one day before the first day of the month or a day later.[82]
The year 4596, the seventeenth year of a cycle of 19 years, was a leap year. According to the modern calendar it was a full year of 385 days of the type זשה with Passover on Thursday 6 April. It appears from the letter of the *Resh Galuta* that in reality the year was defective of the type זחג and Passover was on Tuesday 4 April. The calendar was different than the present-day calendar.
In order to go further we must examine the following passage of the letter:[83]

משום סיהרא דניסן דקא מתיליד **ביממא** דתלתא בשבה בארבע שעות.....

- Bornstein followed the reading of Mann and understood that the Molad of Nissan was on Tuesday at 4 Jewish hours: 3 – 4 – 0 in our notations, Monday at 10 p.m. about 12 hours before our modern Molad. This explains that there was no *Molad Zaken* in Tishri 4597 and therefore the year was defective.
- Jaffe did not read ארבע שעות but assumed ארבע ידות or ארבע דנקות. He understood that the Molad was 40 minutes in the **morning** thus a Molad 3 – 12 – 720, very near to the Molad used at that time after the adaptation of the Braita of Samuel in 776.We understand now why Jaffe championed the *Iggul* of Rav Nahshon; it allowed him getting a good coincidence between the assumed Molad used by the Palestinians and the Molad mentioned by the *Resh Galuta*. This explains also why there was no *Molad Zaken* in Tishri 4597 and the year was defective and had 383 days. Now according to this understanding of Jaffe, the *Resh Galuta* was aware of the effective Molad of 3 – 12 – 720 and the *keviah* sent from Palestine was correct and incontestable. Why was he then justifying the decision sent from Palestine and championing the unity of the communities of Israel as if he was facing opposition and objection against the *keviyah* sent from Palestine? In order to answer this question Jaffe must invoke the problem of *Molad zaken* in Shevat.[84] The Molad of Shevat 4597 would indeed be (3 – 12 – 720) + (2 – 4 – 438) + (6 – 2 – 1012) = 4 – 20 – 10: The problem of *Molad zaken* in Shevat was in the news and the Palestinians decided not to pay attention to it and not delay Rosh ha-Shannah 4597 to Saturday.
- Stern (2001) proved irrefutably that the reading is ארבע שעות.He understands that the Molad was at four hours in the **morning** thus the Molad was 3 – 16 – 0. This Molad was very near the modern Molad 3 – 15 – 811 and perhaps it was exactly the same but the *Resh Galuta* rounded it off. Thus the Molad was already the same

[82] See B. Erakhim 9b. See Ajdler, J. *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Jerusalem 1996, p. 221.
[83] For a complete transcription of the letter of the Resh Galuta see :
1. Mann, J. note 80.
2. Bornstein, H. J. Ha-Tekufah Vol 14-15 Warsaw 1922, p. 346.
3. Kasher, M, 1949 Torah Shelema XIII, p. 170.
4. Sar Shalom, R, 1985: Shearim le-Luah ha-Ivri p. 27.
5. Stern, S, 2001: Calendar and Community pp. 277-283 (with Xerox copy of the original).

[84] See Jaffe (1931) pp. 98-102.

as the modern Molad and the *Resh Galuta* knew this Molad. The question is then: why was this year defective? Stern answers that the postponement of *Molad Zaken* was not yet in observance.[85]

It must be noted that all these positions are untenable:

- Bornstein does not explain the aim of the letter of the *Resh Galuta*. Indeed this letter is certainly not a letter of announcement of the *keviyah* of the year 4596. It does even not mention that the year 4596 is a leap year. On the other hand he doesn't explain and justify the discrepancy with regard to the modern Molad.
- Jaffe founded his explanation and his elaborated theory on an incorrect reading.
- Stern understands that the Molad is the same as today but the rule of *Molad Zaken* did not yet exist. It would be introduced only in about 838 C.E. The position of Stern seems unacceptable for many reasons.
    1. It seems difficult to imagine that a rule like *Molad Zaken*, of which the origin is "as obscure as is its rational",[86] would have been introduced so late at a moment when it seems that the Babylonians could already have been associated with the calendar committee and without their objection. Furthermore we do not see a plausible motivation for such an innovation.
    2. It is certainly less problematic to behold the rules and to adapt the Molad in function of the scientific knowledge than to change the rules which are sanctified by their age.
    3. If we consider[87] that the work of al-Kwarismi about the Jewish calendar was genuine, it would mean that in about 825, the rules of the calendar, including *Molad Zaken* were known, the only unknown elements were the epoch of the Molad and of the cycle of 19 years.[88]
    4. The assumption of Stern that the present-day Molad was already the same in 836 and in 506 C.E. and even earlier is in contradiction with the theory that the Molad was derived from Ptolemy's Almagest in about 838 C.E. after the completion of an Arabic translation.[89]
    5. Stern does not provide a plausible explanation of the purpose of the letter of the *Resh Galuta*. He does not explain the reason of the objections raised against the *keviyah* sent from Palestine.

Because of all these arguments I propose another explanation. It rests on the general theory of the evolution of the Molad of Jaffe but it deviates from his interpretation of the letter of the *Resh Galuta* and its purpose.

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[85] Stern (2001) p. 196. Stern had already used the same argument in order to explain the *keviyah* of the year 4266. Again he assumed that the Molad in Tishri 4266 was the same as today or very near to it and he explains that at this time the postponement of *Molad Zaken* was not yet in observance. See Stern (2001) p. 195.
[86] Sten (2001) p. 195.
[87] This is far from certain. See Langermann (1987) and Sar Shalom (1988) in Sinaï n°106, pp.26-51.
[88] See Stern (2001) p.185.
[89] See Stern (2001) p. 209 about the death of R. Ahai bar R. Huna on Sunday 4 Adar 4266.

We assume that in Tishri 776 C.E. the Molad was fixed at 4 – 0 – 0 according to the observation of the Braita of Samuel and in March 836 the Molad was still based on the Molad of Tishri 776 and was about 3 – 12 – 664 or 3 – 12 – 720.
In fact the lunation was 29 – 12 – 793 or, according to Jaffe 29 – 12 – 793.2962.
In the first assumption the Molad Nissan 4596 was (3 – 15 – 811) – (0 – 3 – 363) = 3 – 12 – 448. In the second assumption we must reduce the difference 3 – 363 by 0.2946 * number of months elapsed between Tishri 4537 and Tishri 4596 or by 0.2946 * (56890 – 56103) = 232 hal. In this last assumption the Molad would then be 3 – 12 – 680 very near to the Molad calculated by Jaffe.[90]

This Molad of Nissan was thus certainly before the limit of 3 – 13 – 642 and therefore there was no *Molad Zaken* in the following month of Tishri; the leap year 4596 was a defective year of 383 days and Pessah was on Tuesday and not on Thursday.
Under the caliph al-Mamun (786-833) the son of the celebrated Harun al-Rashid (766-809) there was a cultural renaissance and the translation of Ptolemy's Almagest appeared in two versions; an older one by al-Hassan ibn Quraysh and another dated 827/828 by al-Hajjaj. This letter would be a piece of evidence of the first critics against the Palestinian authority. Some influential scholars had studied the new translation of the Almagest and had probably deduced from the table of mean conjunctions of the Almagest that the mean conjunction of Ptolemy of March 836 was 3 – 14 – 1041 in Alexandria and after transformation to Bagdad time it was indeed close to 3 – 16, corresponding to 10 a.m. or 4 hours in the morning as indicated in the letter of the *Resh Galuta*. They argued that the molad being about 3-16, there must be a *Molad Zaken* in Tishri 4597 and therefore the year 4596 should be an abundant year of 385 days and Passover should be on Thursday. These scholars contested thus the *keviyah* sent from Palestine on the basis of the data found in the Almagest which had just been translated into Arabic. The scenario could have been that the Exilarch, who was not aware of the true Molad used by the calendric calculators or *meabrim* and accepted the Molad 3 – 16 proposed by the readers of the Almagest, had to advocate in favor of the Palestinian's *keviyah* and against those people who contested the fixing of the year on the basis of the Ptolemaic conjunction. This allows understanding why this letter advocated in favor of the Palestinian's decision and the primacy and the unity of the communities. This letter was thus not a letter announcing the *keviyah* to the communities; it was a letter advocating for the unity of the communities around the *keviyah* sent from Palestine. It is probable that concurrently the Exilarch expressed his doubts to the Palestinians. It is likely that the Exilarch's intervention led to a common meeting in around 838 in the course of which the new Molad was adopted, in order to solve the contradiction between the Palestinian Molad and the Molad deduced from the Almagest.

In my opinion the rules of the calendar were already fixed long ago but the Molad was still the object of changes and adjusting. The postponement of *Molad Zaken* was like the other postponements, old history. Except the postponement A, all the postponements belonged already to the calendar of Hillel at the inception of the fixed calendar.

VI. Our Present Molad is derived from the Almagest.

[90] Thus the *Resh Galuta* knew already the Molad used by the Palestinian *meabrim*.

At a period when the evolution of the Jewish calendar was not yet imagined, Slonimski had already remarked the dependence of our Molad on the table of mean conjunctions of the Almagest. Slonimski had remarked that the first conjunction of the table of Ptolemy corresponds to the conjunction of Nissan 3014.
The epoch of the Almagest is 1 Toth, year 1 of the Era of Nabonassar corresponding to Wednesday 26 February – 746 C.E. at noon.
The first conjunction of the table of mean conjunctions[91] in Ptolemy's Almagest is 24 Toth; 44'17'' corresponding to Toth 24, 17h 42m 48s after noon or Saturday 22 March – 746 at 5h 42m 48s a.m. (after midnight) or 11h 770.40 hal in Jewish hours corresponding after rounding off to 7 – 11 – 770. Ptolemy's table gives also the distance of the common position of mean sun and mean moon, at the moment of the mean conjunction, from the sun's apogee. For this first mean conjunction this distance was: 288°; 38' 50". After addition of the sun's apogee of 65°; 30' we get the common mean longitude of 354°; 08' 50". This conjunction preceded thus slightly the equinox; it was thus certainly the mean conjunction of Nissan 3014.
Now if we calculate the modern Molad of Nissan 3014 we find that it was on 7 – 12 – 540, thus Saturday at 6h 30m a.m. in round figures. Slonimski considered that this coincidence could not be a mere chance. He considered that our modern Molad was deduced from the Almagest by the addition of 850 halakim. It is a noticeable point that the number of lunations between the Molad *Weyad*[92] and the Molad of Nissan 3014 is 235 * 158 + 12 * (7 – 1) + 13*4 + 6 = 37260. It gives a shift of the Molad of 24300 hal = 22.5 hours and therefore the epoch or Molad of *Weyad*[93] was (7 – 12 – 540) – (0 – 22 – 540) = 6 – 14.

VII. The Meeting ועד המאוחר Between Palestinians and Babylonians in About 838 C.E

Bornstein and Jaffe assumed that a meeting was held in Palestine with the participation of the Babylonian specialists.[94] Their participation could have been motivated by the fact

[91] Ptolemy's Almagest, G.J. Toomer, London 1984, p.278. On page 275, in the text it calculated that the conjunction was 23; 44, 17 days after the epoch, which was noon of Toth 1 of the era of Nabonassar. The astronomical day began at noon. By contrast Ptolemy tabulated 24; 44, 17 with the meaning: the 24th day of Toth , 44, 17 after noon (the whole day being 60 parts, 44, 17 represents 0.738055555 of a day of 24 hours, i.e. 17h 42m 48s). Apparently the convenience of this notation to the user became so obvious that he adopted it also in the Handy Tables. This is probably also the origin of the inclusive reckoning for dates adopted in the Jewish calendar. For example 6 – 12 – 540, the Molad of Nissan 3014 means Friday at 12h 540 *hal*. In many calculations it would be more convenient to use the exclusive and homogenous notation 5 – 12 – 540 giving the time elapsed since the beginning of the week at Sunday 0h but the custom of the *meabrim* is to use the inclusive notation and designate the beginning of the week by 1 – 0 – 0 instead of 0 – 0 – 0 (after the beginning of the week).
[92] The Molad of Tishri 2 AMI (Tishri of the second year of the era of *Beharad*). Originally the Aera Mundi was counted from the second year; it was the Era of *Weyad*, 2 AMI or 1 AMII.
[93] During a long period this Molad was the epoch of the Molad. Ibn Ezra, in his *Sefer ha-Ibbur* related this Molad to the Biblical passage in Deut XXIII, 13.
[94] We have no real evidence of such a meeting. Bornstein and Jaffe based themselves on the contents of a letter addressed by the Babylonians to the Palestinians at the occasion of the R' Sa'adia-Ben Meir dispute, mentioning the existence of such a meeting which would have given to the Babylonian scholars all the

that the Babylonians had provoked this meeting in order to debate about the discrepancies observed between the *keviyah* sent from Palestine on the basis of their Molad, and the mean conjunction found in the Almagest. This was the beginning of the active participation of the Babylonians to the fixing of the calendar.
We have seen that the Council of intercalation adopted in 776 C.E. a new Molad; its epoch was 4 – 0 – 0, Tuesday 17 September 776 at 6 p.m. The modern Molad of this month of Tishri 4537 is 4 – 3 – 363; thus a difference of 3 – 363 = 3.3361 hours.
It is likely that the purpose of the meeting was to reform the molad in order to bring it in concordance with the Almagest, which was the authoritative reference. In fact we have no real piece of evidence proving the reality of this meeting and therefore no information about its decisions; However, from the elements of the dispute between R' Sa'adia Gaon and Ben Meir and from the different exchanges of letters between both parties which were found in the Cairo Geniza, Bornstein and Jaffe deduced that the object of the different between both parties was a difference of 642 halakim between their Moladot. The Molad of the Palestinians was 642 halakim less than that of the Babylonians. It seems thus that they adapted at this meeting the Molad according to the table of the Almagest. However it seems that without paying too much attention to this point, they made the adaptation differently. They did not realize that this difference would bring in the future such a dispute. The Molad of the Almagest for Nissan 3014 was after rounding off, 7 – 11 – 770 in Alexandria. According to Ptolemy's Geography the difference of longitude between Alexandria and Jerusalem is 5°; 30' corresponding to 22m or 396 halakim. The Molad in Jerusalem was thus 7 – 12 – 86. The Babylonians added 454 other halakim in order to get a rounded off number: 7 – 12 – 540, from which they deduced the epoch of the era of the creation: (7 – 12 – 540) – (0 – 22 – 540) = 6 – 14.[95] By contrast

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elements allowing them to perform by themselves all the calendar calculations. See Jaffe *Korot* (1931) p.187 and Bornstein, *Makhaloket*, 1904, pp. 88-89.
However such a meeting makes sense. We have seen that the letter of the *Resh Galuta* was probably a piece of evidence of the contestation against the *keviyah* sent from Palestine because it was in contradiction with the table of conjunctions of the Almagest. This problem justified a meeting with the Babylonians, the authors of the contestation. Besides, only such a meeting could explain how the Babylonians acquired the knowledge allowing them to make independent calculations of the *keviyah* and contradict the Palestinians at the occasion of the dispute between Ben Meir and Sa'adia Gaon in 922.

[95] Nowadays we consider exclusively the era of *Beharad*. But before the eleventh century the era of the creation was counted from the second year, it was the era of *Weyad*. This era is already mentioned in the Talmud Avoda Zara 9b. All the dates in the Talmud are expressed in AMII. In B. Avoda Zara 9b it writes:
403 years of the era of the Destruction = 4231 AMII
Thus 1 Era of the destruction= 4231 – 402 = 3829 AMII = 3830 AMI = 70 C.E.
It seems interesting at his point to give the chronology of the first year of *Beharad*. This era was probably introduced because it placed the epoch of this era at the beginning of a cycle of 19 years.
The *tekufah* of Samuel of Tishri: the *tekufah* of Samuel was on 24 September at 3 a.m.
The Molad *Beharad* 2 – 5 – 204 was on Sunday 6 October – 3760 at 23h 11m 20s.
1 Tishri AMI was Monday 7 October – 3760.
30 Marheshvan 1 AMI was Thursday 5 December – 3760.
30 Kislev 1AMI was Saturday 4 January – 3759.
The *tekufah* of Samuel of Nissan was on Wednesday 22 Adar at 0h or Tuesday 25 March – 3759 at 6 p.m.
26 Adar 1 AMI was Sunday 30 March – 3759.
29 Adar 1 AMI was Wednesday 2 April – 3759.
24 Ellul 1 AMI was Sunday 21 September – 3759.
Molad *Weyad* or 6 – 14 was on Friday 26 September – 3759 at 8 a.m.
1 Tishri 2 AMI was Saturday 27 September – 3759.

the Palestinians subtracted the remainder of six months i.e. 2 – 4 – 438 from the Molad of Nissan 3014 : 7 – 12 – 86 and found 5 – 7 – 728 for the Molad of Tishri 3014. They rounded off this Molad to 5 – 7 – 540 by subtracting 188 halakim. This led them to a rounded off Molad for Nissan of Tohu: (5 – 7 – 540) – (0 – 22 – 540) = 4 – 9 – 0.
Apparently the participants did not find an agreement for a common decision.
Thus Palestinians and Babylonians left each other with different Moladot, the Babylonians added 454 halakim to the conjunction of Ptolemy while the Palestinians subtracted 188 halakim. The Palestinians, who considered themselves as the principal concerned, let probably the problem open in the hope that new observations would help solving it definitively.

Jaffe supposed that at the end of the ninth century the members of the Palestinian council of intercalation were made aware of the observations of Al-Battani: the determination of the equinox of 19 September 882 and the observation of the lunar eclipse of 21 July 882.[96]
The observation of the equinox[97] confirmed them that the observation of 776 was acceptable and it informed them that the limit of Passover of March 17 connected to the new system of a regular cycle of intercalation was now acceptable and justified.[98] The observation of the lunar eclipse[99] would have persuaded the members of the council of intercalation that the mean conjunctions preceded the mean conjunctions of Ptolemy and therefore the rounding off adopted by the Palestinians seemed justified to them by contrast to the rounding off adopted by the Babylonians. Apparently Palestinians did not inform Babylonians of these last developments.

The problem is that there is no proof that the members of the council of intercalation knew already the treatise of Al-Battani. Furthermore the details of the observation of the

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In Vayikra Rabbah XXIX, 1 it writes that the creation began on Sunday 25 Ellul; this seems in contradiction with our table giving Sunday 24 Ellul. Apparently this passage is anterior to the rule **lo DU Rosh.** The 1 Tishri 2 AMI was on Friday and therefore the preceding Sunday was the 25 Ellul. Similarly the Sunday 26 Adar 1 AMI was in this ancient calendar Sunday 27 Adar, the year 1 AMI being an abundant year of 355 days. This day would be Sunday 25 Adar 1 AMI if this year of Tohu was a defective year of 353 days. But this is contrary to our calendar. In other words the ancient traditions placing the beginning of the creation of the world on Sunday 25 Ellul or on Sunday 25 Adar are anterior to our calendar and don't agree with it.

[96] Jaffe had apparently no access to the original treatise of Al-Battani and knew these observations through secondary sources like the information provided by *Yessod Olam* of R' Isaac Israeli. See *Yessod Olam*, ma'amar IV, chap. 7, p. 12a for the lunar eclipse.

[97] See Al-Battani Vol 1 pp. 42 and 210. The equinox occurred on 19 September 1h 15m a.m. ar-Raquah or 18 September 22h 39m UT. The modern value is 23h 05m. This observation is considered as one, if not the most, exceptional astronomical observation of history.
This observation justifies already Passover on March 17.

[98] The equinox "observed" by Al-Battani was on 19 September at 1h 15m a.m. ar-Raquah or at about 0h 48m. The following vernal mean equinox was then on 18 March at about 4 p.m. Thus Nissan 16 may fall on March 18 and Passover may fall on March 17. The observation of Al-Battani supported the cycle of intercalation adopted. This is the meaning of the statement of R' Juda ha-Levi in *Sefer ha-Kuzari*, book IV, chap 29 that the *tekufah* of Adda is in agreement with the observation of Al-Battani.

[99] See Al-Battani, Vol 1, pp. 57 and 230. The lunar eclipse was on Tuesday 23 July 883 at 8h 06m p.m. or Wednesday 15 Av 4643.

lunar eclipse are insufficient to know the mean conjunction[100] and Al-Battani is unlikely to have published his works before the beginning of the tenth century. It is however exact that the comparison of the table of conjunctions of the Almagest and that of Al-Battani allows concluding that the mean conjunctions of Al-Battani preceded those of Ptolemy by 31 minutes if we take into consideration the longitudes of ar-Raquah of 73°; 15' and Alexandria of 60°; 30'.[101]
Thus the astronomical treatise of Al-Battani would arbiter in favor of the Palestinian position. But it is not sure that the Treatise of Astronomy of Al-Battani was known by the Palestinian council of intercalation before the outbreak of the dispute. Anyhow it seems likely that the whole discussion between Palestinians and Babylonians about the Molad was forgotten and two concurrent and contradictory methods of calculation of the *keviyah* coexisted until the outbreak of the dispute in 922 C.E, without that the protagonists remembered the origin of the discrepancy.

VIII. The Dispute of R' Sa'adia Gaon and Ben Meir.

On *Hoshana Rabba* 921 C.E. The Palestinian Gaon Ben Meir or his son proclaimed on the Mount of Olives that the months of Marheshvan and Kislev of 4682 would be defective. As a result Passover 922 would fall on Sunday instead of the following Tuesday if the year had been made full. In 922 the Jews of Palestine and probably the communities in Egypt celebrated Passover on Tuesday, two days before the Jews of Babylonia. This split between the communities of Palestine and Babylonia caused a considerable agitation throughout the Jewry. References to this event can be found in non Jewish documents. The Syrian Elias of Nissibis[102] wrote that in the year 1232 of the Seleucid era dissension broke out between the Jews of the West (Palestine) and those of the East (Babylonia) with regard to the calculation of their holiday. The Jews of the West celebrated the New Year[103] on a Tuesday and those of the East celebrated it on the next Thursday.[104] Similarly the Karaïte Sahal ben Mazliah[105] referred also this event and sought to prove from this controversy that the rabbinic calendar calculations were groundless. According to the Babylonian Molad, in Tishri 4683 there was the postponement *Gatrad* and in Tishri 4684 there was the postponement *Yah*, therefore the *keviyah* of the three years was then: השג, הכז, בחג By contrast, the Molad of the Palestinians was 642 hal less and there was no postponement in Tishri 4683 or in Tishri 4684 and the keviyah of the three years was החא, גכה, זשג Furthermore the astronomical situation was exceptional on Rosh ha-Shanah 4683: the true conjunction occurred about 1.5 hours after sunset on Monday evening, the lunar latitude was about 5°, and,

[100] We know that the relative position of the two bodies, the sun and the moon, may vary 1.9° + 5.4° = 7.3° from their mean value near the conjunction. As the hourly motion of D, the elongation moon-sun is 0.51°, the maximum interval between the mean new moon and the true new moon is 14.3 hours. At the moment of the full moon the situation is similar between the true and the mean full moon.
[101] The time difference between ar-Raquah and Alexandria is thus 51 minute. However on p.42 Vol 1, it writes in the main text that this difference is 40 minutes; this would reduce the difference to 20 minutes.
[102] See Bornstein, *Divrei Yemei ha-Ibbur ha-Aharonim*, ha-Tekufah, Vol 16, Warsaw 1923 pp. 237-238.
[103] Of 4683 or September 923.
[104] Otsar Israel: entry Ben Meir, written by Jaffe.
[105] See Bornstein, *Divrei Yemei ha-Ibbur ha-Aharonim*, ha-Tekufah, Vol 16, Warsaw 1923 pp. 237.

exceptional fact, the moon was seen on Tuesday evening in Egypt, in Palestine and even in Babylonia.

Year	Year	Babylonian Molad	Keviyah	Palestinian Molad	Keviyah
922-923	4682*	4 – 11 – 932	השג	4 – 11 – 290	החא
923-924	4683	3 – 9 – 441	הכז	3 – 8 – 879	גכה
924-925	4684	7 – 18 - 237	בחג	7 – 17 – 675	זשג

Table 4: the years 4682, 4683 and 4684 according to the Palestinians and the Babylonians.

The vision of the new lunar crescent was thus one day before the first day of Rosh ha-Shannah adopted by the Babylonians. The Karaïtes, who sanctified the first day of Tishri at the moment of the vision of the new moon, celebrated their Rosh ha-Shannah on Wednesday. This was also an exceptional event: never before had the Karaïtes celebrated Rosh ha-Shannah before the Babylonian Rabbis. This event made a great stir and agitation in Egypt and the pupils of Rabbi Sa'adia Gaon were distraught. The letters exchanged between them and Sa'adia Gaon were preserved in the Cairo geniza.

The Palestinian community saw with this vision the proof of the correctness of the calculation of Ben Meir and his *keviyah*. The truth is that the Talmud accepts such an inevitable situation: it is possible that the new crescent is seen one day before the *Keviyah*.[106]

Maimonides wrote about this problematic of the first visibility of the lunar crescent one day before the *yom ha-keviyah*,[107]

ודבר זה הלכה למשה מסיני הוא, שבזמן שיש סנהדרין קובעין על פי הראייה ובזמן שאין שם סנהדרין קובעין על פי הישבון הזה שאנו מחשבין בו היום ואין נזקקים לראייה, אלא פעמים שיהיה יום שקובעין בו בחשבון זה הוא יום הראייה או קודם לו ביום או אחריו ביום, וזה שיהיה אחר הראייה ביום פלא הוא, ובארצות שהן למערב ארץ ישראל.....

It is thus a Mosaic tradition from Sinai that in times when there was a (Palestinian) Sinedrium, declaration of New Moon Days was based on visual observation, while in times when no Synedrium existed, this declaration was based on calculations such as we are using today and no attention was paid to observation of the new crescent. Rather the day established by calculation might well coincide with the day in which the day in which the new moon became visible , but it might sometimes be the day before it or the day after[108] *it. The latter case, however, when the calculated New Moon Day happened to be*

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[106] B. Erakhim 9b. See J. Ajdler (1996): *Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam*, Sifriati 1996, pp. 225-226.
This passage has raised many difficulties.

[107] *Hilkhot Kiddush ha-Hodesh* V; 2.

[108] *It can in fact last until two and even three days later. This passage is contradicted by another difficult passage in HKH VII: 7-8; see J. Ajdler Hilkhot Kiddush ha-Hodesh al-pi ha-Rambam, Sifriati 1996, p.226-227. Jaffe in Korot (1931) p. 197 in the bottom note proposed already to correct the text and write* « או ביום או ביומים ». Ibn Ezra in his commentary on Vaykra XXIII, 3 writes also that it happens sometimes that in Tishri the *keviyah* is on Thursday and the new moon is seen only on Friday evening.

the day after the new moon became visible, occurred only rarely[109]*, and then in the countries west to Palestine.*[110]

It seems however more likely to understand that, according to Maimonides' statement, the first vision of the lunar crescent before the *yom ha-keviyah*, the first day of the month, is a very rare event. However in areas situated west to Israel, the possibility of an early vision of the lunar crescent before the *yom ha-keviyah*, the first day of the month, is less exceptional.[111]

The conclusion of the R' Saadia-Ben Meir controversy at the advantage of the Babylonians had a tremendous consequence at the level of the unity of the Jewish people. Before 922 C.E. the Jewish calendar was communicated by the Palestinian Gaon on an annual or multi-annual basis.
It appears that from about 838 onwards, the Babylonians were able to make their own calculations and during the period of about eighty years preceding 922 C.E. they always agreed with the *keviyah* sent from Palestine. However the removed communities in Europe and Africa were probably not informed in time of the calendar data and were not able to keep the festivals at the same time as the two great centers of Palestine and Babylonia.
However, Spain and Kairouan, two centers having narrow bonds with Babylonia, were probably informed in time. Only after the end of the dispute, the rules of the calendar and the four gates table became universally known and only then the complete unity of the Jewish communities of the Diaspora in the celebration of their festivals was finally achieved.

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[109] It is probably more correct to understand that the visibility of the new moon before the yom ha-keviyah is exceptional, but in the countries west to Palestine it is less exceptional.

[110] Translation of Solomon Gandz in Sanctification of the New Moon, Yale Judaica Series: Volume XI, p.22-23.

[111] R' Raphael ha-Levi from Hanover writes in his book " כללי סוד העיבור" still in manuscript in the library of Jews College in London: **ובמדינות שהן למערב ארץ ישראל**, פירוש במדינות שהן למערב ארץ ישראל יהיה עת הראייה מאוחר לעת הראייה שבארץ ישראל, ואז אפשר אם היה המולד ג יח, או ה יח, או ז יח, לראות הירח באותו המדינות בליל ד' והקביעה יהיה ביום ה', כי בהקביעה הן שווין, ולפי זה במדינות ההן אין הפלא כל כך גדול במו שהוא בארץ ישראל, אף על פי שהראייה בחוץ לארץ אינה מועלת כי אם בארץ ישראל והוא לא כתב זה אלא לדמיונו
What about the exceptional character of this early vision one day before the *yom ha-keviyah*, he adds : והוא פלא כי צריך להיות מולד אמצעי בתשרי ב" **גטרד**" וצריך שנה פשוטה וגם צריך להיות הקיבוץ מוקדם לאמצעי כדי שיהיה ריוח בין רגע קיבוץ עד עת הראייה יתר מן כ"ד שעות, וצריך שיהיה רוחב הירח צפונית חמש מעלות, ודבר זה שיהיו כל התנאים הממהרים הראייה ביחד הוא פלא גדול ואפשק שלא המצא תמצא בחמש מאות שנחם. וכבר יגעתי ומצאתי תאמין שחקרתי וחפשתי בחיפוש אחר חיפוש משנת ד' אלפים עד שנת ה' אלפים ליצירה ולא מצאתי רק במשל אחד בשנת **ד"א תרפ"ג** שהיה המולד אמצעי בתשרי **ג ט תמא** ונדחה לחמישי ונראה הירח בליל ד' והיה יום הראייה ביום ד' יום אחד מוקדם ליום הקביעה.
This passage, which is a quotation from an unpublished manuscript from R' Raphael ha-Levi from Hanover is an exceptional piece of evidence of his exceptional calculation abilities (and patience) and of the fiability of Maimonides' visibility criterium. Imagine that Raphael Hanover, who had not the least idea of the R' Saadia-Ben Meir dispute, discovered this year 4683, among thousand years, in which exceptionally the new crescent was visible one day before the Babylonian *keviyah*.
Ibn Ezra notes in his commentary on Vaykra XXIII; 3 that this early visibility of the moon can happen in Nissan or in the tree former months. However, he considers incorrectly this early vision of the moon one day before the *keviyah*, to be a commonplace and he writes that it happens rather frequently.

A second consequence, not less important, of the supremacy of the Babylonian community, was that, parallel to the fact that the Jewish calendar became universally known, it became also definitively stiff and rigid. As long as the Babylonian community accepted the Palestinian *keviyah*,[112] the council of intercalation, acting with much secrecy, had the possibility to adapt and improve the calendar. From this time onwards, the Jewish communities could participate to development of the study of the Jewish calendar. It seems that the custom to count the Jewish calendar according to the era of Tohu (*Beharad*), beginning the era with a year L+1, following a leap year, at the beginning of a cycle of 19 years of the proleptic Jewish calendar, instead of the era of the creation (*Weyad*) beginning the era with a year L – 1, preceding a leap year, was introduced by the Jews of Spain and Italy.[113] Similarly the *tekufah* of Adda, a system of mean equinox and solstices fixed rigidly to the cycle of 19 Jewish years and having a good coincidence with the mean equinox and solstices during the 10th and 11th century, was probably introduced in Spain and it was thoroughly studied by the Spanish *meabrim*. Finally the four gates table, a Babylonian discovery, was generalized by the French Tossafist Ritsva,[114] of the 12th century and gave birth to the table of the 61 lines, a table giving the *keviyah* of all the 19 years of a cycle by the simple knowledge of the Molad Tishri of its first year.

It is interesting to note that this important event of 922-924 remained unknown until the beginning of the twentieth century, until the discovery and the study of the documents of the Cairo geniza. It is a fact the R' Sherira Gaon and R' Hai Gaon did not mention the event at all. At the first glance we could think that the leaders of the Babylonian community did not want to let a remembrance of this schism for the posterity; it could have thrown a shadow on the authority of the Jewish calendar and on the doctrine of its sinaïtic origin taught by R' Sa'adia Gaon. This is however not true. We know that R' Sa'adia Gaon wrote two books: ספר הזיכרון and ספר המועדים. The first book was intended to be read publicly in order to recall the event. The second book was probably a treatise of the festivals and the Jewish calendar and it mentioned probably also the events of the famous dispute of 922- 924[115]in order to prevent the possibility of a new schism in the future. It was the *mahaloquet* that prompted him to write the first and probably the second book. R' Sa'adia's works on the calendar are lost, although they appear to have been well known in the middle ages (Rashi, R' Tam and R' Jacob ben Shimshon[116] refer to it); it is a mystery why these two books did not survive.

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[112] As we still see in the letter of the *Resh Galuta*.

[113] The principle of beginning the counting of the Jewish years one year before the era of *Weyad* was already discussed by R' Sa'adia Gaon and R' Hai Gaon but it was rejected by them (see Bornstein, *Mahaloket* 1904, p. 127. It must be remembered that the counting from the year of *Weyad* corresponds to the counting of the Talmud (B. Avoda Zara 9b) according to the era of the creation.

[114] R' Isaac ben Abraham, elder brother of R' Samson ben Abraham of Sens. This attribution was demonstrated by Bornstein.

[115] Encyclopedia Judaica Vol. 14, entry Sa'adiah, p.544 bottom, affirms, without evidence or reference, that the *Sefer ha-Moadim* gave a complete account of the dispute.

[116] For details about R' Jacob ben Shimshon, the "secretary" of Rashi after R' Shemaya, see Abraham Grossman: *Hakhmei Sarfat ha-Rishonim*, Magnes, Jerusalem, 1996; pp.411-426.

By contrast it is evident that the Palestinian side was not interested to speak about this event and indeed they did never mention this dispute again. It is even noticeable that in Tishri 4686, the Molad was 5 – 18 – 214 and a new schism should have appeared about the keviyah of 4686. Indeed according to the Babylonians Rosh ha-Shannah 4686 was on Saturday and the year had the *keviyah* זחא. But for the Palestinians the Molad must occur 642 hal before at: 5 – 17 – 652 and Rosh ha-Shannah should have been on Thursday, with the keviyah השא. They were confronted exactly to the same problem as four years before. In fact there is no information left about a new dispute about the *keviyah* of that year. It seems that the Palestinian Gaon adopted the Babylonian Molad and proclaimed the *keviyah* as usual, as if nothing occurred. Later in the *Megilat Abiathar*,[117] the Palestinian Gaon did not mention anything about the incident but he still claimed the Palestinian authority on the calendar.[118]

The present day calendar is the calendar of the Babylonians since about 838 C.E. that emerged after the dispute of 922-924. This calendar did not change any more.
We offer hereunder two tables showing the weak point of the present calendar, i.e. that the Jewish year is shifting with regard to the Gregorian calendar, in the direction of the summer. This brings us to contemplate again a slight adaptation of the Jewish calendar in order to remain in agreement with the solar year. This subject is beyond the scope of the present paper. It was already thoroughly examined in two other papers[119].

[117] See *Megilat Abiathar*, Schechter JQR Vol XIV (1901-1902) pp. 449-474.
[118] It is also likely that the Palestinians went on calculating the Molad according to their more ancient methods referring to Nissan. Indeed Bornstein discovered that R' Jacob ben Shimshon used methods of calculation similar to that of the Palestinians in the time of Ben Meir. Similarly the four gates table in Mahzor Vitry (Vol.2 end) is constructed according to Nissan. It appears clearly that the French Rabbis were under the influence of the Babylonian but also the Palestinian Gaonim. We know that the German Jewish establishment was of Palestinian origin and had ties with Palestine. We are aware of the responsum by the Palestinian Gaon Elijah ben Solomon ha-Cohen, R' Abiathar's father, to R' Meshulam ben Moses of Mainz in 1070. It was signed also by R' *Abiathar ha-revi'i*, then the fourth in rank in the yeshivah . Grossman has discovered in the Library of the JTS the following passage: דוד בן אברהם קיבל אילו המסורת של חשבון מר' שבתי בר כרמי שקיבל מר' אליהו הכהן זצ"ל הרביעי שבחבורה בן אדונינו אביתר הכהן ראש ישיבת גאון יעקב תתמ"ח ליצירה.
See Grossman, Hakhmei Tsarfat ha-Rishonim, Magnes 1996, p. 423. This document, dated 1088, makes sense: In 1081, while his father was still alive, R' Abiathar was appointed gaon and his son Elijah (named as his still alive grand-father) was appointed the fourth in rank in the yeshiva.
[119] Ajdler (2011). The Future of the Jewish Calendar. BDD 25.
Ajdler (2013/1). The Gregorian Revolution of the Jewish Calendar. BDD 27.

IX. The Present-day Jewish Calendar

N	Year	Jewish Year	Nissan 16 Julian	Nissan 16 Gregorian	Tishri 21 Julian	Tishri 21 Gregorian
1	839	4599-4600	April 4	April 8	October 3	October 7
2	840	4600-4601	March 24	March 28	Sept. 22	Sept. 26
3	841	4601-4602	April 11	April 15	Oct. 10	Oct. 14
4	842	4602-4603	March 31	April 4	Sept. 29	Oct. 3
5	843	4603-4604	March 21	March 25	Sept. 19	Sept. 23
6	844	4604-4605	April 7	April 11	Oct. 6	Oct. 10
7	845	4605-4606	March 27	March 31	Sept. 25	Sept. 29
8	846	4606-4607	April 16	April 20	Oct. 15	Oct. 19
9	847	4607-4608	April 6	April 10	Oct. 5	Oct. 9
10	848	4608-4609	March 25	March 29	Sept. 23	Sept. 27
11	849	4609-4610	April 12	April 16	Oct. 10	Oct. 14
12	850	4610-4611	April 2	April 6	Oct. 1	Oct. 5
13	851	4611-4612	March 22	March 26	Sept. 20	Sept. 24
14	852	4612-4613	April 10	April 14	Oct. 9	Oct. 13
15	853	4613-4614	March 29	April 2	Sept. 27	Oct. 1
16	854	4614-4615	March 18	March 22	Sept. 16	Sept. 20
17	855	4615-4616	April 7	April 11	Oct. 6	Oct. 10
18	856	4616-4617	March 27	March 31	Sept. 25	Sept. 29
19	857	4617-4618	April 14	April 18	Oct.13	Oct. 17

Table 5: The dates of Nissan 16 and the following Tishri 21 during the 243rd cycle

N	Year	Jewish Year	Nissan 16 Gregorian	Tishri 21 Gregorian
1	2036	5796-5797	April 13	October 12
2	2037	5797-5798	April 1	September 30
3	2038	5798-5799	April 21	October 20
4	2039	5799-5800	April 10	October 9
5	2040	5800-5801	March 30	September 28
6	2041	5801-5802	April 17	October 16
7	2042	5802-5803	April 6	October 5
8	2043	5803-5804	April 26	October 25
9	2044	5804-5805	April 13	October 12
10	2045	5805-5806	April 3	October 2
11	2046	5806-5807	April 22	October 21
12	2047	5807-5808	April 12	October 11
13	2048	5808-5809	March 30	September 28
14	2049	5809-5810	April 18	October 17
15	2050	5810-5811	April 8	October 7
16	2051	5811-5812	March 29	September 27
17	2052	5812-5813	April 15	October 14
18	2053	5813-5814	April 4	October 3
19	2054	5814-5815	April 24	October 23

Table 6: The dates of Nissan 16 and Tishri 21 in the 304th cycle. We note a shift of a few days.

Appendix

I. Converting easily a Jewish date into a civil date by using the Julian day.

The classical methods for converting a Jewish date into a civil date are long and dull. The principle rests on the calculation of the *tekufah* of Samuel of September with regard to Tishri 1 and on the fact that the *tekufah* of Tishri falls always on September 24 in the Julian calendar. Louis A, Resnikoff[120] described an algorithm based on the same principle applicable on pocket calculators. Another method of computation makes use of the formula of Gauss[121] giving the date of Nisan 15 in the Julian calendar.[122]
We propose here a simple method in which we calculate the molad as a moment of the week and as a precise moment in history thanks to the Julian day. The method is conceptually very simple but it must however be applied with care and precision.

Let us consider a concrete example: Nisan 15, 5751.

1. The characteristics of the Jewish year A = N + 1 = 5751.

    a. The rank of the year 5751 in the cycle of 19 years.

$[5751]_{19} = 13$; the year 5751 is the 13^{th} year of the cycle 303 of 19 years; it is a regular year preceding a leap year.

    b. The Molad of the year 5751.

The number of Jewish months preceding the Molad of year 5751 is given by the fundamental formula of the Jewish calendar:[123]

$\mathbf{F_t = INT\ [(235N+1)/19]}$ = INT [(235 * 5750+1)/19] = 71118.

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[120] Scripta Mathematica Vol. IX, pp. 191-196 and 274-277.
[121] Gauss, Werke VI Bd. 1874, pp. 80-81. Berechnung des Judischen Osterfestes.
Zach's Monatliche Correspondenz zur beforderung der Erd und Himmelskunde, Mai 1802, p. 435.
Different authors tried to demonstrate this formula:

- Ableitung der gausschen formel zur bestimmung des Judischen Osterfestes, M. Hamburger, Crelles Journal fur die reine und angewandte Mathematik, Band 116 (1896).
- Computation of the dates of the Hebrew New Year and Passover, Ida Rhodes, Comp. & Maths with Appls. Vol 3, pp. 183-190, Pergamon Press 1977.
- A short and elegant demonstration has been proposed by the author of this paper in J. Ajdler (2013/1).

[122] Other formulas were proposed, for example:

- Eine algemeine Formel fur die gesamte judischen Kalenderberechnung, Slonimsky aus Bialystock, Crelles Journal fur die reine und angewandte Mathematik, Band 26 (1844).
- Beitrage zur Chronologie, Nesselman in Königsberg, Crelles Journal fur die reine und angewandte Mathematik, Band 28 (1844).

[123] See: Mathematical appendix in "The Gregorian Revolution of the Jewish Calendar", J. Ajdler (2013/1), pp. 17 - 76. See also J. Loewinger (1986).

The Molad expressed as a part of the week is:

Mol = $[31524 + 71118 * 765443]_{181440}$ [124]= $[31.524 + 71118 * 39673]_{181440}$ = 103938 hal.= 4 – 0 – 258 = (5) – 0 – 258.
This Molad is thus after 4 days and 258 *halakim* or at the beginning of the fifth day at 0h 258 *halakim* i.e. Wednesday at 18h 258 hal. Tishri 1 falls on Thursday.

The four gates table gives then the *keviyah* of the year: הכז. Rosh ha-Shanah is a Thursday and Pesah is on a Saturday.

This result can also be reached directly by calculating the Molad of the years 5751 and 5752 and the days of Tishri 1 of these two years by the application of the four rules of postponement.
F_t = INT [(235 * 5751 + 1)/19] = 71130.
Mol = $[31524 + 71130 * 765443]_{181440}$ = 35694 hal = 1 – 9 – 54 = (2) – 9 – 54. Tishri 1 falls on Monday. The shift of Tishri 1 between 5751 and 5752 is thus four days and the number of days lying between these two days, exclusive of the two days of Tishri 1, is 3.[125] Therefore the year 5751 is a regular year and its length is 354 days. Thus Rosh ha-Shannah falls on Thursday because of the rules of the *dehiyot* (postponements) and the length of the year is 354 days.

c. The year 5751 is thus an ordinary[126] year; it is a regular[127] year of 354 days beginning on a Thursday.

Nisan 15 is the 192nd day of this year and it falls on a Saturday.[128]

2. The Jewish calendar and the Julian day.

The Julian period's epoch is Monday, January 1, - 4712 at noon. At this moment the number of elapsed day of the Julian period was 0 days. The Julian day n° 1 began on Monday at noon and ended on Tuesday at noon. Similarly, until the twentieth century, the astronomical days began at noon of the civil days of the same name.

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[124] $[A]_B$ is the remainder of the division of A by B.
[125] This is the algorithm described by Maimonides in *Hilkhot Kiddush ha-Hodesh VIII*, 7 and 8. He counts the number of days between the two days of Tishri 1, exclusive of the two days of Tishri 1. The length of the year is thus 353, 354 or 355 days according whether this difference is 2, 3 or 4 for a common year, 383, 384 and 385 according whether this difference is 4, 5 or 6 for a leap year. By contrast R. Abraham bar Hiya counts the shift of Rosh ha-Shannah between the two years, i.e. he counts the day of Rosh ha-Shannah of one year + the number of days between. Therefore the length of the year is 353, 354 or 355 days according whether the difference is 3, 4 or 5 for a common year and 383, 384 or 385 according whether the shift is 5, 6 or 7 for a leap year.
[126] An ordinary year has 12 month and a leap year has 13 months.
[127] A regular year has 354 or 384 days, a defective year has 353 or 384 days and a full year has 355 or 385 days according to whether the year is a regular or a leap year.
[128] See the fourteen possible calendars of the Jewish calendar: *Yesodei ha-Ibbur*, Hayim Zelig Slonimski, Warsaw 1852, end of the book. Shearim le-luah ha-Ivri, Rahamim Sar Shalom, Natania 5744, p. 35. *Ha-Luah ve-Shimusho ba-Kronologia*, A. A. Akabia, Magnes, Jerusalem 1953, pp. 50-53.

The Molad of *Beharad*, beginning of the Jewish era AMI, was on Sunday October 6, - 3760 at 23h 204hal; Jerusalem mean time. This moment belonged already to the second Jewish day of the week, which began at 18h, hence (2) – 5 – 204. It means the second day at 5 h and 204 *halakim*. It could be written as 1 – 2 – 204, meaning 1 day 5 h and 204 *hal* after the beginning of the week or 31524 *hal* after the beginning of the week.[129]

Expressed in Julian days, the molad of Beharad was 347997. 466203703703.
On Sunday, October 6, - 3760 at noon, 347 997 days of the JP[130] had elapsed and on Monday, October 7, - 3760 = Tishri 1, 1 AMI, at noon, 347 998 days of the JP had elapsed. Tishri 1, 1 AMI began thus at 347997.25 JD and ended at 347998.25 JD. Tishri 1 corresponded in its majority to the day 347998 of the JP.[131]

There is a second style of the Jewish calendar AMII, beginning on Tishri 1, 2 AMI.
The molad of this year was *Weyad*: 6- 14.
The first day of this year was Tishri 1, 1 AMII = Tishri 1, 2 AMI; it corresponds to Saturday, September 27, - 3759 or 348353 JD, beginning at 348352.25 JD and ending at 348353.25 JD.
We note also that Elul 25, 1 AMI = Monday, September 22, - 3759 = 348348 JD.
Elul 24, 1 AMI = Sunday, September 21, - 3759 = 348347 JD.

3. The year 5751 and the civil year.

Expressed in Julian days, the molad of 5751 is given by the formula:[132]
Mol= 347997.466203703 + 29.530594135804 * 71118 = 2448154.25995370370 JD
This molad is thus on a civil Wednesday 18h 258 hal or on a Jewish Thursday at 0h 258 hal.
Rosh ha-Shanah is thus Thursday, from 2448154.25 JD until 2448155.25 JD.
Tishri 1, 5751 corresponds thus to 2448155 JD and Nisan 15 = 2448155 + 191 = 2448346 JD. This day corresponds to Saturday, March 30, 1991.[133]

We add 0.5 to the JD: 2448346.
α = INT [(2,448,346 – 1,867,216.25) / 36,524.25] = 15

A = 2,448,346 + 1 + 15 – Int (15/4) = 2,448,359
We calculate then:
B= A + 1524 = 2,449,883
C = INT [(2,449,883 – 122.1) / 365.25] = 6,707
D = INT (365.25 * 6,707) = 2,449,731
E = INT [(2,449,883 – 2,449,731) / 30.6001] = 4

[129] See note 91.
[130] Julian Day.
[131] Julian Period.
[132] This formula gives the same result as the formula of Shram.
[133] For the conversion of a Julian day into a civil date see Astronomical Algorthms, Jean Meeus, Willman-Bell, 1991, p. 59. Idem for the determination of the weekday.

The day of the month is:
B – D – INT (30.6001 * E) + F = 2,449,883 – 2,449,731 – 122 + 0.5 = 30.5[134]
The month number m is E – 1 = 3 = March.
The year is C – 4716 = 6707 – 4716 = 1991
The weekday is given by the JD at 0h + 1.5 = 2,448,345.5 + 1.5 = 2,448,347
$[2,448,347]_7 = 6$; it is a **Saturday, March 30, 1991.**

4. Nisan 15, 5751 expressed as a day of the Jewish period.

If we consider that Tishri 1, 1AMI was the first day of the Jewish period, then
1 Jewish Period = 347998 JD = Monday.
Nisan 15, 5751AMI = 2448346 – 347997 = 2100349 Jewish Period or J.P.

II. The derivate Postponements.

1. The postponement 3 – 9 – 204 or **ג ט רד בפשוטה** .

If the Molad of Tishri of an ordinary year is 3 – 9 – 204 or greater, then the Molad of the following Tishri is 7 – 18 or greater. If we apply the general rules we will begin Tishri of the present year on Tuesday and Tishri of next year on Monday. The shift of Rosh ha-Shannah from one year to the other will be 6 days and therefore the ordinary year must be a multiple of 7 plus 6, thus necessarily 356 days. This is impossible; the Jewish ordinary year must have 353, 354 or 355 days. In order to solve this difficulty we must impose to postpone the first day of Rosh ha-Shannah to Thursday as soon as the molad is 3 – 9 – 204 in an ordinary year.

2. The Postponement 2 – 15 – 589 or **ב טו תקפט אחר עיבור.**
If the Molad of Tishri following a leap year 2 – 15 – 589 or more the Molad Tishri of the preceding year is 3 – 18 or more. If we apply the general rules the 1 Tishri of the leap year is Thursday and the 1 Tishri of the following year is Monday. The shift from one year to the other is 4 days. The number of days of the leap year must be a multiple of 7 plus 4, it is necessarily 382 days. This is impossible; the number of days of a leap year is 383, 384 or 385 days. In order to solve this difficulty we must impose to postpone the first day of Rosh ha-Shannah of a year following a leap year from Monday to Tuesday as soon as the Molad reaches 2 – 15 – 589 and this will bring the number of days of the leap year to 383 days.

3. The Calendar of Hillel, from about 648 C.E. till 776 C.E.

The reasoning is the same. The limit 3 – 9 – 204 in an ordinary year becomes 3 – 9 – 3 or ג ט ג בפשוטה.
Similarly the limit 2 – 15 – 589 after a leap year becomes 2 – 15 – 8 after a leap year or ב טו ח אחר עיבור.

[134] March 30 at noon is 30.5.

4. The Calendar of Hillel from 359 until about 648.

1 Tishri could be on Sunday. By a similar reasoning it is easy to demonstrate that the two derivate postponements are:
1 – 9 – 3 in an ordinary year or א ט ג בפשוטה.

2 – 15 – 8 after a leap year or ב טו ח אחר עיבור.

III. The Four gates Table.

The four gates table is a Babylonian invention from the 9th century. It represents a higher degree of sophistication and knowledge of the rules of the calendar. It allows knowing the *keviyah* of a year by the knowledge of its Molad and its rank in the cycle of 19 years. Maimonides did not describe this method nor in *Hilkhot Kiddush ha-Hodesh*. He must find the day of 1 Tishri of two consecutive years in order to find the characteristics of the first year. R' Abraham ibn Ezra worked on the same way in his Sefer ha-Ibbur.
We have a mention of the four gates table in a letter of R' Sa'adia Gaon related to the dispute.[135] R' Sa'adia Gaon gave also the detailed rules of the four gates table. We have also a description of the four gates table in a poem of R' Yose ben al-Naharwani.[136] The Four gates was thus a well established knowledge in Babylonia.
The four gates table was thoroughly examined by R' Abraham bar Hiya in *Sefer ha-Ibbur*[137] and in R' Isaac Israeli's *Yessod Olam*. In the supplement at the end of the second volume of Mahzor Vitry[138] we find the table of the four gates according to the molad of the preceding Nissan.

1. Construction of the Four Gates Table for the Calendar of Hillel 359-648.

We depart from the daily limits of each type of year.

We have already 5 limits for each category of year. In order to find the four additional limits we proceed as follows:

[135] See Bornstein : *Divrei Yemei ha-Ibbur ha-Aharonim*, ha-Tekufah 16, p. 247. He accuses Ben Meir of copying the Babylonian four gates table and adapting the different limits by the addition of 642 *hal*.
[136] Epstein, A. (1901) : La querelle au sujet du Calendrier entre Ben Meir et les académies Babyloniennes, REJ 42, pp. 173-210.
[137] Pp. 63-69.
[138] This supplement begins after page 798. It is likely that this chapter was greatly influenced, if not copied from the Sefer ha Ibbur by R' Jacob ben Samson which was part of his great composition: the *Sefer Elkoshi*. In Mahzor Vitry we find also the commentary on Avot by R' Jacob ben Samson. Abraham Berliner, on pp. 15-16 of the calendar supplement to Mahzor Vitry seems to ignore that the book of R' Jacob ben Samson has the general name of Sefer Elkoshi and he assumes that the author of the manuscript was called Nahum according to Nahum I; 1. Anyhow, it seems that R' Jacob ben Samson exerted an important influence on different parts of the Mahzor Vitry.

Type	Sunday	Monday	Tuesday	Thursday	Saturday
L	7 – 18	1 – 18	2 – 18	3 – 18	5 – 18
L+1	7 – 18	1 – 9 – 3	2 – 15 – 8	3 – 18	5 – 18
L-1	7 – 18	1 – 9 – 3	2 – 18	3 – 18	5 – 18
L+- 1	7 – 18	1 – 9 – 3	2 – 15 – 8	3 – 18	5 – 18

Table 7: The Calendar of Hillel. Table of the different limits in Tishri according to the weekdays.

- We subtract from the limits of the first line the remainder of 12 months 4 – 8 – 12. We get the complementary limits of the third line L – 1:
  3 – 9 – 3, 4 – 9 – 3, 5 – 9 – 3and 6 – 9 – 3.
- We subtract from the limits of the second line the remainder of 13 months 5 – 21 – 8 and we get the additional limits of the line L:
  1 – 20 – 7, 2 – 11 – 10, 4 – 20 – 7 and 6 – 20 – 7
- We subtract from the limits of the third line the remainder of 12 months 4 – 8 – 12 and we get the additional limits of the line L+1:
  3 – 9 – 3, 4 – 0 – 6, 5 – 9 – 3 and 6 – 9 – 3.
- We subtract from the limits of the first line the remainder of 12 months 4 – 8 – 12. We get the complementary limits of the third line L+ – 1:
  3 – 9 – 3, 4 – 9 – 3, 5 – 9 – 3 and 6 – 9 – 3.

2. Construction of the Four Gates Table for the Modern Calendar.[139]

We depart from the daily limits of each type of year.

Type	Monday	Tuesday	Thursday	Saturday
L	7 – 18	2 – 18	3 – 18	5 – 18
L+1	7 – 18	2 – 15 – 589	3 – 9 – 204	5 – 18
L-1	7 – 18	2 – 18	3 – 9 – 204	5 – 18
L+-1	7 - 18	2 – 15 - 589	3 – 9 – 204	5 – 18

Table 8: The Modern Calendar. Table of the different limits in Tishri according to the weekdays.

We have already 4 limits for each category of year. In order to find the three additional limits w proceed as follows:

- We subtract from the first line the remainder of 12 months 4 – 8 – 876. We get the complementary limits of the third line L – 1:
  1 – 9 – 204, 5 – 9 – 204 and 6 – 9 – 204.
- We subtract from the second line the remainder of 13 months 5 – 21 – 589 and we get the additional limits of the line L:
  1 – 20 – 491, 4 – 11 – 695 and 6 – 20 – 491
- We subtract from the third line the remainder of 12 months 4 – 8 – 876 and we get the additional limits of the line L+1:
  1 – 9 – 204, 5 – 9 – 204 and 6 – 0 – 408.

[139] The Construction of the Four Gates Table for the Calendar of the Period 648- 776 can be easily deduced from the four gates table by replacing 3 – 9 – 204 by 3 – 9 – 3 and 2 – 15 – 589 by 2 – 15 – 8.

- We subtract from the limits of the first line the remainder of the length of 12 months 4 – 8 – 876. We get the complementary limits of the third line L+ – 1: 1 – 9 – 204, 5 – 9 – 204 and 6 – 9 – 204.

3. The Four Gates table for the calendar of Hillel: 358-648.

The Four Gates Table						לוח ארבע שערים	
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 2	1r אכג	7 – 18 – 0 1 – 9 – 2	1r אכג	7 – 18 – 0 1 – 9 – 2	1r אכג	7 – 18 – 0 1 – 17 – 14	1R אכה
1 – 9 – 3 2 – 17 - 14	2f בשה	1 – 9 – 3 2 – 17 - 14	2f בשה	1 – 9 – 3 2 – 17 - 14	2f בשה	1 – 18 – 0 1 – 20 – 6	2D בחה
2 – 18 – 0 3 – 9 – 2	3r גכה	2 – 15 – 8 3 – 9 – 2	3r גכה	2 – 15 – 8 3 – 9 – 2	3r גכה	1 – 20 – 7 2 – 11 – 9	2R בכו
3 – 9 – 3 3 – 17 – 14	3f גשו	3 – 9 – 3 3 – 17 – 14	3f גשו	3 – 9 – 3 3 – 17 – 14	3f גשו	2 – 11 – 10 2 – 17 – 14	2F בשז
3 – 18 – 0 4 – 9 – 2	5d החו	3 – 18 – 0 4 – 0 – 5	5d החו	3 – 18 – 0 4 – 9 – 2	5d הכו	3 – 18 – 0 4 – 9 – 2	5d הכו
4 – 9 – 3 5 – 9 – 2	5r הכז	4 – 9 – 3 5 – 9 – 2	5d הכו	4 – 9 – 3 5 – 9 – 2	5d החו	2 – 18 – 0 3 – 17 – 14	3R גכז
5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 – 17 – 14	5F השג
5 – 18 – 0 6 – 9 – 2	7d זחא	5 – 18 – 0 6 – 9 – 2	7d זחא	5 – 18 – 0 6 – 9 – 2	7d זחא	5 – 18 – 0 6 – 20 – 6	7D זחג
6 – 9 – 3 7 – 17 – 14	7f זשג	6 – 9 – 3 7 – 17 – 14	7f זשג	6 – 9 – 3 7 – 17 – 14	7f זשג	6 – 20 – 7 7 – 17 – 14	7F זשה

Table 9: The Four Gates table for the calendar of Hillel: 358-648.

4. The Four gates table for the presumed calendar of Hillel: 648-776.

The Four Gates Table						לוח ארבע שערים	
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 9 – 2	2d בחג	7 – 18 – 0 1 – 20 – 6	2D בחה
1 – 9 – 3 2 – 17 - 14	2f בשה	1 – 9 – 3 2 –15 – 7	2f בשה	1 – 9 – 3 2 –15 – 7	2f בשה	1 – 20 – 7 2 –17 – 14	2F בשז
2 – 18 – 0 3 – 9 – 2	3r גכה	2 – 15 – 8 3 – 9 – 2	3r גכה	2 – 15 – 8 3 – 9 – 2	3r גכה	2 – 18 – 0 3 – 17 –14	3R גכז
3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 9 – 3 5 – 9 – 2	5r הכז	3 – 18 – 0 4 – 11 – 9	5D החא
5 – 9 – 3 5 – 17 – 14	5f השא	5 – 9 – 3 5 –17– 14	5f השא	5 – 9 – 3 5 –17 –14	5f השא	4 – 11 –10 5 – 17 –14	5F השג
5 – 18 – 0 6 – 9 – 2	7d זחא	5 – 18 – 0 6 – 0 – 5	7d זחא	5 – 18 – 0 6 – 9 – 2	7d זכא	5 – 18 – 0 6 – 20 – 6	7D זחג
6 – 9 – 3 7 – 17 – 14	7f זשג	6 – 0 – 6 7 –17– 14	7f זשג	6 – 9 – 3 7 –17– 14	7f זשג	6 – 20 – 7 7 –17– 14	7F זשה

Table 10: The Four gates table for the presumed calendar of Hillel: 648-776.

5. The Four Gates table for the modern calendar.

The Four Gates Table						לוח ארבע שערים	
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 20 – 490	2D בחה
1 – 9 – 204 2 – 17 - 1079	2f בשה	1 – 9 – 204 2 –15 – 588	2f בשה	1 – 9 – 204 2 –15 – 588	2f בשה	1 – 20 – 491 2 –17- 1079	2F בשז
2 – 18 – 0 3 – 9 – 203	3r גכה	2 – 15 – 589 3 – 9 – 203	3r גכה	2 – 15 – 589 3 – 9 – 203	3r גכה	2 – 18 – 0 3 – 17–1079	3R גכז
3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 18 – 0 4 – 11 – 694	5D החא
5 – 9 – 204 5 – 17 – 1079	5f השא	5 – 9 – 204 5 –17– 1079	5f השא	5 – 9 – 204 5 –17 –1079	5f השא	4 – 11 – 695 5 – 17–1079	5F השג
5 – 18 – 0 6 – 9 – 203	7d זחא	5 – 18 – 0 6 – 0 – 407	7d זחא	5 – 18 – 0 6 – 9 – 203	7d זכא	5 – 18 – 0 6 – 20 – 490	7D זחג
6 – 9 – 204 7 – 17 – 1079	7f זשג	6 – 0 – 408 7 –17– 1079	7f זשג	6 – 9 – 204 7 –17– 1079	7f זשג	6 – 20 – 491 7 –17– 1079	7F זשה

Table 11: The Four gates table for the modern calendar.

6. The Four Gates table for the modern calendar according to the Molad of the preceding Nissan.

The Four Gates Table						לוח ארבע שערים	
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
5 – 13 – 642 6 – 4 – 845	2d בחג	5 – 13 – 642 6 – 4 – 845	2d בחג	5 – 13 – 642 6 – 4 – 845	2d בחג	5 – 13 – 642 6 – 16 – 52	2D בחה
6 – 4 – 846 7 – 13 – 641	2f בשה	6 – 4 – 846 7 – 11 – 150	2f בשה	6 – 4 – 846 7 – 11 – 150	2f בשה	6 – 16 –53 7 – 13 – 641	2F בשז
7 – 13 – 642 1 – 4 – 845	3r גכה	7 – 11 – 151 1 – 4 – 845	3r גכה	7 – 11 – 151 1 – 4 – 845	3r גכה	7 – 13 – 642 1 – 13 – 641	3R גכז
1 – 4 – 846 3 – 4 – 845	5r הכז	1 – 4 – 846 3 – 4 – 845	5r הכז	1 – 4 – 846 3 – 4 – 845	5r הכז	1 – 13 – 642 2 – 7 – 256	5D החא
3 – 4 – 846 3 – 13 – 641	5f השא	3 – 4 – 846 3 – 13 – 641	5f השא	3 – 4 – 846 3 – 13 – 641	5f השא	2 – 7 – 257 3 – 13 – 641	5F השג
3 – 13 – 642 4 – 4 – 845	7d זחא	3 – 13 – 642 3 –19 –1049	7d זחא	3 – 13 – 642 4 – 4 – 845	7d זכא	3 – 13 – 642 4 – 16 – 52	7D זחג
4 – 4 – 846 5 – 13 – 641	7f זשג	3 –19 –1050 5 – 13 – 641	7f זשג	4 – 4 – 846 5 – 13 – 641	7f זשג	4 – 16 – 53 5 – 13 – 641	7F זשה

Table 12: The Four Gates table for the modern calendar according to the Molad of the preceding Nissan.

IV. The Problematic of the Molad Zaken in Shevat in a leap year.

1. Molad Zaken in Shevat.

The remainder of the four first months, Tishri, Marheshvan, Kislev and Tevet is 6 – 2 – 1012. If the Molad of Tishri is 1 – 15 – 67 the molad of the following Shevat is 7 – 17 – 1079. As soon as the Molad Tishri reaches 1 – 15 – 68 the Molad Shevat reaches 7 – 18 and the Molad of Shevat is *Zaken*. If we examine the four gates table we acknowledge that this can happen only in a leap year. Similarly, in a leap year, after 3 – 15 – 67 the Molad Shevat becomes *Zaken*. This is also the case, in a leap year, when the Molad Tishri becomes greater than 6 – 15 – 67. The Molad of Shevat is *Zaken* because it occurs after noon of the first day of Shevat. On the other weekdays, the limits 2 – 15 – 68, 4 – 15 – 68, 5 – 15 – 68 and 7 – 15 – 68 have not the same consequences because the leap year becomes then full and therefore there is no Molad Zaken because it occurs on the last day of Tevet. The four gates tables shows us also that when the Molad reaches the

limit 1 – 20 – 491, 3 – 18 and 6 – 20 – 491 the leap year which was until now defective or regular, becomes full and therefore there is no more *Molad Zaken*.
We have thus a *Molad Zaken* in Shevat of a leap year if the Molad is between 1 – 15 – 68 and 1 – 20 – 490, the two boundaries included. Similarly we have a *Molad Zaken* in Shevat in a leap year if the Molad is between the 3 – 15 – 68 and 3 – 17 1079 and if it is between 6 – 15 – 68 and 6 – 20 – 490, the two boundaries included. Indeed:

If 1 – 15 – 67 < M < 1 – 20 – 491, the leap year is defective and has the keviyah **בחה**. The length of four Jewish lunations is 118d 2h and 1012hal. The length of the four first months is 117 = 16 * 7 + 5 days. 1 Tishri is a Monday and 1 Shevat is Saturday. The Molad Shevat is (1 – 15 – 68) + (6 – 2 – 1012) = 7 – 18; it is thus *Zaken*.
As soon as M = 1 – 20 – 491 the year becomes full and 1 Shevat shifts backwards by one day and there is no more *Molad Zaken*.

If 3 – 15 – 68 < M < 3 – 18, the leap year is regular and has the keviyah is **גכז**. The length of the four Jewish lunations is 118d 2h and 1012hal. The length of the four first months is 118 = 16 * 7 + 6. 1 Tishri is Tuesday and 1 Shevat is Monday. The Molad of Shevat is (3 – 15 – 68) + (6 – 2 – 1012) = 2 – 18; the Molad is *Zaken*. As soon as M = (3 – 18) the first day of Tishri shifts from Tuesday to Thursday and there is no more *Molad Zaken* in Shevat.

If 6 – 15 – 68 < M < 6 – 20 – 491, the leap year is defective and has the keviyah is **.זחג**
The length of the four Jewish lunations is 118d 2h and 1012hal. The length of the four first months is 117 = 16 * 7 + 5. 1 Tishri is Saturday and 1 Shevat is Thursday. The Molad of Shevat is (6 – 15 – 68) + (6 – 2 – 1012) = 5 – 18; the Molad is *Zaken*. As soon as M = (6 – 20 – 491) the leap year which was defective becomes full and there is no more *Molad Zaken*.

We note further that if M = 1 – 15 – 68 then the Molad of next Tishri is:
(1 – 15 – 68) + (5 – 21 – 589) = 7 – 12 – 657. Similarly if M = (3 – 15 – 68) then the Molad of next Tishri is 2 – 12 – 657 and, on the same way, if M = (6 – 15 – 68) then the Molad of next Tishri is 4 – 12 – 657.

2. Prevention of the occurrence of a Molad Zaken

If we want to prevent the occurrence of a *Molad Zaken* in a leap year we must in the four gates table replace the limits 1 – 20 – 491 by 1 – 15 – 68, 3 – 18 by 3 – 15 – 68 and 6 – 20 – 491 by 6 – 15 – 68.
We must make other changes; the Molad of the next Tishri corresponding to the limits 1 – 15 – 68, 3 – 15 – 68 and 6 – 15 – 68 are 7 – 12 – 657, 2 – 12 – 657 and 5 – 12 – 657. Thus in the lines L + 1(מוצאי עיבור)and L + – (ביני עיבור) we must replace 7 – 18 by 7 – 12 – 657, 2 – 15 – 589 by 2 – 12 – 657 and 5 – 18 by 5 – 12 – 657.

The construction of the four gates table creates an additional keviyah in the leap years: **.גשא**

3. Historical evidence of the existence of the keviyah **גשא**.

1. R' Abraham bar Hiya.

In his *Sefer ha-Ibbur*,[140] he mentions twice the *keviyah* גשא. He first mentions the *keviyah* as a possible *keviyah*[141] but he writes later that this possible theoretical *keviyah* did not find a practical application because this was not necessary.[142]

2. *Massekhet Sofrim*.[143]

In *Massekhet Soferim* XX, 12 it deals with the reading of the Torah on both days of *Rosh Hodesh* Tevet when *Rosh Hodesh* falls on Sunday and Monday.[144]
There are two days of *Rosh Hodesh* if the year is regular or full. In the first assumption the first day of *Rosh Hodesh* is Tishri 89. But if the year is full then the first day of *Rosh Hodesh* is Tishri 90. The first assumption implies that 1 Tishri was four weekdays before the first day of *Rosh Hodesh*. Thus if the first day of *Rosh Hodesh* is Sunday, 1 Tishri is on Wednesday: this is impossible. The only possibility is then that we are in a full year גש. If it is an ordinary year it has 355 = M7 + 5 days and Rosh ha-Shannah of next year is on Friday, this is impossible. It must then be a leap year of 385 = M7 days and next year will also begin on Tuesday. Pessah of this year will be two days before, on Sunday and the *keviyah* is then **גשא**.This *keviyah* does not exist today but we can assume that it once existed or, at least, that it was once taken into consideration.

3. *Sefer ha-Pardes*.[145]

Sefer ha-Pardes is one of the books issued by the "school of Rashi"; Berliner assumed that it was composed by R' Shemaya.
In *Sefer ha Pardes*, about the Shabbat and festivals readings,[146] it writes that if Sukkot is on Tuesday and Marheshvan and Kislev are full there will be 29 Sabbaths and we won't be obliged to read two sections together. The year considered is a full leap year beginning on Tuesday. It has 385 days and the next year begins also on Tuesday.
Pessah will be two days before the day of Rosh ha-Shanah of next year, on Sunday. It is a year **גשא**.

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[140] Ed. Filipowski, London 1851.
[141] P. 63.
[142] P. 65.
[143] The reference to *Massekhet Sofrim* was mentioned for the first time by Hayim Bornstein in *Divrei Yemei ha-Ibbur ha Aharonim*, Ha-Tekufah 16, Warsaw, 1923, p. 283.
[144] In the text of *Massekhet Sofrim* published in the Vina Romm edition and in the *Massekhet Soferim* edited in Mahzor Vitry, ed. Simon Horowitz, Nuremberg 1923, Vol. 2, p.716 there is an additional interpolation: שאין חשבון ראש חדש מיום שני אלא בזמן שהשנים כסדרן. The signification of this interpolation is that *Rosh Hodesh* Tevet has two days only if the year is regular (Marheshvan defective and Kislev full) or full (Marheshvan and Kislev full). This interpolation is not necessary at all and Gra suppressed it.
[145] The reference to *Sefer ha-Pardes* was mentioned for the first time by Hayim Bornstein in *Divrei Yemei ha-Ibbur ha Aharonim*, Ha-Tekufah 16, Warsaw, 1923, p. 273.
[146] *Sefer ha-Pardes*, ed. R' H.L. Ehrenreich, Budapest 1924 and Bnei Berak 1990, p.340 five lines from bottom.

Apparently these two quotations are remnants of ancient calendar rules which were not adapted or corrected and which could reach us.[147]

4. There was a *Molad Zaken* in Shevat 4596*.

The year 4596 is a leap year. Its modern Molad is 6 – 22 – 660 and the current molad was about 3h 20 m before: 6 – 19 – 297. This Molad is greater than 6 – 15 – 68 and there was a *Molad Zaken* in Shevat 4596. According to the exegesis of the letter of the *Resh Galuta* made by Jaffe, the *Resh Galuta* knew the keviyah and therefore he must find a plausible reason to the writing of this letter. Jaffe supposed that the object of the discussion was whether we accept *Molad Zaken* in Shevat or not. Once we know today that the reading of Jaffe in the letter of the *Resh Galuta* was incorrect,[148] there is no more any justification to this approach. The *Resh Galuta* did not know the Molad of the Palestinians and the problem raised was probably the contradiction between the Molad of the Palestinians and the mean conjunction deduced from the Almagest.

[147] We note that the Gra corrected the reading in Soferim XX, 12 but he did not react and note the impossibility of this configuration. It is thus normal, because of the difficulty of the subject, that the copyists copied without amending the text and let survive these interesting passages.
[148] Jaffe read: ארבע ידות instead of ארבע שעות.

V. The Four gates table of the modern calendar in the assumption that we want to prevent *Molad Zaken* in Shevat.

The Four Gates Table						לוח ארבע שערים	
Ordinary Years						Leap Years	
L – 1		L + 1		L+ – 1		L	
ערבי עיבור		מוצאי עיבור		ביני עיבור		שנות עיבור	
2 – 5 – 10 – 13 – 16		1 – 4 – 9 – 12 – 15		7 – 18		3 – 6 – 8 – 11 – 14 17 – 19	
Molad	Kev	Molad	Kev	Molad	Kev	Molad	Kev
7 – 18 – 0 1 – 9 – 203	2d בחג	7 – 12 –657 1 – 9 – 203	2d בחג	7 – 12 –657 1 – 9 – 203	2d בחג	7 – 18 – 0 1 – 15 – 67	2D בחה
1 – 9 – 204 2 – 17 - 1079	2f בשה	1 – 9 – 204 2 –12 – 656	2f בשה	1 – 9 – 204 2 –12 – 656	2f בשה	1 – 15 – 68 2 –17- 1079	2F בשז
2 – 18 – 0	3r	2 – 12 – 657	3r	2 – 12 – 657	3r	2 – 18 – 0 3 – 9 – 203	3R גכז
3 – 9 – 203	גכה	3 – 9 – 203	גכה	3 – 9 – 203	גכה	3 – 15 – 68 3 – 17–1079	3F גשא
3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 5 – 9 – 203	5r הכז	3 – 9 – 204 3 – 9 – 203	5r הכז	3 – 18 – 0 4 – 11 – 694	5D החא
5 – 9 – 204 5 – 17 – 1079	5f השא	5 – 9 – 204 5 – 12 – 656	5f השא	5 – 9 – 204 5 –12 – 656	5f השא	4 – 11 – 695 5 – 17–1079	5F השג
5 – 18 – 0 6 – 9 – 203	7d זחא	5 –12 – 657 6 – 0 – 407	7d זחא	5 – 12 – 657 6 – 9 – 203	7d זכא	5 – 18 – 0 6 – 15 – 67	7D זחג
6 – 9 – 204 7 – 17 – 1079	7f זשג	6 – 0 – 408 7 – 12– 656	7f זשג	6 – 9 – 204 7 –12 – 656	7f זשג	6 – 15 – 68 7 –17– 1079	7F זשה

Table 13: The Four gates table of the modern calendar in the assumption that we want to prevent *Molad Zaken* in Shevat.

VI. The fictitious or proleptic Gregorian calendar extrapolated until the era of *Beharad*.

The Gregorian calendar offers a good approximation of the tropical year; it is slightly too long but the difference is only one day after 3333 years. Thus an extrapolation of the Gregorian calendar backwards until the Jewish era mundi would give us a much better apprehension of the situation of some dates with regard to the seasons. It can be demonstrated that during the period – 3800 – – 3700 the difference between the two calendars was – 30 days instead of +13 days during the period 1900 – 2100.
Thus the tekufah of Samuel, which was Tuesday 25 March – 3759 at 6 p.m. was in the fictitious Gregorian calendar 30 days earlier on Tuesday 23 February and the tekufah of Adda , which was on Tuesday 1 April – 3759 was in the fictitious Gregorian calendar on 2 March. If we consider that the Gregorian calendar accumulates a difference of nearly two days in 6000 years with regard to the tropical year we can consider that the true

equinox was then, at the beginning of the Jewish era of *Beharad*, rather on March 22 and the mean equinox, very roughly, on March 24. Therefore the *tekufah* of Samuel was 29 days before the mean equinox and the *tekufah* of Adda was 22 days before.

Passing from the Julian calendar to the Gregorian calendar			
– 4000 until – 3900	– 32	– 1000 until – 900	– 9
– 3900 until – 3800	– 31	– 900 until – 800	– 8
– 3800 until – 3700	– 30	– 800 until – 700	– 8
– 3700 until – 3600	– 29	– 700 until – 600	– 7
– 3600 until – 3500	– 29	– 600 until – 500	– 6
– 3500 until – 3400	– 28	– 500 until – 400	– 5
– 3400 until – 3300	– 27	– 400 until – 300	– 5
– 3300 until – 3200	– 26	– 300 until – 200	– 4
– 3200 until – 3100	– 26	– 200 until – 100	– 3
– 3100 until – 3000	– 25	– 100 until – 0	– 2
– 3000 until – 2900	– 24	0 until – 100	– 2
– 2900 until – 2800	– 23	100 until 200	– 1
– 2800 until – 2700	– 23	200 until 300	0
– 2700 until – 2600	– 22	300 until 400	1
– 2600 until – 2500	– 21	400 until 500	1
– 2500 until – 2400	– 20	500 until 600	2
– 2400 until – 2300	– 20	600 until 700	3
– 2300 until – 2200	– 19	700 until 800	4
– 2200 until – 2100	– 18	800 until 900	4
– 2100 until – 2000	– 17	900 until 1000	5
– 2000 until – 1900	– 17	1000 until 1100	6
– 1900 until – 1800	– 16	1100 until 1200	7
– 1800 until – 1700	– 15	1200 until 1300	7
– 1700 until – 1600	– 14	1300 until 1400	8
– 1600 until – 1500	– 14	1400 until 1500	9
– 1500 until – 1400	– 13	1500 until 1600	10
– 1400 until – 1300	– 12	1600 until 1700	10
– 1300 until – 1200	– 11	1700 until 1800	11
– 1200 until – 1100	– 11	1800 until 1900	12
– 1100 until – 1000	– 10	1900 until 2100	13

Table 14: Passing from the Julian to the Gregorian Calendar.

VII. The historical tables of Moladot and Keviyot proposed by Jaffe.

1. The period 359 – 648 C.E: **לוח א'**
2. The period 648 – 776 C.E: **לוח ב'**
3. The period 781 – 848 C.E: **לוח ה'**

לוח א. מולדות תשרי וקביעות ועבורים מראשית התיסדות חשבון העבור בניסן שנת ד"א קי"ט עד ד"א תי"ח. יתרונות המולדות לחדש א יב יא, לששה חדשים ב ד ו, לשנה פשוטה ד ח יב ולמעוברת ה כא ח. הרחיות: א) דו; ב) י"ח שעות; ג) א ט ג בפשוטה; ד) ב טו ח אחר עבור. הקביעות תשע בפשוטה: אכג, בשה, גכה, גשו, החו, הכז, השא, זחא, זשג, ותשע במעוברת: אכה, בחה, בכו, בשז, גכז, החא, השג, זחג, זשה,—הפסח לא יקדם את יום 19 במרס. שנות העבור מסומנות בכוכב כזה *.

	מחזור ריז פסח שנה ד"א קה 345	ריח פסח קכד 364	ריט פסח קמג 383	רכ פסח קסב 402	רכא פסח קפא 421	רכב פסח ר 440	רכג פסח ריט 459	רכד פסח רלח 478
א		ד יז א הכז 3_4	ז ט ו זשג 4_4	ג א יא גכה 3_4	ה יח א זחא 3_4	א י ו בשה 4_4	ד ב יא הכז 4_4	ו יט א זשג 4_4
ב		ב א יג בשה 24_3	ד יח ג הכז 23_3	ז י ח זשג 24_3	ג ב יג גכה 23_3	ה יט ג זחא 23_3	א יא ח בשה 24_3	ד ג יג החו 23_3
ג*		ו י י זחג 11_4	ב ג — בכו 11_4	ד יט ה החא 10_4	ז יא י זשה 12_4	ג ד — גכז 11_4	ה כ ה זחג 11_4	א יב י אכה 10_4
ד		ה ח ג הכז 31_3	א — ח אכג 31_3	ג טז יג גשו 31_3	ו ט ג זשג 1_4	ב א ח בשה 1_4	ד יז יג הכז 31_3	ז י ג זשג 31_3
ה		ב יז — בשה 20_3	ה ט ה השא 21_3	א א י אכג 20_3	ג יח—א)החו 20_3	ו י ה זשג 21_3	כ ב י בשה 21_3	ד יט — הכז 20_3
ו*		ז א יב זשה 9_4	ב יח ב גכז 8_4	ה י ז השג 9_4	א ב יב אכה 8_4	ג יט ב החא 8_4	ו יא ז זחג 7_4	ב ג יב בכו 8_4
ז		ה כג ה זחא 28_3	א טו י בשה 29_3	ד ח — החו 27_3	ז — ה זשג 29_3	ב טז י גכה 28_3	ה ט — הכז 27_3	א א ה אכג 27_3
ח*		ג ח ב גכז 16_4	ו — ז זחג 16_4	א טז יב אכה 15_4	ד ט ב החא 15_4	ז א ז זשה 17_4	ב יז יב בשז 16_4	ה י ב השג 16_4
ט		ב ה י בשה 5_4	ד כב — הכז 5_4	ז יד ה זשג 5_4	ג ו י גכה 4_4	ה כג — זחא 4_4	א טו ה בשה 6_4	ד ז י הכז 5_4
י		ו יד ז זשג 26_3	ב ז יב בשה 25_3	ד כג ב הכז 25_3	ז טו ז זשג 25_3	ג ז יב גכה 24_3	ו — ב זחא 24_3	א טז ז בשה 26_3
יא*		ג כג ד החא 13_4	ו טו ט זחג 12_4	ב ז יד בכו 12_4	ה — ד השג 14_4	ז טז ט זשה 13_4	ג ח יד גכז 12_4	ז א ד זחג 12_4
יב		ב כ יב גכה 2_4	ה יג ב השא 2_4	א ה ז אכג 1_4	ג כא יב החו 1_4	ו יד ב זשג 3_4	ב ז ז בשה 2_4	ד כב יב הכז 1_4
יג		ז ה ט זשג 22_3	ב כא יד גכה 22_3	ה יד ד השא 22_3	א ו ט אכג 21_3	ג כב יד החו 21_3	ו טו ד זשג 23_3	ב ז ט בשה 22_3
יד*		ד יד ו החא 9_4	ז ו יא זשה 10_4	ב כג א גכז 10_4	ה טז ו השג 10_4	א ז יא אכה 9_4	ד — א החא 9_4	ו טז ו זחג 9_4
טו	ג (מו׳ניסן)ג 30_3	ג יא יד גשו 30_3	ו ד ד זחא 29_3	א כ ט בשה 30_3	ד יב יד הכז 30_3	ז ה ד זשג 30_3	ב כא ט גכה 29_3	ה יג יד השא 29_3
טז	ה ד ו* השג 18_4	ז כ יא אכג 19_3	ג יג א גשו 19_3	ו ה ו* זחג 17_4	א כא יא בשה 19_3	ד יד א הכז 19_3	ז ו ו זשג 19_3	ב כב יא*גכז 17_4
יז	ד א יד הכז 7_4	ה ה ח* השג 7_4	ז כא יג* אכה 7_4	ה ב יד הכז 6_4	ו ו ח* זחג 6_4	א כב יג* בכו 6_4	ד טו ג* החא 6_4	א כ ד בשה 7_4
יח	א י יא בשה 28_3	ד ג א החו 26_3	ו יט ו זשג 27_3	ב יא יא בשה 27_3	ה ד א הכז 26_3	ז כ ו אכג 26_3	ג יב יא גשו 26_3	ז ה א זחא 26_3
יט*	ה יט ח זחג 15_4	א יא יג אכה 14_4	ד ד ג החא 14_4	ו כ ה זשה 15_4	ב יב יג בשז 15_4	ה ה ג השג 15_4	ז כא ח אכה 14_4	ג יג יג גכז 13_4

	מחזור רכה פסח שנה ד"א רנז 449	רכו פסח רעו 516	רכז פסח רצה 535	רכח פסח שיד 554	רכט פסח שלג 573	רל פסח שנב 592	רלא פסח שעא 611	רלב פסח שצ 630
א	ב יא ו בשה 3_4	ה ג יא הכז 2_4	ז כ א אכג 3_4	ג יב ו גשו 3_4	ו ד יא זחא 2_4	א כא א בישה 3_4	ד יג ו הכז 3_4	ז ה יא זשג 3_4
ב	ו כ ג זשג 24_3	ב יב ח בשה 23_3	ה ד יג הכז 22_3	ז כא ג אכג 23_3	ג יג ח גשו 23_3	ו ה יג זחא 22_3	א כב ג בשה 23_3	ד יד ח הכז 23_3
ג*	ד ה —החא 11_4	ו כא ה זשה 12_4	ב יג י בשז 11_4	ה ו — השג 11_4	ז כב ה אכה 11_4	ג יד י גכז 10_4	ו ז — זחג 10_4	א כג ה בכו 10_4
ד	ג ב ח נכה 30_2	ה יח יג זחא 31_3	א יא ג בשה 1_4	ד ג ח הכז 31_3	ו יט יג זשג 31_3	ב יב ג בשה 31_3	ה ד ח הכז 30_3	ז כ יג אכג 30_3
ה	ז יא ה זשג 20_3	ג ג י גכה 19_3	ה כ — זחא 20_3	א יב ה בשה 21_3	ד ד י החו 19_3	ו כא — זשג 20_3	ב יג ה בשה 20_3	ה ה י הכז 19_3
ו*	ד כ ב החא 7_4	ז יב ז זשה 8_4	ג ד יב גכז 7_4	ה כא ב זחג 8_4	א יג ז אכה 7_4	ד ה יב החא 7_4	ו כב ב זשה 8^4	ב יד ז בשז 8_4
ז	ג יז י גשו 28_3	ו י — זשג 29_3	ב ב ה בשה 28_3	ד יח י הכז 27_3	ז יא — זשג 28_3	ג ג ה גכה 27_3	ה יט י זחא 27_3	א יב — בשה 28_3
ח*	א ב ז אכה 15_4	ג יח יב החא 16_4	ו יא ב זחג 15_4	ב ג ז בכו 15_4	ד יט יב החא 14_4	ז יב ב זשה 16_4	ג ד ז גכז 15_4	ה כ יב זחג 15_4
ט	ז — — זשג 5_4	ב טז ה גכה 4_4	ה ח י הכז 4_4	א א — אכג 4_4	ג יז ה גשו 4_4	ו ט י זשג 5_4	ב ב — בשה 5_4	ד יח ה הכז 4_4
י	ד ח יב החו 24_3	ז א ב זשג 25_3	כ יז ז בשה 24_3	ה ט יב השא 25_3	א ב ב אכג 24_3	ג יח ז החו 24_3	ו י יב זשג 25_3	כ ג ב בשה 25_3
יא*	א יז ט אכה 12_4	ד ט יד החא 12_4	ז כ ד זשה 13_4	ב יח ט גכז 12_4	ה י יד השג 13_4	א ג ד אכה 12_4	ג יט ט השא 12_4	ז יא יד זחג 11_4
יב	ז טו ב זשג 1_4	ג ז ז גכה 1_4	ה כג יב זחא 1_4	א טז ב בשה 2_4	ד ח ז הכז 1_4	ז — יב זשג 2_4	ב יז ב גכה 1_4	ה ט ז השא 1_4
יג	ד כג יד הכז 21_3	ז טז ד זשג 21_3	ג ח ט גכה 21_3	ו — יד זחא 21_3	א יז ד בשה 22_3	ד ט ט הכז 21_3	ז א יד זשג 22_3	כ יח ד גכה 21_3
יד*	ב ח יא בכו 9_4	ה א א השג 10^4	ז יז ו זשה 9_4	ג ט יא גכז 9_4	ו ב א זחג 9_4	א יח ו בחה 8_4	ד י יא החא 8_4	ז ג א זשה 10_4
טו	א ו ד אכג 29_3	ג כב ט החו 29_3	ו יד יד זשג 30_3	כ ז ד בשה 29^3	ד כג ט הכז 29_3	ז טו יד זשג 29_3	ג ח ד גכה 28_3	ו — ט זחא 28_3
טז	ה טו א*השג 17_4	א ז ו* אכה 17_4	ג כג יא* החא 17_4	ו טז א זשג 19_3	ב ח ו* בכו 16_4	ה — יא* השג 18_4	ז יז א* זשה 17_4	ג ט ו* נכז 16_4
יז	ד יב ט הכז 6_4	ז ד יד זשג 6_4	כ כא ד גכה 6_4	ד — יג* החא 6_4	א ה יד אכג 5_4	ג כב ד החו 5_4	ו יד ט זשג 7_4	כ ז יד בשה 6_4
יח	א כאו בשה 27_3	ד יג יא הכז 26_3	ז ו א זשג 26_3	ב כב ו גכה 26_3	ה יד יא השא 26_3	א ז א אכג 25_3	ג כג ו החו 25_3	ו טו יא זשג 27_3
יט*	ו ו ג זחג 14_4	א כב ח בכו 14_4	ד יד יג החא 13_4	ז ז ג זשה 14_4	ב כג ח גכז 14_4	ח טו יג השג 14_4	א ח ג אכה 13_4	ד — ח החא 13_4

א) בשנת ד"א קפה חל מולד תשרי בי"ח שעות בצמצום.

ב) אם היו דוחין משום מולד זקן בשבט (שזה יקרה רק בשנה מעוברת), אז היו משתנות הקביעות בשנות ד"א קנג, רנ"א לקביעות זשה; בשנות ד"א קס"ד, רלה, רמב, שמ—לקביעות השג; בשנת שם"ה—לקביעות בכ"ו, ובשנות ד"א קס"ט, רסז—לקביעות חדשה אשו. זולת השנים הנזכרות אלו אירע מולד תשרי שאחר עבור טן ב יב ט עד ב טו ח, אז היתה במעוברת עוד קביעות חדשה גשא, אלא שבמקרה לא אירע כן במשך הזמן של הלוח א'. כן שנוי הקביעות של כל אחת מהשנים הנזכרות היה גורר אחריו ממילא גם שנוי קביעות השנה הבאה אחריה.

לוח ב. לשנות ד"א שצ עד ד"א תקל"ז.

יתרונות הטולדות לחדש א יב יא; לשנה פשוטה ד ח יב ולמעוברת ה כא ח. הדחיות: א) אדו; ב) י"ח שעות; ג) ג ט ג בפשוטה; ד) ב טו ח אחר עבור. הקביעות שבע בפשוטה: בחג, בשה, גכה, הכז, השא, זחא, זשג, ושבע במעוברת: בחה, בשז, גכז, החא, השג, זחג, זשה. הפסח לא יקדם את יום 19 במרס. שנות העבור מסומנות בכוכב כזה *).

	מחזור רלב פסח שנה ד"א שצ 630	רלג פסח תט 649	רלד פסח תכח 668	רלה פסח תמז 687	רלו פסח תסו 706	רלז פסח תפה 725	רלח פסח תקד 744	רלט פסח תקכג 763
א	ז ה יא זשג 3_4	ב כב א גכה 2_4	ה יד ו השא 2_4	א ו יא בחג 2_4	ג כג א הכז 3_4	ו טו ו זשג 3_4	ב ז יא בשה 2_4	ה — א הכז 2_4
ב	ד יד ח הכז 23_3	ז ו יג זשג 23_3	ב כג ג גכה 22_3	ה טו ח השא 22_3	א ז יג בחג 22_3	ד — ג הכז 23_3	ו טז ח זשג 23_3	כ ח יג בשה 22_3
ג*	א כג ה בשז 11_4	ד טו י תשג 12_4	ז ח — זשה 11_4	ג — ה גכז 10_4	ה טז י השג 10_4	א ט — בחה 10_4	ד א ה החא 10_4	ו יז י זחג 9_4
ד	ז כ יג בחג 30_3	ג יג ג הכז 31_3	ו ה ח זשג 1_4	א כא יג בשה 31_3	ד יר ג הכז 30_3	ז ו ח זשג 30_3	כ כב יג גכה 30_3	ה טו ג תשא 30_3
ה	ה ה י הכז 19_3	ז כב— בחג 19_3	ג יד ה הכז 20_3	ו ו י זחא 19_3	א כג— בשה 20_3	ד טו ה הכז 19_3	ז ז י זשג 19_3	ג — — גכה 19_3
ו*	ב יד ז בשז 8_4	ה ו יב השג 8_4	ז כג ב בחה 7_4	ג טו ז גכז 6_4	ו ז יב זחג 7_4	ב — ב בשז 8_4	ד טז ז השג 8_4	ז ח יב זשה 7_4
ז	א יב—בשה 28_3	ד ד ח הכז 28_3	ו כ י זשג 28_3	כ יג — בשה 27_3	ה ה ה הכז 26_3	ז כא י בחג 27_3	ג יד — הכז 28_3	ו ו ה זחא 26_3
ח*	ה כ יב זחג 15_4	א יג ב בחה 14_4	ד ה ז החא 15_4	ו כא יב זשה 16_4	ב יד ב בשז 15_4	ה ו ז השג 15_4	ז כב יב בחה 15_4	ג טו ב גכז 14_4
ט	ד יח ה הכז 4_4	ז י י זשג 4_4	ג ג — גכה 3_4	ה יט ה זחא 4_4	א יא י בשה 5_4	ד ד — הכז 4_4	ו כ ה זשג 4_4	כ יב י בשה 4_4
י	ב ג ב בשה 25_3	ד יט ז הכז 24_3	ז יא יב זשג 24_3	ג ד ב גכה 23_3	ה כ ז זחא 24_3	א יב יב בשה 25_3	ד ה ב הכז 24_3	ו כא ז זשג 24_3
יא*	ו יא יד זחג 11_4	ב ד ד בשז 13_4	ד כ ט השג 13_4	ז יב יד זשה 12_4	ג ח ד גכז 11_4	ח כא ט זחג 12_4	א יג יד בחה 11_4	ד ו ד החא 11_4
יב	ה ט ז השא 1_4	א א יב בחג 31_3	ג יח ב הכז 2_4	ו י ז זשג 2_4	כ ב יב בשה 1_4	ד יט כ הכז 31_3	ז יא ז זשג 1_4	ג ג יב גכה 31_3
יג	ב יח ד גכה 21_3	ה י ט השא 21_3	א ב יד בחג 20_3	ג יט ד הכז 22_3	ו יא ט זשג 22_3	ב ג יד בשה 21_3	ד כ ד הכז 20_3	ז יב ט זשג 21_3
יד*	ז ג א זשה 10_4	ב יט ו גכז 9_4	ה יא יא השג 9_4	א ד א בחה 8_4	ג כ ו החא 9_4	ו יב יא זחג 8_4	כ ח א בשז 9_4	ד כא ו השג 9_4
טו	ו — ט זשג 30_3	א טז יד בשה 30_3	ד ם ד הכז 29_3	ז א ט בזש 29_3	ב יז יד גכה 28_3	ה י ד השא 29_3	א ב ט בחג 28_3	
טז	ג ט ו הכז 19_3	ו א יא*זחג 16_4	א יח א בשה 19_3	ד י ו* החא 16_4	ז ב יא*זשה 17_4	ב יט א* גכז 16_4	ה יא ו* השג 17_4	
יז	ז יח ג*בחה 6_4	ד כג ד הכז 5_4	ו ב יג*זחג 5_4	ג ז יד גכה 5_4	ו — ד זחא 5_4	א טז ט בשה 6_4	ד ח יד הכז 5_4	
יח	ו טו יא זשג 27_3	ב ח א בשה 26_3	ה — ו הכז 25_3	ז טז יא זשג 25_3	ג ט א גכה 25_3	ו א ו זחא 25_3	א יז יא בשה 26_3	
יט*	ד — ח החא 13_4	ו טז יג זחג 13_4	ב ט ג בשז 14_4	ה א ח השג 14_4	ז יז יג זשה 13_4	ג י ג גכז 13_4	ו ב ח זחג 13_4	

קביעות השנים, ד"א תקל"ז — תקמ"א עי' בלוח ג' בגוף הספר עמוד פ"ב.

אם היו דוחין משום טולד זקן בשבט, אז היו משתנות הקביעות בשנים מעוברות אלו: בשנת ד"א שמ"ה—אל בשז (אלו היה מחזור רל נכנם בלוח ב): בשנות ד"א תכז, תקכה — אל זשה, ובשנות ד"א תנ"ב, תקל—לקביעות חדשה גשא.

לוח ד׳. מולדות תשרי וקביעות השנים לעגול החזר חלילה של י״ג מחזורים (ר״ם—רנ״ב) או רמ״ז שנה (ד״א תקמ״ב—תשפ״ח), ההחלת השנה מניסן שאחר תשרי. העגול נוסר על פי העיון שבבריתא דשמואל ולפי ההנחה כי 90216 ימים = 12888 שבועות = 3055 חדשים = 247 שנים = 13 מחזורים בני ״ט שנה. סדר שנות העבור: גו״ח אדז״ט. יתרון המולדות: לחדש א יב יא, לשנה פשוטה ד ח יב, ולמעוברת ה כא ח. אלא שמוסיפים במולד ניסן חלק אחד אחר כל 4 שנים ואחר כל 5 שנים חליפות במקומות שמשכנו קו לסמן ההוספה. הדחיות: א) אדו; ב) י״ח שעות; ג) ג ט ג (ב) בפשוטה; ד) ב טז ח (ט) אחר עבור. הקביעות בפשוטה בחג בשה גכה הכז השא זחא זשג, ובמעוברת בחה בשז גכז החא השג זחג זשה.

שנים מחזור (רט) א 61				(רמא) ב 23				(רטב) ג 44				(רמג) ד 8				(רמד) ה 31				(רמה) ו 54				(רמו) ז 16				(רמז) ח 39			
שנגברו	דא	תקמא	781	דא	תקס	800		דא	תקעט	819		דא	תקצח	838		דא	תרין	857		דא	תרלו	876		דא	תרנה	895		דא	תרעד	914	
			פסח			פסח				פסח				פסח				פסח				פסח				פסח				פסח	
א	ז	כא	ח	בחג	2^4	ג	יד	ג	הכז	3^4	ו	ו	יב	זשג	3^4	א	כג	ו	בשה	ד	טז	־	הכז	2^4	ז	ח	י	זשג	2^4	ג	א	ד	גכה	1^4	ה	יז	יג	השא	2^4
ב	ה	ו	ו	הכז	22_3	ז	כג	־	בחג	22_3	ג	טו	ט	הכז	23_3	ז	ח	ג	זחא	21_3	ב	־	יג	בשה	23^3	ד	יז	ז	הכז	22^3	ז	י	א	זשג	22^3	ג	ב	י	גכה	21^3
ג*	ב	טו	ג	בשז	10^4	ה	ז	יב	השג	11^4	א	־	ו	בחה	10^4	ג	יז	א	גכז	9_4	ו	ט	י	זחג	9^4	ב	ב	ד	בשז	11^4	ד	יח	יג	השג	11^4	ז	יא	ז	זשה	10^4
ד	א	יב	יא	בשה	31_3	ד	ה	ה	הכז	30_3	ו	כב	־	זשג	31_3	ב	יד	ט	בשה	30_3	ה	ז	ג	הכז	29^3	ז	כג	יב	בחג	29^3	ג	טז	ו	הכז	31^3	ו	ט	א	זשג	31^3
ה	ה	כא	ח	זחא	19_3	א	יד	ג	בשה	20_3	ד	ו	יב	הכז	19_3	ו	כג	ז	זשג	20^3	ב	טז	־	בשה	19^3	ה	ח	ט	הכז	18^3	א	א	ד	בחג	18^3	ג	יז	יג	הכז	20^3
ו*	ג	ו	ו	גכז	7^4	ה	כג	־	זחג	7^4	א	טו	ט	בחה	6_4	ד	ח	ג	החא	6^4	ז	־	יב	זשה	8^4	ב	יז	ז	בשז	7^4	ה	י	א	השג	7^4	א	ב	י	בחה	6^4
ז	ב	ג	יד	בשה	27_3	ד	כ	ח	הכז	27_3	ז	יג	ב	זשג	27_3	ג	ה	יא	גכה	26^3	ה	כב	ו	זחא	26^3	א	טו	־	בשה	28^3	ד	ז	ט	הכז	28^3	ז	־	ג	זשג	27^3
ח*	ו	יב	יא	זחג	14^4	ב	ה	ה	בשז	15^4	ד	כא	יד	השג	16^4	ז	יד	ט	זשה	15^4	ג	ז	ג	גכז	14^4	ה	כג	יב	זהג	14^4	א	טז	ו	בחה	14^4	ד	ט	א	החא	14^4
ט	ה	י	ד	השא	4^4	א	ב	יג	בחג	3^4	ג	יט	ח	הכז	4^4	ו	יב	ב	זשג	5^4	ב	ד	יא	בשה	4^4	ד	כא	ה	הכז	3_4	ז	יד	־	זשג	3^4	ג	ו	ט	גכה	3^4
י	ב	יט	א	גכה	24_3	ה	יא	יא	השא	24_3	א	ד	ה	בחג	23_3	ג	כ	יד	הכז	24^3	ו	יג	ח	זשג	25^3	ב	ו	ג	בשה	24^3	ד	כב	יב	הכז	24^3	ז	טו	ו	זשג	23^3
יא*	ז	ג	יד	זשה	12^4	ב	כ	ח	גכז	12^4	ה	יג	ב	השג	12^4	א	ה	יא	בחה	11^4	ג	כב	ו	החא	11^4	ב	ז	ט	בשז	12^4	ו	טז	־	זחג	11^4	ה	־	ג	השג	12^4
יב	ו	א	ז	זשג	2^4	א	יח	א	בשה	1^4	ד	י	י	הכז	1_4	ז	ג	ה	זשג	1^4	ב	יט	יד	גכה	31^3	ה	יב	ח	השא	31^3	א	ה	ב	בחג	31^3	ג	כא	יא	הכז	1^4
יג	ג	י	ד	הכז	22_3	ו	ב	יג	זחא	20_3	א	יט	ח	בשה	21_3	ד	יב	ב	הכז	21^3	ז	ד	יא	זשג	21^3	ב	כא	ה	גכה	20^3	ה	יג	יד	השא	20^3	א	ו	ט	בחג	20^3
יד*	ז	יט	א	בחה	9^4	ג	יא	יא	גכז	8^4	ו	ד	ה	זחג	8_4	א	כ	יד	בשז	9^4	ד	יג	ח	השג	10^4	ז	ו	ב	זשה	9^4	ב	כב	יב	גכז	8^4	ה	טו	ו	השג	8^4
טו	ו	טז	י	זשג	29^3	ב	ט	ד	בשה	29_3	ה	א	יג	הכז	28_3	ז	יח	ז	בחג	28^3	ג	יא	ה	הכז	29^3	ג	יא	ז	זשג	30^3	ו	ג	יא	זשג	30^3	א	כ	ה	בשה	29^3
טז	ד	א	ז	הכז	18_3	ו	יח	א	זשג	18_3	ב	י	י	בשה	18_3	ה	ג	ד	הכז	17^3	ז	יט	יד	בחג	17^3	ג	יב	ח	הכז	18^3	ו	ה	ב	זחא	18^3	ו	כא	יא	בשה	17^3
יז*	א	י	ד	בחה	5^4	ד	ב	יג	החא	5^4	ו	יט	ז	זחג	4_4	ב	יב	ב	בשז	6^4	ה	ד	יא	השג	6^4	ז	כא	ה	בחה	5_4	ג	יג	יד	גכז	4^4	ו	ו	ט	זחג	5^4
יח	ז	יב	זשג	26_3	ג	־	ו	גכה	25_3	ה	יז	א	השא	25_3	א	ט	י	בשה	26^3	ד	ב	ד	הכז	26^3	ו	יח	יג	זשג	26^3	ב	יא	ח	בשה	25^3	ה	ד	ב	הכז	24^3
יט*	ד	טז	ט	השג	14^4	ז	ט	ד	זשה	14^4	ג	א	יג	גכז	13^4	ה	יח	ז	זחג	13^4	א	יא	א	בחה	12^4	ד	ג	יא	החא	13^4	ו	כ	ה	זחג	12^4	ב	יב	יד	בשז	13^4

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